Evaluation of Deformation Mode Change along 3-Dimensional Crack Front Line by CED*

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In an idealized 2-dimensional crack, the deformation constraint is kept constant along the crack front line. Thus, it is expected that the fracture mode is also kept constant along the crack front line. However, the deformation constraint and fracture mode generally change along the crack front line in a 3-dimensional crack, and this deformation mode change cannot be dealt with by applying conventional parameters such as K or J . The CED (crack energy density) was proposed as a parameter which has no restriction on the constitutive equation and is expected to be applicable to almost all kinds of crack problems. This paper purports to demonstrate that the deformation mode change above can be evaluated quantitatively when the CED is applied.

Key Words: Fracture Mechanics, Finite Element Method, 3-Dimensional Crack, CED (Crack Energy Density), Deformation Mode

1. Introduction

The recent development of a computer system has made numerical analysis of 3-dimensional crack problems a practical proposition. However, the crack parameters which should play the main role to deal with 3-dimensional problems have not been discussed sufficiently. The conventional 3-dimensional parameters, for example, the stress intensity factor⁽¹⁾, J -integral⁽²⁾ and so forth, are defined as extensions of 2-dimensional ones. Thus they can keep the clear physical meaning only under special constitutive conditions and certain restrictions are imposed on their applications as is seen in 2-dimensional problems. Moreover, the most typical problem of the 3-dimensional problems has not been taken into account in considering 3-dimensional crack parameters. Idealized states such as plane strain and plane stress are dealt with in 2-dimensional problems. The deformation mode (fracture mode) is assumed to be constant

in the direction of thickness. This assumption is not valid in 3-dimensional problems. The deformation mode (fracture mode) changes along a crack front line in a 3-dimensional crack, even if the crack front line is straight and tensile stress is given perpendicular to the crack plane. It is impossible to treat this problem using conventional crack parameters, although it is important to deal with actual fracture phenomena.

This paper purports to show that the CED (crack energy density) enables us to deal with the deformation mode change above quantitatively along a crack front line, which is an unavoidable problem in 3-dimensional crack problems. The CED was proposed as a parameter which can be applied to all kinds of fracture problems without any restrictions on constitutive condition, dimension and so on^{(3)~(5)}. The fundamental matters of the CED for 3-dimensional cracks have been studied already and it can be applied to 3-dimensional cracks with no objectionable points⁽⁶⁾⁽⁷⁾. The concrete definition of the CED and its expression by domain integral for the notch model, that is, one of the 3-dimensional crack models, are reviewed first in this paper. The CED can generally be

* Received 30th May, 1991. Paper No. 90-0996A

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divided into four components. They comprise the contributions of shear deformations in three arbitrary planes orthogonal to one another, plus the contribution of volume change. The formulation of these components is shown next. Subsequently, 3-dimensional finite element analyses of center-cracked specimens are carried out for elastic and elastic-plastic cases. The availability of the CED for 3-dimensional crack problems is demonstrated not only through the discussion based on the total value of the CED, but also the discussion about the deformation mode at a crack tip based on its components mentioned above. The former corresponds to the discussion regarding the effect of thickness of cracked specimen in terms of conventional crack parameters, and the latter is not possible without the introduction of the components of the CED in this study.

2. CED and Its Components Corresponding to Shear Deformations in Each Plane

In this section, the concrete definition of the CED and its representation by domain integral are given first. The notch model is used as a crack model. Secondly, it is shown that the CED is given as the sum of the contributions of shear deformation in three orthogonal planes and that of volume change.

2.1 Definition of the CED for notch model

The CED is defined as "the strain energy area density, that is, the work done per unit area in the plane containing a crack front line from the initial state through to the present state" ^{(3)~(5)}. Its concrete definition is given by considering the amount above in a practical crack model. The notch model⁽⁶⁾ is employed in this paper. The availability of this model has already been confirmed.

In the notch model, an actual crack in the initial state is replaced by a notch with sufficiently small radius of curvature ρ in the plane perpendicular to the crack front line at each point of the crack front line as shown in Fig. 1. $\Gamma_0(x_1, x_3)$ is a semicircular path in the plane congruent with the shape of the notch tip. The coordinate of its center is $(x_1, x_2, x_3) = (X_1, 0, X_3)$ (X_1 and X_3 represent coordinate values). The gathering of such paths forms a layer in the model. This layer corresponds to the plane containing a crack front line in the definition of the CED and is regarded as a substantial fracture process zone. The extension of a crack is realized by cutting off the center of the layer as shown in Fig. 1. The cut part of the layer indicated by the hatched area in the figure is assumed to have the rigidity only under compression.

The CED for the notch model is defined by Eq. (1) at each point in the plane containing a crack front line.

$$\mathcal{E}(t, X_1, X_3) = \int_{\Gamma_0(X_1, X_3)} W dx_2 \quad (1)$$

Here W is the strain energy density defined by Eq. (2), where σ_{ij} , ϵ_{ij} and t are stress, strain and time, respectively.

$$W = \int_0^t \sigma_{ij} d\epsilon_{ij} \quad d\epsilon_{ij} = (\partial \epsilon_{ij} / \partial t) dt \quad (2)$$

2.2 Expression of the CED by domain integral

The CED defined as the strain energy area density can be expressed by the domain integral through the conservation law based on the energy conservation law⁽⁶⁾. The integral is given by Eq. (3).

$$\begin{aligned} \mathcal{E}_J(t, X_1, X_3) = & \int_{\Gamma_1} (W dx_2 - T_i u_{i,1} d\Gamma) \\ & - \int_{\Gamma_2 - \Gamma_0(X_1, X_3) + \Gamma_3} T_i u_{i,1} d\Gamma - \int_A (\sigma_{i3} u_{i,1})_{,3} dA \\ & - \int_A \int_0^t (\sigma_{ij,1} d\epsilon_{ij} - d\sigma_{ij} \epsilon_{ij,1}) dA \end{aligned} \quad (3)$$

The CED is evaluated as $\mathcal{E}(t, X_1, X_3) = \mathcal{E}_J(t, X_1, X_3)$. $\Gamma_1 + \Gamma_2 + \Gamma_3$ is an arbitrary path surrounding $\Gamma_0(X_1, X_3)$ as shown in Fig. 1. A is the area of the part enclosed by $\Gamma_1 + \Gamma_2 + \Gamma_3 - \Gamma_0(X_1, X_3)$. T_i , u_i and n_i are traction force, displacement and outward normal unit vector, respectively. The fourth term on the right-hand side of Eq. (3) vanishes for an elastic body in which the constitutive equation is given as $\sigma_{ij} = \partial W / \partial \epsilon_{ij}$ ⁽³⁾. Thus, $\mathcal{E}_J$ -integral coincides with J -integral under this condition⁽⁶⁾.

2.3 Contribution of shear deformation of the CED

The strain energy density in Eq. (1) is generally separated into four parts expressed in the form of Eqs. (4)~(7).

$$W_{12} = \int_0^t \left\{ \frac{1}{3} (\sigma_{11} - \sigma_{22}) (d\epsilon_{11} - d\epsilon_{22}) + 2\sigma_{12} d\epsilon_{12} \right\} \quad (4)$$

$$W_{23} = \int_0^t \left\{ \frac{1}{3} (\sigma_{22} - \sigma_{33}) (d\epsilon_{22} - d\epsilon_{33}) + 2\sigma_{23} d\epsilon_{23} \right\} \quad (5)$$

$$W_{31} = \int_0^t \left\{ \frac{1}{3} (\sigma_{33} - \sigma_{11}) (d\epsilon_{33} - d\epsilon_{11}) + 2\sigma_{31} d\epsilon_{31} \right\} \quad (6)$$

$$W_V = \int_0^t \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) (d\epsilon_{11} + d\epsilon_{22} + d\epsilon_{33}) \quad (7)$$

That is,

$$W = W_{12} + W_{23} + W_{31} + W_V. \quad (8)$$

The quantity given by Eq. (4) can be grasped as the contribution of shear deformation in the x_1 - x_2 plane to the strain energy density. W_{23} and W_{31} are thought

Fig. 1 Notch model

of as the contributions of shear deformations in x_2-x_3 and x_3-x_1 planes, respectively. The rest of the strain energy density expressed by Eq.(7) is the contribution of volume change to W . In particular, when the $x_1-x_2-x_3$ coordinate system is taken so that the directions of coordinate axes coincide with those of principal stresses and principal strains, the meanings of the above components are easily understood. For example, W_{12} is given by Eq.(9) in this case.

$$W_{12} = \int_0^t \frac{1}{3} (\sigma_1 - \sigma_2) (d\epsilon_1 - d\epsilon_2) \quad (9)$$

Here σ_1, σ_2 and σ_3 are principal stress components and ϵ_1, ϵ_2 and ϵ_3 are principal strain components. $\sigma_1 - \sigma_2$ in Eq.(9) is the maximum shear stress in the x_1-x_2 plane as is shown in Fig. 2.

Corresponding to the separation of strain energy density above, the CED by Eq.(1) can also be divided into the contributions of shear deformations and volume change as

$$\mathcal{E}_{12}(t, X_1, X_3) = \int_{\Gamma_0(x_1, x_3)} W_{12} dx_2 \quad (10)$$

$$\mathcal{E}_{23}(t, X_1, X_3) = \int_{\Gamma_0(x_1, x_3)} W_{23} dx_2 \quad (11)$$

$$\mathcal{E}_{31}(t, X_1, X_3) = \int_{\Gamma_0(x_1, x_3)} W_{31} dx_2 \quad (12)$$

$$\mathcal{E}_v(t, X_1, X_3) = \int_{\Gamma_0(x_1, x_3)} W_v dx_2 \quad (13)$$

$$\mathcal{E} = \mathcal{E}_{12} + \mathcal{E}_{23} + \mathcal{E}_{31} + \mathcal{E}_v \quad (14)$$

It is possible to evaluate the deformation mode at a crack tip by means of these components of the CED. In 3-dimensional crack problems, it is meaningful that the coordinate system are chosen so that the direction of axes x_1, x_2 and x_3 coincide with those of principal stresses and principal strains at crack tip when $\rho \rightarrow 0$. The components defined in this coordinate system are considered important to discuss the problem.

3. Evaluation of Deformation Mode Change along Crack Front Line

The elastic and elastic-plastic finite element analyses of a center-cracked specimen are carried out to verify the validity of the discussion by use of the CED in 3-dimensional problems. The variation of the total

Fig. 2 Maximum shear stress

value of the CED along a crack front line is evaluated first. This corresponds to the discussion about the effect of the thickness of a cracked specimen based on conventional crack parameters. Moreover, the deformation mode change along a crack front line is studied by paying attention to components of the CED defined in Section 2. This is only possible based on the CED.

3.1 Finite element analyses

The object of analysis is a center-cracked specimen under a uniform tensile stress in x_2 direction. Dimensions of the specimen and load conditions are shown in Fig. 3. As to the thickness of the specimen, although six values of D ($2D$ is the thickness) are actually taken for analysis ($D=0.5, 1.0, 5.0, 10.0, 20.0, 60.0$ mm), the results for $D=0.5, 1.0, 10.0, 60.0$ mm only are shown here due to lack of space. Symmetry makes it sufficient to analyze 1/8 of the specimen. The specimen is discretized by eight nodes hexahedron isoparametric elements. The elastic analysis is characterized by Young's modulus $E=210.7$ GPa and Poisson's ratio $\nu=0.3$. Mises-type associated flow rule is adopted in elastic-plastic analyses. Yield stress σ_Y is 392 MPa and strain hardening rate H' is $E/100$. The total value of the CED is evaluated by the domain integral of Eq.(3). The components of the CED given by Eqs.(10)~(13) are evaluated directly by using the stresses and strains at the Gauss points closest to the notch tip in the elements forming the notch tip. Strictly speaking, although the stresses and the strains at the notch tip should be used, those above are used in the evaluation for convenience's sake. As each component of the CED is evaluated directly by using the values at the notch tip, its accuracy is considered to be

Fig. 3 Center-cracked specimen

slightly less than that of total CED by domain integral. Thus, it is confirmed that the sum of each component almost coincides with the total CED by Eq.(3).

3.2 Correspondence of each component of the CED to deformation mode at crack tip

Each component of the CED given by Eqs.(10)~(13) is evaluated by employing the $x_1-x_2-x_3$ coordinate system in which the x_3 axis is taken so as to be

parallel to the crack front line as shown in Fig. 3. In this case, the quantities evaluated by $\mathcal{E}_{12}$ and $\mathcal{E}_{23}$ correspond to the shear deformations in the x_1-x_2 and x_2-x_3 planes characterized by the slips along the dotted lines in Figs. 4(a) and (b) respectively. $\mathcal{E}_{31}$ is small in the problem here, so the ratios of $\mathcal{E}_{12}$ and $\mathcal{E}_{23}$ to $\mathcal{E}$ are compared with those from plane stress and

Fig. 4 Correspondence of shear component of the CED to deformation at crack tip

Fig. 5 Variation of the CED along crack front line (elastic)

plane strain analyses when considering the deformation mode change along the crack front line.

3.3 Elastic analyses

Elastic analyses are carried out under uniform tensile stress $\sigma=9.8$ MPa. The variations of the CED along the crack front line are shown in Fig. 5. The CED is evaluated by the domain integral. In the margin of the figure, the values of the CED for corresponding plane stress and plane strain states are indicated on the left and the right, respectively. These values of the CED are evaluated through the value of stress intensity factor K as $\mathcal{E}=K^2/E$ for the plane stress state and $\mathcal{E}=(1-\nu^2)K^2/E$ for the plane strain state. In the elastic case, the CED theoretically coincides with the J -integral; the results of the numerical analyses prove it. Therefore, it should be noted that the discussion about the effect of plate thickness by conventional crack parameters corresponds to the discussion which pays attention to the total value of the CED. In the discussion based on conventional parameters, it was considered that the plane strain

state is realized in the inner part of a thick cracked specimen, because the value of K or J approaches that of plane strain. On the other hand, the value of J or K is different from that of plane stress in the part close to the free surface of a specimen. The value of the crack parameter decreases in the part close to the free surface of a specimen. This phenomenon is called the boundary layer effect⁽⁶⁾ and has been discussed as one peculiar to 3-dimensional cracks. Although the value of the crack parameter varies as mentioned above, it is well known that the fracture mode near the free surface is of the plane stress type. These facts complicate the discussion of the boundary layer effect.

In addition to the discussion based on the total value of the parameter, the deformation mode change along the crack front line is evaluated by paying attention to each component of the CED. Figures 6 (a)~(d) show the ratios of $\mathcal{E}_{12}$, $\mathcal{E}_{23}$ and $\mathcal{E}_{31}$ to $\mathcal{E}$ at each point of a crack front. In the figures, black triangles and black squares represent $(\mathcal{E}_{12} + \mathcal{E}_{23} + \mathcal{E}_{31})/\mathcal{E} = (\mathcal{E} - \mathcal{E}_V)/\mathcal{E}$ and $\mathcal{E}_{12}/\mathcal{E}$ respectively. As $\mathcal{E}_{31}$

(a) $D=0.5$ mm

(b) $D=1.0$ mm

(c) $D=10.0$ mm

(d) $D=60.0$ mm

Fig. 6 Ratio of shear component to the CED (elastic)

is small enough to neglect in this case, the difference between a black triangle and a black square is nearly equal to $\mathcal{E}_{23}/\mathcal{E}$. The far left and far right symbols in the figures indicate the ratios for plane stress and plane strain states, respectively. These are evaluated by finite element analysis using the finite element division and material constants corresponding to the 3-dimensional analysis here. It is known from the figures that the ratios of the components of the CED are almost constant in the neighborhood of the surface, not depending on the thickness of the specimen D . The ratios are nearly equal to those of plane stress case. When D is sufficiently small, the ratios change slightly along the crack front line. This means the plane stress state is almost realized throughout the specimen of such small D . As the value of D increases, the ratios of the components around the center of the specimen change. These ratios gradually approach those of the plane strain condition with the increase of D . The plane strain state is realized around the center of the thick specimen. Although a uniform stress is applied in the part far from the crack tip, the substan-

tially applied load to the crack tip decreases with the increase in D in the neighborhood of the free surface because of the change in the constraint condition of deformation. This seems to cause the decrease of the total CED with the increase in D in the neighborhood of the surface. On the other hand, although the ratios of the components of the CED are nearly equal to those of the plane strain state around the center of the specimen of $D=10.0$ mm, the total value of the CED there is a little higher than that of the plane strain case (see Fig. 5(c)). This is also caused by a substantial change in the applied load. These facts indicate that the total value of the crack parameter is not enough to discuss the 3-dimensional problems. The constraint condition change is to be evaluated by using the components of the CED.

3.4 Elastic-plastic analysis

The load is applied until σ reaches 245 MPa ($\sigma/\sigma_Y = 0.625$). Fully plastic deformation is ultimately realized in every cracked specimen. The variation of total value of the CED with the increase in load is shown in Figs. 7(a)~(d). In the figures, the range of the CED

Fig. 7 Variation of the CED with load

along the crack front line is indicated by the shaded region. The broken line and the dotted line indicate the results of plane stress and plane strain analyses, respectively. The variations of the CED along the crack front line at $\sigma/\sigma_Y=0.5$ and $\sigma/\sigma_Y=0.625$ are shown in Figs. 8 and 9, respectively. In elastic-plastic problems, the CED does not generally coincide with the J -integral. In a special case when a monotonously increasing load is applied and the associated flow rule is employed, the fourth term in the right-hand side of Eq. (3) is small and therefore the CED and the J -integral take almost the same value.

The results of J are not shown in the figures as the difference between the CED and J is also small in the analysis here. The results of corresponding plane stress and plane strain analyses are shown by the far left and far right symbols, respectively. Unlike elastic problems, there are two measures to compare 3-dimensional results with 2-dimensional ones. One concerns uniform tensile stress and the other concerns

displacement at load point. So, in addition to the results obtained from constant stress condition indicated by $\blacktriangleright$ and $\blacktriangleleft$, the results when the averaged displacement at the loading face in the 3-dimensional analysis is applied as constant displacement condition are also indicated by $\triangleright$ and $\triangleleft$. It should be noted here that these values of 2-dimensional cases are shown just for reference. Even if the differences between these values and the values of the CED are small at the center or at the surface of the specimen, it does not necessarily mean that the situations at the crack tip are similar to each other. The situation at the crack tip can be evaluated reasonably when the ratios of each component of the CED are also considered. As to the ratios of each component of the CED, the difference between constant stress and constant displacement conditions was small, so only the results from the constant stress condition are shown in Figs. 10 and 11.

It is known from Figs. 8 and 9 that the increment

(a) $D=0.5$ mm

(b) $D=1.0$ mm

(c) $D=10.0$ mm

(d) $D=60.0$ mm

Fig. 8 Variation of the CED along crack front line
(Elastic-Plastic, $\sigma/\sigma_Y=0.5$)

Fig. 9 Variation of the CED along crack front line
(Elastic-Plastic, $\sigma/\sigma_Y=0.625$)

Fig. 10 Ratio of shear component to the CED
(Elastic-Plastic, $\sigma/\sigma_Y=0.5$)

(c) $D=10.0$ mm (d) $D=60.0$ mmFig. 10 Ratio of shear component to the CED
(Elastic-Plastic, $\sigma/\sigma_Y=0.5$) (a) $D=0.5$ mm (b) $D=1.0$ mm (c) $D=10.0$ mm (d) $D=60.0$ mmFig. 11 Ratio of shear component to the CED
(Elastic-Plastic, $\sigma/\sigma_Y=0.625$)

of the CED is greater in the neighborhood of the surface than in the neighborhood of the center with the increase of plastic deformation. This shows that the shear deformation in the x_2-x_3 plane which causes necking or shear lip becomes prominent in the neighborhood of the surface with the increase of plastic deformation. This shear deformation causes the marked increase of the CED. It is observed from Figs. 10 and 11 that the ratio of $\mathcal{E}_{23}$ increases with plastic deformation around the surface. This confirms the above supposition.

4. Concluding Remarks

It is shown that the CED defined as the strain energy area density can be separated into four components. The components consist of the contributions of the shear deformations in three orthogonal planes defined by arbitrary rectangular coordinates and volume deformation. It is possible to evaluate the deformation mode at the crack tip by using these components. This kind of discussion cannot be carried out by the conventional crack parameters. This advantage of the CED is demonstrated through numerical analyses. The plane stress state and the plane strain state are regarded as standards of deformation in the analyses. It is thought to be possible to express shear fracture criteria based on corresponding components of the CED. Further investigation into the above components is necessary for the development of such criteria.

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