A Method of Obtaining Strain Distributions of Circumferentially Notched Cylindrical Bars by means of a Hardness Test*

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Plastic strain distributions at the net section of notched bars subjected to large deformation are obtained using the Vickers microhardness test and the numerical calculation proposed. Each notched bar is loaded up to $0.7 P_{\max}$ ($P_{\max}$: maximum tensile load), $0.8 P_{\max}$, $0.9 P_{\max}$, $0.95 P_{\max}$, $P_{\max}$, $0.98 P_{\max}$ beyond $P_{\max}$ and $0.95 P_{\max}$ beyond $P_{\max}$. Strains at a point are calculated by making an assumed curve of radial displacement touch the real curve of radial displacement distribution at that point. The distributions of the equivalent and axial plastic strains are concave at all load levels tested. The equivalent plastic strain is only slightly greater than the axial plastic strain, even at the notch root where the difference between these two strains is the largest. The magnitude of the axial plastic strain at the notch root at $P_{\max}$ is about twice as large as that of the axial plastic strain at $P_{\max}$ of the unnotched specimen.

Key Words: Material Testing, Notch, Hardness, Plasticity, Large Deformation, Strain Distribution, Numerical Calculation

1. Introduction

Accurate strain distribution in large deformation is required to discuss the initiation strain of ductile fractures or the reason for notch strengthening of ductile materials. At present the finite element method is the most powerful method of obtaining strain distribution in large deformation⁽¹⁾. Strain distributions obtained by FEM are, however, affected by the constitutive equation employed and a quantitative expression for anisotropy caused by deformation. This suggests that we should experimentally obtain the accurate strain distribution in large deformation. In many studies on the initiation strain of ductile fracture, circumferentially notched cylindrical bars have been used. In those notched bars, ductile fracture initiates at the net section. It is therefore important to obtain the accurate strain distribution at the net section in large deformation.

Earl et. al. obtained strain distributions at the net section in large deformation by measuring the radial displacement of segregation bands on a longitudinal section⁽²⁾. They fitted a hyperbolic sine function to the measured values of radial displacement over the whole range of net radius and calculated the distributions of strain components using this curve. This method, however, provides a low accuracy of strain

distributions for the following reasons: (1) measurement of very small displacement of segregation bands leads to an error, resulting in an inaccurate curve of radial displacement and (2) this inaccurate curve produces an error in strain calculations, especially in the calculation of radial strain obtained from differentiating this curve. We could realize that the radial displacement distribution is too complex to be expressed over the whole range of net radius by a simple and single function. Consequently, their results have low reliability.

There is a good relation between the hardness number and equivalent plastic strain⁽³⁾. If this relation of a material is known, we can obtain the equivalent plastic strain distribution of a notched bar machined from the same material by means of a hardness test. Furthermore, careful consideration makes it possible to derive the distributions of strain components from this equivalent plastic strain distribution. This indicates that a hardness test is a powerful means of obtaining the distributions of strain components in large deformation.

In this paper, a method of obtaining strain distributions in large deformation by means of a hardness test are discussed and the strain distributions thus obtained are presented.

Notations

R_n : net radius prior to deformation ($= d_n/2$. see Fig. 1)

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R : arbitrary radius at the net section prior to deformation (see Fig. 1)

r_n : net radius after deformation (see Fig. 1)

r : radius R after deformation (see Fig. 1)

U_n : radial displacement at the notch root ($= r_n - R_n$, see Fig. 1)

U : radial displacement at radius R ($= r - R$, see Fig. 1)

ϵ_{eq} : equivalent plastic strain

$\epsilon_r, \epsilon_\theta, \epsilon_z$: radial, circumferential and axial plastic strains

2. Experimental

The circumferentially notched cylindrical specimens shown in Fig. 2 were used in the experiment. The ratio of net diameter to gross diameter d_n/D_n is 0.8 and the initial notch radius ρ_0 is 2 mm. Each notched specimen was loaded up to $0.7 P_{max}$ (P_{max} : maximum tensile load), $0.8 P_{max}$, $0.9 P_{max}$, $0.95 P_{max}$, P_{max} , $0.98 P_{max}$ beyond P_{max} , $0.95 P_{max}$ beyond P_{max} and $0.91 P_{max}$ beyond P_{max} . After unloading, notched specimens were cut along the net section and each cut surface was polished carefully. Vickers microhardness tests were carried out under a weight of 1 kgf and a holding time of 30 seconds. Hardness measurements were made along two mutually perpendicular diameters on the polished surface. Specimens were machined from an austenitic stainless steel SUS 304.

Figure 3 shows the relation between Vickers

Fig. 1 Dimensions of notched specimen and notations of undeformed and deformed notches

Fig. 2 Circumferentially notched cylindrical specimen ($d_n/D_n = 0.8$)

hardness number VHN and axial plastic strain ϵ in uniaxial tension. The solid line is the VHN- ϵ curve used to calculate strain distributions. This curve is also the VHN- ϵ_{eq} curve under multiaxial state of stress⁽⁴⁾.

Mean true stress at the net section $P/\pi(r_n)^2$ versus mean true strain at the net section $2\ln(R_n/r_n)$ curve is shown in Fig. 4. The corresponding curve of the unnotched specimen is the true stress-true strain curve of SUS 304 employed. The mean true strain at P_{max} of the notched specimen is greater than that at P_{max} of the unnotched specimen. This indicates that because of the strain concentration at the notch root, the axial strain at the notch root at P_{max} is considerably greater than the axial strain at P_{max} of the unnotched specimen. The details are discussed later.

At $0.98 P_{max}$ beyond P_{max} , some fine cracks were observed at the notch root and in its immediate vicinity. As the tensile load decreased, these cracks grew gradually and coalesced to such an extent that they were visible to the naked eye. In particular, the crack extension was remarkable during the decrease from $0.95 P_{max}$ beyond P_{max} to $0.91 P_{max}$ beyond P_{max} . This crack extension produces much greater reduction in actual cross-sectional area than deformation does. Consequently, most of the decrease from $0.95 P_{max}$ beyond P_{max} to $0.91 P_{max}$ beyond P_{max} is due to the crack extension.

3. Hardness Number Distributions

Distributions of the hardness number at the net section are shown in Fig. 5. In all load levels tested, the maximum hardness number occurs at the notch root and the minimum hardness number at the center of the net section. At the load levels of $0.7 P_{max}$, $0.8 P_{max}$ and $0.9 P_{max}$, the distributions of hardness number are uniform near the center of the net section. The

Fig. 3 Hardness number-axial plastic strain curve in uniaxial tension

range of this uniform distribution decreases with increasing deformation.

As previously described, the decrease from $0.95 P_{\max}$ beyond $P_{\max}$ to $0.91 P_{\max}$ beyond $P_{\max}$ is caused mainly by the extension of ductile cracks near the net section. It follows that during this decrease in load, deformation can hardly occur near the net section. Consequently, the distribution of hardness number at $0.91 P_{\max}$ beyond $P_{\max}$ was approximately the same as that at $0.95 P_{\max}$ beyond $P_{\max}$. Solid lines in Fig. 5 are the VHN- r/r_n curves used to calculate strain distributions. The broken line is the distribution of hardness number of the material before deformation.

4. A Method of Obtaining Radial Displacement from Hardness Number Distribution

As discussed in the introduction, the radial displacement distribution cannot be fitted by a single function over the whole range of net radius. Thus, we

Fig. 4 Mean true stress-mean true strain curves of notched and unnotched specimens

Fig. 5 Hardness number distributions at the net section

divide radius R_n equally into n segments and make an assumed function in each segment touch the real curve of the radial displacement distribution only at a point of division. The radial displacement distribution obtained by this method approaches the real distribution as the number of divisions is increased.

The radial displacement in i th segment $x_{i-1} \leq x \leq x_i$ is assumed to be expressed by an equation of the form

$$u - u_{i-1} = a_{i-1}(x - x_{i-1}) + b_i(x - x_{i-1})^m, \quad (1)$$

where

$$u = \frac{U}{U_n}, \quad x = \frac{R}{R_n}.$$

The values of a_{i-1} and u_{i-1} have already been determined in the previous segment $x_{i-2} \leq x \leq x_{i-1}$. The condition whereby Eq. (1) touches the real curve of radial displacement distribution at x_i determines the value of the coefficient b_i . The value of the exponent m is held constant over the whole range of $0 \leq x \leq 1$ and is given before calculation.

The circumferential and radial strains at x_i are given by

$$(\varepsilon_\theta)_i = \ln\{1 + (U/R)_i\}$$

$$(\varepsilon_r)_i = \ln\{1 + (dU/dR)_i\},$$

where from Eq. (1) we obtain

$$(U/R)_i = \frac{U_n}{R_n} \cdot \frac{1}{x_i} \{u_{i-1} + a_{i-1}(x_i - x_{i-1}) + b_i(x_i - x_{i-1})^m\}$$

$$(dU/dR)_i = \frac{U_n}{R_n} \{a_{i-1} + mb_i(x_i - x_{i-1})^{m-1}\}. \quad (2)$$

Volume constancy gives the axial strain component

$$(\varepsilon_z)_i = -(\varepsilon_\theta)_i - (\varepsilon_r)_i.$$

The equivalent plastic strain at radius $r_i (= R_i + U_i)$ is given by

$$(\varepsilon_{eq})_i^2 = \frac{4}{3} \{(\varepsilon_\theta)_i^2 + (\varepsilon_\theta)_i(\varepsilon_r)_i + (\varepsilon_r)_i^2\}.$$

At radius r_i , this equivalent plastic strain should be equal to the experimental equivalent plastic strain given by substituting $(VHN)_{r=r_n}$ obtained from the VHN- r/r_n curves shown in Fig. 5 into the VHN- ε curve shown in Fig. 3. This condition determines the value of U_i , and, consequently, the value of the coefficient b_i .

From the equation determined by substituting $i+1$ into i in Eq. (1), we can derive the coefficient a_i in the next segment $x_i \leq x \leq x_{i+1}$ of

$$a_i = \frac{R_n}{U_n} \left(\frac{dU}{dR} \right)_{x=x_i} = \frac{R_n}{U_n} \left(\frac{dU}{dR} \right)_i,$$

where $(dU/dR)_i$ is calculated from the radial displacement curve in the previous segment $x_{i-1} \leq x \leq x_i$, namely, Eq. (1).

The coefficient a_0 of the radial displacement curve in the first segment is obtained from Eq. (2):

$$a_0 = \frac{R_n}{U_n} \left(\frac{dU}{dR} \right)_0 = \frac{R_n}{U_n} \{\exp(\varepsilon_r)_0 - 1\}.$$

At the center of the net section,

$$(\varepsilon_\theta)_0 = (\varepsilon_r)_0 = -\frac{1}{2}(\varepsilon_z)_0 = -\frac{1}{2}(\varepsilon_{eq})_0.$$

The value of a_0 is therefore given by the following equation:

$$a_0 = \frac{R_n}{U_n} \left[\exp \left\{ -\frac{1}{2}(\varepsilon_{eq})_0 \right\} - 1 \right].$$

We can obtain the equivalent plastic strain at the center of the net section $(\varepsilon_{eq})_0$ by substituting $(VHN)_{r=0}$ obtained from the $VHN-r/r_n$ curves into the $VHN-\varepsilon$ curve.

Using the initial values $u_0=0$, a_0 , R_n , r_n and b_1 , which is given an appropriate value, we can determine the value of b_i successively from the first segment at the center of the net section toward the last segment at the notch root. Finally, we obtain the values of r_n and U_n . In the first calculation, the experimental value of r_n is given as an initial value of r_n . The calculated value of r_n is generally different from the initial value of r_n . Changing the initial value of r_n , we must iterate this calculation until the calculated value of r_n agrees with the initial value of r_n . When these two values agree with each other, we obtain the radial displacement distribution which the distribution of the hardness number provides.

For this method of calculation, the following two problems should be solved.

(1) How many divisions should be employed for the calculation?

(2) What value of m should be employed?

To solve these two problems, the effect of both the number of divisions and the value of m on the value of U_n of the load level of P_{max} was examined. Under the constant value $m=2.0$, the value of U_n for $n=100$ agreed with the value of U_n for $n=200$. The values of U_n for $n=50$, 25, 10 and 5 were 0.01%, 0.03%, 0.22% and 0.89% greater than the value of U_n for $n=200$, respectively. The effect of the number of divisions on the value of strain components was approximately the same as that on the value of U_n .

The effect of the values $m=1.5$, 2.0, 3.0 and 4.0 on the value of U_n was examined under the conditions $n=50$, 100 and 200. Under the condition of $n=50$, the value of U_n for $m=1.5$ was 0.4% greater than that for $m=2$, and the values of U_n for $m=3$ and $m=4$ were 0.42% and 0.62% less than that for $m=2$, respectively. Under the condition of $n=100$, the value of U_n for $m=1.5$ was 0.21% greater than that for $m=2$, and the values of U_n for $m=3$ and $m=4$ were 0.21% and 0.31% less than that for $m=2$, respectively. Under the condition of $n=200$, the value of U_n for $m=1.5$ was 0.10% greater than that for $m=2$, and the values of U_n for $m=3$ and $m=4$ were 0.11% and 0.17% less than that for $m=2$, respectively. Under

the condition of $n=200$, the difference in the value of exponent m had little effect on the calculated results.

We can conclude that the effect of the differences in the number of divisions n and the value of exponent m on the calculated results is quite small under the conditions of $n \geq 100$ and $m=2$. This conclusion was the same for other load levels. Consequently, in this investigation, the calculation was carried out under the conditions of $n=200$ and $m=2$.

5. Strain Distributions

Comparisons of the experimental value of net radius $(r_n)_{exp}$ with the calculated value of net radius $(r_n)_{cal}$ are shown in Table 1. There is a good agreement between these two values. For load levels beyond P_{max} , the difference between these two values increases with increasing deformation. This is because cracks at the notch root grew and coalesced to such an extent that they were visible to the naked eye. Although for load levels beyond P_{max} , the hardness measurements were made along the diameter parallel to cracks, the distribution of hardness number is not free from the influence of the crack extension.

Figure 6 shows the comparison between three radial displacement distributions obtained from two different methods for the load level of $0.95 P_{max}$ prior to P_{max} . Two curves with open symbols were calculated on the assumption that the radial displacement distribution can be fitted by a single function over the whole range of the net radius. On the other hand, the curve with solid circles was obtained from the numerical calculation proposed. The single functions used⁽²⁾ are

$$u = ax + bx^3 + cx^5$$

and

$$u = \frac{\sinh(kx)}{\sinh k}.$$

The values of three coefficients a , b and c in the polynomial were determined by using the values

Table 1 Comparison of experimental value of net radius with calculated value of net radius

Applied load	$(r_n/R_n)_{exp}$	$(r_n/R_n)_{cal}$	$\frac{(r_n/R_n)_{exp}}{(r_n/R_n)_{cal}}$
0.7 P_{max}	0.970	0.969	1.001
0.8 P_{max}	0.935	0.935	1.000
0.9 P_{max}	0.912	0.915	0.997
0.95 P_{max}	0.879	0.875	1.005
1.0 P_{max}	0.796	0.792	1.005
0.98 P_{max} beyond P_{max}	0.742	0.732	1.014
0.95 P_{max} beyond P_{max}	0.708	0.682	1.038

$$\left(\frac{dU}{dR}\right)_{x=0.1} = \frac{R_n}{U_n} \{\exp(\varepsilon_r)_{0,n} - 1\} \text{ and } u_{x=1} = 1,$$

where the values of U_n and $(\varepsilon_r)_{0,n}$ are those obtained from the numerical calculation proposed. Similarly, the value of the coefficient k in the above hyperbolic sine function was determined by the value $(dU/dR)_{x=1}$. If we determine the value of the coefficient k using the value $(dU/dR)_{x=0}$, approximately the same curve as that of the polynomial is obtained. Figure 6 indicates that the radial displacement distribution cannot be expressed by a simple and single function over the whole range of net radius and that the numerical calculation proposed is valid.

Figure 7 shows the radial displacement distribution for various load levels. The rate of increase in radial displacement becomes greater beyond $0.95 P_{\max}$ prior to $P_{\max}$. The increment of radial displacement is smaller in the load range from $0.98 P_{\max}$ beyond $P_{\max}$ to $0.95 P_{\max}$ beyond $P_{\max}$ than in the load range from $P_{\max}$ to $0.98 P_{\max}$ beyond $P_{\max}$. This conflicts with the fact that during the unstable deformation beyond $P_{\max}$, the magnitude of deformation corresponds to the magnitude of decrease in load. This discrepancy is attributed to the crack extension, which produces greater decrease in load than deformation does, as described in Section 2.

Strain distributions of $0.80 P_{\max}$, $P_{\max}$ and $0.95 P_{\max}$ beyond $P_{\max}$ are shown in Figs. 8(a)~(c). The difference between equivalent and axial plastic strains is quite small, even at the notch root where the difference is the largest. The maximum equivalent and axial plastic strains occur at the notch root for all load levels tested. The distribution of the absolute of the radial strain is similar in shape to the distributions of the equivalent and axial plastic strains. The circumferential plastic strain distribution is rather mild in comparison with the equivalent, axial and radial

Fig. 6 Comparison of present result with curve-fitted results

strain distributions. For the load level of $0.8 P_{\max}$, a strongly deformed region is limited near the notch root while for the load levels of $P_{\max}$ and $0.95 P_{\max}$ beyond $P_{\max}$, a strongly deformed region develops toward the center of the net section.

Variations of strains at the center of the net section and at the notch root with mean true strain $2 \ln(R_n/r_n)$ are shown in Fig. 9. The broken line in the figure is the variation of axial plastic strain of unnotched specimen with mean true strain. For the strain range beyond $P_{\max}$ of the unnotched specimen, or beyond necking, the equation $\varepsilon = 2 \ln(R_n/r_n)$ (R_n and r_n are minimum radii of the necking region prior to and after deformations, respectively) is approximately valid. Thus, the broken line was extended beyond $P_{\max}$. It is clear that the equivalent and axial plastic strains at the notch root are much greater than the axial plastic strain of the unnotched specimen for the same mean true strain. The radial strain at the notch root is also greater than the axial strain of the unnotched specimen.

The notch strength ratio, or the ratio of notch tensile strength to unnotch tensile strength, was 1.212 for this shape of notch. In previous papers⁽⁵⁾⁽⁶⁾, the author revealed that two principal factors are indispensable to notch strengthening. One of these two factors is that the axial strain near the notch root at $P_{\max}$ becomes much greater than the axial strain at $P_{\max}$ of the unnotched specimen. The present results show that the equivalent and axial plastic strains at the notch root at $P_{\max}$ are 0.80 and 0.779, respectively, and are about twice as large as the axial plastic strain at $P_{\max}$ of the unnotched specimen 0.399. This provides a quantitative proof of the above factor.

6. Conclusions

It is clarified that strain distributions of circum-

Fig. 7 Radial displacement distributions

(a) $0.8 P_{\max}$ (b) $P_{\max}$ (c) $0.95 P_{\max}$ beyond $P_{\max}$

Fig. 8 Strain distributions at the net section

ferentially notched cylindrical bars subjected to large deformation can be obtained using both the Vickers

Fig. 9 Variation of strains at the notch root and at the center of the net section with mean true strain

microhardness test and the numerical calculation. Constitutive equation is not used in calculating strain distributions. The strain distributions obtained are, therefore, approximately the same as the actual strain distributions.

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