A Stochastic Theory of Propagation of Elastic Waves in Porous Solids for Nondestructive Pore Characterization*

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A new theory of propagation of elastic waves in porous materials was developed for pore characterization. The idea is based on the arrival probability of a direct ray, reflecting ray, and creeping ray. The theory gives a good account of the nature of elastic waves propagated in porous solids not only qualitatively, but also quantitatively. The authors show a rather good correlation between pore characteristics (porosity and pore size) and ultrasonic characteristics (waveform and arrival time), and propose a nondestructive technique based on the presented theory for evaluating the porosity and pore size.

Key Words: Elastic Wave, Nondestructive Inspection, Probabilistic Method, Inverse Problem, Ultrasonics, Pore, Void, Cavity, Bubble

1. Introduction

It is well known that pores (bubbles) are produced in steels by hydrogen attack, creep, and so forth. Since mechanical properties of materials decrease with the nucleation and growth of these cavities, the detection and sizing of pores by nondestructive techniques has recently become an important subject for the assessment of the integrity and quality of various engineering materials.

If pores are produced in the interior of materials, three typical changes are caused in the elastic waves propagated through the materials. The first is the delay of arrival time. The second is the decrease in amplitude, and the third is the frequency drop. By use of these changes in elastic waves, an ultrasonic velocity method^{(1)~(3)}, ultrasonic damping method^{(4)~(6)}, back-scattering method^{(7),(8)} and ultrasonic spectrum analysis method⁽⁹⁾ have been proposed for the nondestructive pore characterization. It has become evident by these methods that there exists a strong correlation between porosity (volume fraction) and ultrasonic characteristics. However, the propagation mechanism of ultrasonic rays in porous solids is still too vague to quantitatively relate the ultrasonic charac-

teristics (delayed time, amplitude, frequency, etc.) to pore characteristics (porosity, pore size, etc.). As for theoretical studies, Biot⁽¹⁰⁾ has developed a theory of propagation of elastic waves in a system composed of a porous elastic solid saturated by a fluid. After his work, some theoretical analyses^{(11)~(14)} based on the Biot theory have been performed, and have predicted the decrease in both velocity and amplitude of elastic waves with the increase in porosity. However, these analyses have some difficulties in their application to nondestructive pore characterization for the following reasons:

(1) Their theory contains many physical parameters whose quantities, especially frame bulk modulus, cannot be determined before pore characteristics are known⁽¹⁴⁾.

(2) It is so complicated to solve the wave equations used in their theory that we cannot easily deal with the waveform changes.

(3) The wave equations are based on the assumption that pore characteristics are uniform throughout the medium, so that it is difficult to ascertain the local pore distributions.

It is apparent that propagation characteristics in a small region where a pore does not exist are the same of those of a no-pore medium. We have assumed that the changes in wave propagation are caused only in a local area around pores where acoustic rays reflect and diffract. According to these concepts, we

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have developed a new theory of propagation of elastic waves in porous solids using a probabilistic method without wave equations. Our theory is so simple that we can easily predict waveform changes, and it is applicable to even inhomogeneous porous materials such as those whose local porosities and both sizes and shapes of pores are not uniform. In the present paper, we shall introduce this theory, and propose an ultrasonic method for predicting porosities and pore sizes aiming at establishing a nondestructive approach for quantitative characterization of porous materials.

2. Theory

We have developed the simple model shown in Fig. 1, considering the adaptability to various pore distributions. The internal structure of porous solids may be approximated by a number N of porous planes arranged at an equal distance D . Suppose that gaseous pores of radius r distribute in these planes with the area fraction ϕ . Although we illustrated only circular pores in Fig. 1, the model is adaptable to any shapes of pores. We take here the incident rays as plane waves of the P-wave which do not transmit through gaseous pores, but are allowed to travel around the pore surfaces. We also assume that the damping loss is negligible.

2.1 Influence of multiple reflections

In this section, we suppose that when elastic rays impinge a pore, they entirely reflect in the reverse direction of incidence. Now consider the elastic waves which emit from the i -th plane towards the left direction, as shown in Fig. 2. Since the probability of

Fig. 1 Theoretical model of propagation of elastic waves in porous solids

reflections at each plane must be equal to the area fraction ϕ , the arrival probability of direct rays becomes $(1-\phi)^{N-i}$. Now let $F_{N,i}(j \cdot \Delta t)$ be the arrival probability of the elastic ray which requires more time of the amount $j \cdot \Delta t$ than that of the direct rays. Then, from the above result, the probability $F_{N,i}(0 \cdot \Delta t)$ is written as

$$F_{N,i}(0 \cdot \Delta t) = (1-\phi)^{N-i} \quad (i=0, 1, \dots, N)$$

where Δt is the time necessary for a wave to move back and forth between two adjacent planes separated by distance D , and is given by $2D/V_p$. Next, we shall determine the arrival probability of a wave whose time lag is $j \cdot \Delta t$. Of these waves, consider a ray whose first reflecting point is the k -th plane and whose second reflecting point is the m -th plane, as is illustrated by path ③ in Fig. 2. If we denote the arrival probability of this ray by P_{km} , the probability of a $j \cdot \Delta t$ delayed wave is expressed as

$$F_{N,i}(j \cdot \Delta t) = \sum_{k=\text{MAX}(2,i+1)}^N \sum_{m=\text{MAX}(1,k-j)}^{k-1} P_{km}$$

Here $\text{MAX}(a,b)$ denotes the larger value between a and b , and P_{km} is determined as follows.

The arrival probability from the emission at the i -th plane to the first reflection at the k -th plane is given by $(1-\phi)^{k-i-1} \cdot \phi$, and that from the first reflection to the second reflection at the m -th plane is given by $(1-\phi)^{k-m-1} \cdot \phi$. Hence, the probability from the emission at the i -th plane to the second reflection at the m -th plane becomes $(1-\phi)^{2k-m-i-2} \cdot \phi^2$, and the traveling time is given by $\{(k-i)D + (k-m)D\}/V_p$. Note that the direct ray which emits from the m -th plane requires more time of the amount $(i-m)D/V_p$ than that of the direct ray emitted from the i -th plane. From these relations, P_{km} is expressed by the product of the probability that a second reflection will take place at the m -th plane, $(1-\phi)^{2k-m-i-2} \cdot \phi^2$, and the probability that the elastic rays which emit from the m -th plane will arrive at the left end with time lag $j \cdot \Delta t - \{(k-i)D + (k-m)D\}/V_p - (i-m)D/V_p = (j-k$

Fig. 2 Reflecting paths of acoustic rays

$+m) \cdot \Delta t$. The latter probability may be written as $F_{N,m}([j-k+m] \cdot \Delta t)$. Hence,

$$P_{km} = (1-\phi)^{2k-m-i-2} \cdot \phi^2 \cdot F_{N,m}([j-k+m] \cdot \Delta t).$$

From these results, the arrival probability $F_{N,i}(j \cdot \Delta t)$ is expressed as

$$F_{N,i}(j \cdot \Delta t) = \sum_{k=\text{MAX}(2,i+1)}^N \sum_{m=\text{MAX}(1,k-j)}^{k-1} (1-\phi)^{2k-m-i-2} \cdot \phi^2 \cdot F_{N,m}([j-k+m] \cdot \Delta t).$$

This can be calculated starting with initial value $F_{N,i}(0 \cdot \Delta t) = (1-\phi)^{N-i}$.

Finally, let us consider the arrival probability that elastic rays which emit from the right end will arrive at the left end with time lag $j \cdot \Delta t$. This probability can be calculated by replacing i by zero in the above equation. For example, in Fig. 3, we show the calculated results of $F_{N,0}(j \cdot \Delta t)$ for $N=100$ and $\phi=0.25, 0.5$ or 0.75 . It appears that the elastic wave decreases its amplitude and increases its arrival time delay with a higher area fraction ϕ .

2.2 Influence of creeping rays

In this section, we suppose that when incident rays impinge a pore, they entirely creep around the pore surface. These waves are known as creeping rays^{(8),(12)}. Hurst and Temple⁽¹⁵⁾ have reported that the creeping ray speed depends on the cavity radius and frequency of incident rays. Here, we let the creeping ray speed be V_c . Then, when waves impinge a pore, they require more time of amount $\Delta t = r/V_c$ on the average to travel around the pore surface. This is the case of a circular pore. In another case, we use the corresponding Δt for each shape of the pore. For example, Δt is given by $\pi r/(2V_c) - r/V_p = r(\pi - 2\alpha)/(2V_c)$ for a spherical pore of radius r , where α denotes the velocity ratio (V_c/V_p).

Now consider a wave which propagates number N of planes with time lag $j \cdot \Delta t$. This wave must creep at number j of planes and pass through the other planes without impinging the pores. Since the probability that the incident rays will creep at a plane is equal to the pore area fraction ϕ , the arrival probability of the $j \cdot \Delta t$ delayed wave, $F_N(j \cdot \Delta t)$, is given by

$$F_N(j \cdot \Delta t) = \phi^j (1-\phi)^{N-j} {}_N C_j \quad (1)$$

Here, ${}_N C_j$ denotes the number of possible combinations in choosing number j of planes from number N of planes.

In Fig. 4, we present the arrival probabilities for $N=100$ and different ϕ . Like the arrival probability of the reflecting waves shown in Fig. 3, time lags increase with an increase in area fraction ϕ . However, the probability does not decrease as much as that of the reflecting rays.

2.3 Pore response function

In this section, we shall describe the method for predicting the waveforms passing through porous

solids. Here we consider only creeping rays because, as is clear from the above results, the arrival probability of the reflecting rays is negligible when compared with that of the creeping rays. If we suppose that the wave which passed through a nonporous material ($\phi=0$) has an amplitude 1, pulse width 0 and time lag 0, the waveform after passing through a porous material must take the same form as that of the arrival probability $F_N(j \cdot \Delta t)$ represented by Eq. (1). This indicates that $F_N(j \cdot \Delta t)$ corresponds to, so to speak, an impulse response for a group of pore planes. Since, in discrete waveform analysis, the unit impulse is represented by a rectangular pulse of amplitude Δt^{-1} and pulse width Δt , we should use $F_N(j \cdot \Delta t) \cdot \Delta t^{-1}$ as the actual impulse response. This function which we shall call the pore response function, $h(t)$, is expressed as

$$h(t) = F_N(j \cdot \Delta t) \cdot \Delta t^{-1}, (j=0, 1, \dots, N) \quad (2)$$

Using $h(t)$, the relationship between the arrival waveform which passed through the porous materials, $y(t)$, and that which passed through the nonporous materials, $y_0(t)$, may be written in terms of a convolution integral as

$$y(t) = \int_0^\infty y_0(t-\tau) \cdot h(\tau) d\tau. \quad (3)$$

we can thus predict $y(t)$ using Eqs. (1) to (3) by

Fig. 3 Arrival probability of multiple reflecting rays

Fig. 4 Arrival probability of creeping rays

determining $y_0(t)$ and $h(t)$. Conversely, if we can ascertain the waveforms of both the nonporous and porous materials, the pore distribution may be estimated from the deconvolution of $y(t)$ by $y_0(t)$.

3. Experimental Confirmation of the Theory

Using perforated aluminium plates, we examined the quantitative correspondence between our theory and measurements. The hole radius and pitch length of each specimen are tabulated in Table 1. The dimensions of the specimens and the arrangement of the transducers are shown in Fig. 5. A couple of broadband-type AE transducers (AET 500 TYPE) of a sensitive diameter of 20 mm were mounted on the specimen. The amplifier (NF-TYPE 9913) has a flat frequency response up to 20 MHz. Considering the reproducibility of the measurements, grease was not used for the mounting of the transducers, and ultrasonic testings were done underwater. Transient elastic pulses were generated on one side of the specimen by giving a 10 V step signal to the transmission transducer. The responses were detected at the opposite side with a reception transducer.

Figure 6 shows the response signal and the frequency spectrum of the electronic equipment including a couple of transducers without a propagation

Table 1 Hole diameters and pitch lengths of the specimens

Specimen No.	Diameter d (mm)	Pitch p (mm)	Number of Holes
1	—	—	0
2	0.5	4.0	7×13
3	0.5	2.0	13×25
4	1.0	4.0	7×13
5	1.0	2.0	13×25

Fig. 5 Dimension of the specimen and the arrangement of ultrasonic transducers

medium. The ultrasonic waveforms after passing through each specimen are shown in Fig. 7 by solid lines. We can see that the amplitudes and frequencies of the arrival waves fall with both the increase in hole sizes and decrease in pitches. It also appears that the ultrasounds arrive later as holes distribute densely.

We shall apply our theory to these experimental results. The correspondence to the model in Fig. 1 gives the following relations:

$$\phi = d/p$$

$$N = 7, 13$$

$$\Delta t = \pi d / (4 V_c) - d / (2 V_p) \\ = d(\pi - 2\alpha) / (4 V_c)$$

where d is the hole diameter, p is the pitch length, and $\alpha = V_c/V_p$. By substituting these parameters for Eq. (1), the arrival probabilities of the ultrasounds can be predicted, as shown in Fig. 8. Using Eqs. (2) and (3) and the detected waveform through the nonperforated specimen, we can predict the arrival waveforms through perforated specimens. In Fig. 7, we present the predicted waveforms by the broken line. Figure 7(a) shows the waveforms when creeping speed V_c is equal to P-wave speed V_p (6100 ms^{-1}). Figures 7(b) and 7(c) show the waveforms for $V_c/V_p = 0.5$ or 0.4 , respectively. The predicted waveforms for $V_c/V_p = 0.4$ to 0.5 agreed well with those of measured signals. Hurst and Temple⁽¹⁵⁾ have derived the following approximation for creeping wave speed:

$$V_c = V_p [1 + 0.928(\pi \cdot f \cdot d / V_p)^{2/3}]^{-1},$$

Fig. 6 Signal response of the electronic equipment including transducers

Fig. 7 Comparison of measured and calculated signals passing through the perforated specimens. Broken curves show the calculated waveforms for (a) $V_c/V_p=1$, (b) $V_c/V_p=0.5$ and (c) $V_c/V_p=0.4$

where f is the frequency of elastic waves, and d is the diameter of cavity curvature. In Fig. 9, we plot the above relationships between the cavity diameter and creeping velocity for different f . Since the mean frequency of the ultrasounds used in this experiment is estimated as 1 to 2 MHz from Fig. 6, the velocity ratio V_c/V_p is estimated as 0.3 to 0.5 from Fig. 9. This suggests that the values 0.4 and 0.5 of V_c/V_p used in the calculation of waveforms in Figs. 7 (b) and (c) are reasonable.

Next, let us determine the pore response functions using the detected signals shown by solid lines in Fig. 7. To minimize the analytical errors, deconvolution was performed in a frequency domain using

numerical filters and numerical windows⁽¹⁶⁾. The curved lines in Fig. 10 show the deconvoluted results, and the bar graphs represent the theoretical solutions of Eq. (2) for $V_c/V_p=0.45$. The pore response functions obtained from the measured signals agreed well with those of the theory.

4. A Method for Predicting Porosity and Pore Size

4.1 In the case when pores distribute on a number of cross sections

Now consider the case of hydrogen-attacked steels, in which circular defects are produced on grain boundaries⁽¹⁷⁾. Considering the correspondence of this defect distribution to the model in Fig. 1, we shall take

Fig. 8 Arrival probabilities of delayed waves

Fig. 10 Comparison of the measured and calculated pore response functions of the perforated specimens

Fig. 9 The variation of the creeping wave velocity as a function of cavity diameter and wave frequency (from Hurst and Temple's⁽¹⁵⁾ approximation) Fig. 11 The variation of the pore response function as a function of pore area fraction ϕ and pore radius r for the model shown in Fig. 1

D as the mean grain size, N as the number of grains crossed by the wave path, ϕ as the area fraction of defects occupied in a grain surface, and r as the mean radius of the defects.

In Fig. 11, we present the theoretical pore response functions for $N=100$, $V_p=6000 \text{ ms}^{-1}$ and $\alpha = V_c/V_p=0.5$. If we let t_p be the time when the pore response function obtains maximum value, it may be approximated by $N \cdot \phi \cdot \Delta t$ as is seen from Fig. 11 and Fig. 4. Since Δt for a circular pore is given by r/V_c , t_p may be rewritten as

$$t_p = N \cdot \phi \cdot \Delta t = N \cdot \phi \cdot r / V_c \quad (4)$$

We can therefore predict the product of defect radius r and area fraction ϕ by knowing t_p , N and V_c . The individual quantities of ϕ and r may be determined so as to coincide the measured amplitude or pulse width of the pore response function with those of our theory. Since these measured amplitudes depend on the attachment condition of the transducers, we recommend the use of pulse widths.

If we let the mean pulse width of the pore

response function be W , and the maximum amplitude be H , the following expression applies:

$$W \cdot H = \int_{-\infty}^{\infty} h(t) dt = 1.$$

Noting that H is approximated by $h(t_p)$, we can theoretically predict the relationships between ϕ and r and W and H from Eqs. (2) and (4), as shown in Fig. 12.

4.2 In the case where pores distribute uniformly

Pore distributions of materials such as powder-sintered materials are considered to be uniform in the interior of the solids. In these cases, we shall consider the model of spherical pores of radius r separated by equal distance p , as shown in Fig. 13. Correspondence to the model in Fig. 1 gives the following relations

$$\begin{aligned}\phi &= \pi r^2 / p^2, \\ N &= L/p, \\ \Delta t &= (\pi - 2\alpha)r / (2V_c),\end{aligned}$$

where L is the propagation distance, and $\alpha = V_c / V_p$. For such materials, the volume fraction of pores, say porosity ψ , is a more meaningful quantity than is the area fraction ϕ . Since ψ is given by $4\pi r^3 / (3p^3)$, the above expressions are rewritten as:

$$\begin{aligned}\phi &= \pi(4\pi/3\psi)^{-2/3}, \\ N &= (4\pi/3\psi)^{-1/3} \cdot L/r, \\ \Delta t &= (\pi - 2\alpha)r / (2V_c).\end{aligned}\quad (5)$$

In Fig. 14, we present the pore response functions for $L=10$ mm, $V_p=6000$ ms⁻¹ and $\alpha=0.45$ using Eq. (2). From Eq. (4), the arrival time lag becomes

$$\begin{aligned}t_p &= N \cdot \phi \cdot \Delta t \\ &= (\pi - 2\alpha)L \cdot 3\phi / (8V_c).\end{aligned}\quad (6)$$

This indicates that t_p is independent of pore sizes, as

is seen from Fig. 14. We can therefore predict porosities from time lag t_p . The pore radius can be estimated by coinciding the measured mean pulse width with that of the theoretical pore response function, as mentioned previously.

Maximum amplitude H of the pore response function is given by substituting $t_p/\Delta t$ for j in Eq. (2), and mean pulse width W is determined from the relation $W \cdot H = 1$. Then, H and W are given by

$$\begin{aligned}H &= \phi^m (1 - \phi)^{N-m} C_m / \Delta t, \\ W &= \Delta t / \{ \phi^m (1 - \phi)^{N-m} C_m \},\end{aligned}$$

where $m = t_p/\Delta t = 3L\phi/(4r)$. Substituting Eq. (5) for the above equation, in Fig. 15, we present the relationships between ϕ , r and H , W , t_p for $L=10$ mm. The porosity and pore size can be estimated using the top and bottom figures in Fig. 15.

Finally, we shall derive the relationship equation between the porosity and apparent wave speed. Although the ray velocity is usually determined from the arrival time of the wavefront, we shall here determine it from the time t_p when the amplitude of the pore response function is the largest. If the arrival time of the wave which passed through the nonporous materials is t_0 , then the apparent velocity in porous materials, V , is given by $L/(t_0 + t_p)$, where $t_0 = L/V_p$. From these relations and Eq. (6), we can obtain

$$V = V_p \cdot \{1 + 3(\pi - 2\alpha)\phi/8\alpha\}^{-1}. \quad (7)$$

In Fig. 16, the relationship between porosity and the apparent wave speed is presented for different velocity ratios V_c/V_p .

Mihara et al.⁽¹⁾ and Shinke et al.⁽¹⁸⁾ have measured the relationships between the apparent velocity and

Fig. 12 Pore area fraction dependence of arrival time delay, amplitude and mean pulse width of the pore response function

Fig. 13 The model of the wave propagation for uniform distribution of spherical pores

Fig. 14 The variation of the pore response function as a function of porosity ψ and pore radius r for the model shown in Fig. 13

Fig. 15 Porosity dependence of arrival time delay, amplitude and mean pulse width of the pore response function

porosity of sintered materials using 2.25 MHz and 5 MHz ultrasonic transducers, respectively. Their actual experimental results agreed well with our theoretical results shown in Fig. 16 for the range of $\alpha=0.4$ to 0.6. This range of α also corresponds well to that ($\alpha=0.4$ to 0.5) used in our experimental confirmation.

5. Conclusions

In this study, we developed a new theory of propagation of elastic waves in porous solids, and proposed a nondestructive method for predicting porosities and pore sizes. The results are summarized

as follows:

(1) Our theory provides a good account of the nature of elastic waves propagated in porous materials. Compared with the conventional theory, this stochastic theory is very simple, and is applicable to various types of porous materials whose local porosities and pore shapes are different.

(2) The waveform changes including arrival time differences depend substantially on the creeping nature of elastic rays, but depend little on multiple reflections.

(3) The wave speed of creeping rays is predicted

Fig. 16 Relationship between porosity ψ and apparent velocity V for different velocity ratios (V_c/V_p)

to be much smaller than that of incident rays.

(4) The experimental results using perforated specimens agreed well with our theory.

(5) By use of the pore response function, we can estimate porosities and pore sizes from its time delay and mean pulse width.

(6) It has become evident in theory that time delay of elastic waves is proportional to the product of the pore area fraction and pore radius in the case of circular pores distributing in a number of cross sections, and is proportional to porosity in the case of spherical pores distributing uniformly.

(7) The quantitative correspondence of maximum amplitude and mean pulse width of the pore response function to porosities and pore sizes was formulated.

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