Analysis of Effect of Tension in Bar Rolling by Energy Method using Finite-Element Division*

Kazutake KOMORI** and Kazunori KATO***

In bar rolling, a bar is usually formed by applying front and back tensions; thus, it is important to analyze the effects of the front and back tensions on the rolling properties. Furthermore, it is also important for precise rolling to confirm the control equation coefficients, that is, the ratios of the variations of the rolling properties, such as the width spread, forward slip, load and torque, to the changes of the tensions. In this paper, the analysis was performed by the energy method using the finite-element division, which we proposed previously. First, the effects of the tensions on the rolling properties were proved reasonable by the comparison with the experimental formulas. Next, the control equation coefficients for the torque were obtained approximately using the results obtained when no tension was applied. Finally, it was found that as the roll radius increases, the control equation coefficients for all of the rolling properties decrease.

Key Words: Plasticity, Rolling, Energy Method, Finite-Element Division, Tension

1. Introduction

Caliber rolling is performed to obtain a bar from a billet. Many rolling mills are used continuously. A bar is usually formed by applying front and back tensions. In recent years, as bars of high dimensional accuracy are required, size control is greatly needed. It is important to verify the effect of the tension on the rolling properties. Thus, some experimental research has been carried out^{(1),(2)}. Especially, in terms of the control, it is important to confirm the ratios of the variations of the rolling properties to the changes of the tensions. The ratios are called the control equation coefficients. The values of the control equation coefficients are considered to change as the rolling condition changes. However, this has not been

clarified experimentally. The analytical approaches are necessary to confirm this, because in the analysis, each rolling condition can be changed easily. In this paper, the effects of the front and back tensions on the rolling properties and the effects of the rolling conditions on the control equation coefficients are obtained by the energy method using the finite-element division, which we proposed previously^{(3),(4)}.

2. Method of Analysis

2.1 Fundamental equation

In this paper, the energy method using the finite-element division, which we proposed previously^{(3),(4)} is used. That is, the velocity field v is obtained by minimizing the functional Φ in the following equation :

$$\begin{aligned} 2T\omega &\leq \Phi \\ \Phi &= \int 1/(1+M) \cdot \sigma_{eq} \cdot \dot{\epsilon}_{eq}^k dV \\ &+ \int \sigma_{eq}/\sqrt{3} \cdot \Delta v_T^k d\Gamma^k \\ &+ \int \tau_f \cdot \Delta v_T^k dS_c + Q_b v_b^k - Q_f v_f^k \end{aligned} \quad (1)$$

where

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** Department of Mechanical Engineering, Daido Institute of Technology, 2-21 Daidocho, Minami-ku, Nagoya-shi 457, Japan

*** Department of Mechanical Engineering, Faculty of Engineering, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152, Japan

T : true torque of upper and lower rolls
 ω : angular velocity of upper and lower rolls
 V : volume of material in plastic solid
 Γ^k : interface on which the discontinuity of tangential velocity arises
 S_c : contact surface with roll
 k : superscript indicating kinematically admissible velocity field
 M : strain rate exponent
 σ_{ep} : equivalent stress
 $\dot{\epsilon}_{ep}^k$: equivalent strain rate
 Δv_f^k : amount of discontinuity of velocity
 τ_f : frictional stress on roll surface
 Q_B, Q_F : back and front tensions
 v_B^k, v_F^k : velocities of material at entrance and exit cross sections.

The velocity field v is expressed as the summation of several admissible fundamental velocity fields v_i , which are given beforehand:

$$v = \sum a_i v_i \quad (2)$$

Here, v_i is a velocity vector which is composed of x , y , and z components of the velocities at all nodal points. So the number of the total components is $3N$ (N : number of nodes). In this paper, the analysis of the temperature distribution⁽⁵⁾ is performed because a bar is made by the hot rolling, and analysis of the stress distribution⁽⁶⁾ is performed to obtain the load. The description of the method of these analysis is omitted.

2.2 Fundamental velocity fields

The fundamental velocity fields, which are mentioned in the previous section, will be explained concretely. Figure 1 is a finite-element division of the round-oval pass. Figures 1(a), (b), (c) and (d) show an entrance cross section, a longitudinal section, an exit cross section and a plane figure, respectively.

2.2.1 Fundamental velocity fields for whole region The following two velocity fields of the rolling direction are introduced.

$$v_x = h(0, 0)/h(x, 0) \cdot U \quad (3)$$

$$v_x = U \quad (4)$$

where h denotes a z -value on the roll and U denotes a roll velocity in the xz plane.

v_y and v_z are determined under the following conditions: for any cross section perpendicular to the x -axis, u_y is independent of z and u_z changes linearly in the thickness direction.

These velocity fields become v_1 and v_2

2.2.2 Fundamental velocity fields for local regions Figure 2(a) denotes the finite elements; the number of finite elements perpendicular to the paper is only one. We shall refer to such elements as a large element hereafter. As we pile up large elements, the finite-element division shown in Fig.1 is

obtained. We shall also call the region between two continuous large elements a cross section hereafter. Figure 1(d) is shown in Fig. 2(b) again. The plastic deformation region is divided into two regions: the region between the roll contact surfaces (I) and the region near the free side surface (II).

In region (I), the following three kinds of velocity fields are assumed:

$$v_x = \{(z/h)^3 - 1/3\} \cdot U, \quad v_y = 0 \quad (5)$$

the barreling of the free side surface

$$v_x = 0, \quad v_y = \{(z/h)^2 - 1/3\} \cdot y/h \cdot U \quad (6)$$

the warping of the cross section toward the width direction

$$v_x = y^2 \cdot U, \quad v_y = -y^2 \cdot (\partial h / \partial x) \cdot y/h \cdot U \quad (7)$$

In region (II), the following two types of velocity fields are considered:

the uniform deformation toward the rolling direction

$$v_x = U, \quad v_y = 0 \quad (8)$$

the plane strain deformation in the cross section

$$v_x = 0, \quad v_y = -y/h \cdot U \quad (9)$$

The velocity fields of Eqs.(5)~(9) are introduced in each cross section. That is, the compo-

Fig. 1 Coordinates and notations (round-oval pass)

Fig. 2 Fundamental velocity fields

nents of each velocity field in the velocity fields of Eqs. (5)~(9) are given to all nodal points in each cross section separately. Here, the rest of the components of each velocity field are determined by the condition of the incompressibility in each element. That is, in each large element, the components which are denoted by the arrows in Fig. 2(a), are determined so as to satisfy the condition of the incompressibility of the elements, which are denoted by the circles in Fig. 2(a). Accordingly, the fundamental velocity fields are obtained.

2.3 Deformation resistance

As the empirical formula of the deformation resistance of the common carbon steel at an elevated temperature and high strain rate, Shida's experimental formula⁽⁷⁾ is used in the analysis.

$$\sigma_{eq} = \sigma_t(t) \cdot \sigma_\epsilon(\epsilon_{eq}) \cdot \left(\frac{\dot{\epsilon}_{eq}}{10} \right)^{M(t)} \quad (10)$$

where

t : temperature (700–1200°C)

ϵ_{eq} : equivalent strain (0.1–0.7)

$\dot{\epsilon}_{eq}$: equivalent strain rate (0.1–100/s)

In the region where this formula was not defined, the following modifications are performed.

$$\sigma_\epsilon(\epsilon_{eq}) = \sigma_\epsilon(0.1) \quad (\epsilon_{eq} \leq 0.1) \quad (11)$$

$$\sigma_\epsilon(\epsilon_{eq}) = \sigma_\epsilon(0.7) \quad (\epsilon_{eq} \geq 0.7) \quad (12)$$

Though the temperature of the blue shortness of the steel is usually 200–300°C, it is assumed to be 500°C because of the high strain rate in the bar rolling. The following modifications are done below this temperature.

$$\sigma_t(t) = 40 \text{ kgf/mm}^2 \quad (t \leq 500^\circ\text{C}) \quad (13)$$

$$M(t) = 0 \quad (t \leq 500^\circ\text{C}) \quad (14)$$

It is assumed that σ_t and M change linearly between 500 and 700°C.

3. Results

3.1 Analysis of effect of tension

3.1.1 Square-diamond pass The standard rolling conditions are shown under ① in Table 1 ($2h_B$: height of material, $2h_F$: depth of caliber, $2b_B$: width of material, $2R$: distance between two rolls, $2b_K$: width of caliber, τ_f/k : frictional factor, t_R : temperature of roll, t_M : temperature of material). The number of finite elements is 225. The number of the fundamental velocity fields in the previous chapter is 55. The result of the finite-element division in the standard rolling condition is shown in Fig. 3. The standard rolling condition coincides with the condition of the experiment by Noguchi et al.⁽¹⁾ using hot steel. Here the temperature of the material before rolling is assumed to be 920°C, because the temperature decreases after the material is taken out from the

furnace**.

The effects of the front and back tensions on the rolling properties are shown in Fig. 4. Here, $q_F/(\sqrt{3}k_F)$ and $q_B/(\sqrt{3}k_B)$ are the nondimensional front and back tensions, respectively ($q_F = Q_F/S_F$, $q_B = Q_B/S_B$, S_F , S_B : area of exit and entrance cross sections, k_F , k_B : shearing yield stress at exit and entrance cross sections). Figure 4(a) denotes the width spread w ($w = b_F - b_B/b_B$, $2b_B$, $2b_F$: width of material before and after rolling). Figure 4(b) denotes the forward slip ξ ($\xi = (v_F - U)/U$). Figure 4(c) denotes the load P . Figure 4(d) denotes the torque T . In the figures, $\circ$, $\square$ and

Table 1 Standard rolling conditions

	①D-S	②O-R	③R-0	④R-0
h_B (mm)	31	24	5	5
h_F (mm)	21	17	3.75	2.75
b_B (mm)	18	14	5	5
R (mm)	137	137	50	50
b_K (mm)	21	18	6.25	6.88
τ_f/k	1	1	1	1
U (m/s)	0.29	0.29	30	30
t_R (°C)	20	20	20	20
t_M (°C)	920	925	1000	1000

Fig. 3 Finite-element division (diamond-square pass)

Fig. 4 Effects of tensions (diamond-square pass)

** Noguchi, Y., Private Communication

Δ imply the results of the front, back and front and back tensions, respectively. The experimental formulas are obtained by the least square of the results of the experiments. The dotted lines and the dot-dash lines denote the results of the front and back tensions, respectively.

The results of the analysis agree well with those of the experimental formulas by Noguchi et al.⁽¹⁾. That is, the effects of the back tension on the width spread, forward slip and load are bigger than those of the front tension.

The shapes of the contact surface are shown in Fig. 5. The area of the contact surface and width spread decrease with increasing front and back tensions.

3.1.2 Oval-round pass The standard rolling conditions are shown under ② in Table 1. The number of finite elements is 250. The number of fundamental velocity fields is 61. The result of the finite-element division in the standard rolling condition is shown in Fig. 6. The temperature of the material before rolling is assumed to be 925°C**.

The effects of the front and back tensions on the rolling properties are shown in Fig. 7. The results of the analysis agree well with those of the experimental formulas. That is, the effects of the back tension on the width spread and load are bigger than those of the front tension.

The temperature distributions of the standard rolling condition are shown in Fig. 8. The temperature distribution at the contact surface is shown in Fig. 8 (a), and that slightly (0.5 mm) inside the material is shown in Fig. 8 (b). The temperature near the contact surface changes sharply. In Fig. 8 (b), the temperature is lower than the initial temperature in the major part, because the roll velocity is very slow.

Fig. 5 Shapes of contact surface

Fig. 6 Finite-element division (oval-round pass)

3.1.3 Discussion Much research on bar rolling has been performed from the standpoint of control^{(8),(9)}. In this research, the following assumptions are made to analyze the effect of the tension on the rolling properties. (1) The superposition of the effects of the front tension on the rolling properties and those of the back tension is possible. (2) The rolling properties change linearly with linearly increasing tension. The rolling property is generally expressed as follows under these assumptions.

$$A = A_0 \cdot \{1 + a_B \cdot q_B / (\sqrt{3} \cdot k_B) + a_F \cdot q_F / (\sqrt{3} \cdot k_F)\} \quad (15)$$

where A_0 denotes the rolling property without the tension. a_F and a_B are the control equation coefficients of the front and back tensions on the rolling properties (nondimensional value), respectively. Judging from Figs. 4 and 7, these two assumptions are almost appropriate. Then in the next section, the control equation coefficients in the various rolling conditions are obtained.

Fig. 7 Effects of tensions (oval-round pass)

Fig. 8 Temperature distribution (oval-round pass)

3.2 Analysis of control equation coefficients

3.2.1 Effect of roll radius The standard rolling conditions are shown under ③ in Table 1. The round-oval pass is one of the finishing passes. The number of finite elements is 275. The number of fundamental velocity fields is 67. The result of the finite-element division under the standard rolling conditions is shown in Fig. 1. The roll radius R/h_B is changed from 10 to 30.

The effects of the tensions on the rolling properties are shown in Fig. 9. The solid lines, the dotted lines and the dot-dash lines denote the results of R/h_B equals 10, 20 and 30, respectively. Here, the analysis is performed under only three rolling conditions, which are no tensions, $q_B/(\sqrt{3}k_B)=0.4$ and $q_F/(\sqrt{3}k_F)=0.4$, because the assumptions which are mentioned in the

Fig. 9 Effects of tensions (roll radius)

Fig. 10 Control equation coefficients (roll radius)

previous section are almost appropriate.

The control equation coefficients of the front and back tensions α_F and α_B are obtained in Fig. 10 using Fig. 9 and Eq.(15). In Eq.(15), the rolling properties $1+w$ and $1+\xi$ instead of w and ξ are used. The absolute values of all of the control equation coefficients decrease with increasing roll radius.

The physical meaning of the control equation coefficients for the torque is considered below. First, the front tension is assumed to be zero for convenience. The control equation coefficients for the torque are obtained from Eq.(15):

$$\alpha_B = \sqrt{3} \cdot k_B / q_B \cdot (T - T_0) / T_0 \quad (16)$$

where T_0 is the torque without the tension.

Next, the variations of the first, second and third terms of Eq.(1) and the velocity of the entrance cross section are assumed to be zero with changing of the tension. Then, Eq.(16) becomes as follows:

$$\alpha_B = \sqrt{3} \cdot k_B \cdot S_B \cdot v_B / T_0 \quad (17)$$

where v_B is the velocity without the tension. That is, the control equation coefficient of the back tension is obtained by the results without the tension. The control equation coefficient of the front tension is obtained in the same way :

$$\alpha_F = \sqrt{3} \cdot k_F \cdot S_F \cdot v_F / T_0 \quad (18)$$

The control equation coefficients obtained by Eqs. (17) and (18) are supplemented in Fig. 10(d) with the dotted lines. Thus, the physical meaning of the control equation coefficients for the torque is confirmed.

The temperature distributions of the standard rolling condition are shown in Fig. 11. The temperature distribution at the contact surface is shown in Fig. 11(a), and that slightly (0.5 mm) inside the material is shown in Fig. 11(b). In Fig. 11(b), the temperature is higher than the initial temperature in the major part, which is different from the result in Fig.8, for the reason that the roll velocity is very fast.

Fig. 11 Temperature distribution (round-oval pass)

Fig. 12 Effects of tensions (reduction)

Fig. 13 Control equation coefficients (reduction)

3.2.2 Effect of reduction The standard rolling conditions are shown under ④ in Table 1. The reduction $r = (h_B - h_F)/h_B$ is changed from 0.35 to 0.45. The effects of the tensions are shown in Fig. 12. The solid lines, the dotted lines and the dot-dash lines denote the results of $r = 0.35, 0.40$ and 0.45 , respectively.

The control equation coefficients are shown in Fig. 13. In Fig. 13(a), the absolute value of the control equation coefficient of the front tension decreases with increasing reduction, whereas that of the back tension increases. In Fig. 13(b), the absolute values of the control equation coefficients decrease. The results obtained by Eqs.(17) and (18) are supplemented. These results agree well with the results of the analysis.

3.2.3 Effect of height/width ratio of caliber

The standard rolling conditions are shown under ④ in Table 1. The height/width ratio of the caliber h_F/b_K is changed from 0.2 to 0.4. The effects of the tensions are shown in Fig. 14. The solid lines, the dotted lines and the dot-dash lines denote the results of $h_F/b_K = 0.4, 0.3$ and 0.2 , respectively.

The control equation coefficients are shown in Fig. 15. In Fig. 15(a), the absolute value of the control equation coefficient of the front tension decreases with increasing height/width ratio of the caliber, whereas that of the back tension increases. In Fig. 15(b), the absolute values of the control equation coefficients decrease. The results obtained by Eqs.(17) and (18) are supplemented. These results agree well with the results of the analysis.

Fig. 14 Effects of tensions (height/width ratio of caliber)

Fig. 15 Control equation coefficients (height/width ratio of caliber)

4. Conclusions

An analysis of the effect of the tension in the bar rolling was performed by the energy method using the finite-element division. The following results were obtained.

(1) The results of the effects of the front and back tensions on the width spread, forward slip, load and torque agreed well with the experimental formulas obtained by Noguchi et al..

(2) The method of obtaining the control equation coefficients for the torque approximately using the results obtained when no tension was applied was proposed. The approximated results agreed well with the results obtained when tension was applied.

(3) The absolute values of the control equation coefficients for the width spread, forward slip, load and torque decreased with increasing roll radius.

(4) The absolute values of the control equation coefficients for the torque decreased with increasing reduction and height/width ratio of the caliber.

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