A Simulative Experiment for Characterizing the Strength of Functionally Gradient Material*

(The Analysis for the Case of Radial Crushing Test)

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The effects of a gradient distribution of components on the deformation and fracture of functionally gradient materials (FGM) in the corundum/plaster model system were examined as a function of the volume fraction of corundum powder and the multiples of gravity. The radial crushing strength of the FGM was measured under the combined conditions for cast rings with three different volume fractions and three different multiples of gravity. The distribution patterns of stress and strain in FGM rings were analyzed by the conventional curved beam theory considering the change of elastic modulus. The brittle fracture was initiated from the inner surface of the ring on the load action line independent of the distribution of corundum powder. The variation of strength with the gradient distribution of corundum powder and with the centrifugal force was also studied.

Key Words : Bending, Elasticity, Brittle Fracture, Numerical Analysis, Centrifugal Casting, Functionally Gradient Material, Radial Crushing Strength

1. Introduction

A fabrication method of functionally gradient material (FGM)⁽¹⁾, which is a kind of non-homogeneous particulate-reinforced metal matrix composite, has been proposed by the authors. The driving force of the formation of the tailored distribution of ceramics powder is basically the difference in centrifugal force produced by the difference in density. The mixtures consisting of plaster assumed melt and corundum powder are chosen as the experimental model, and several types of corundum distribution profiles are made. The gradient of components is represented by a function of the centrifugal force and the mean volume fraction in a unified manner as reported⁽²⁾. The specimen is shaped as a thick-walled ring, and the experimental materials are brittle and are broken under an elastic deformation. Then, the

effect of gradient distribution on the strength may be evaluated by analyzing the experiment and theory of the ring crushing test. The stress-strain distribution fields are calculated by the conventional curved beam theory considering the distribution of the elastic modulus. The distribution of the elastic modulus is calculated by Wakashima's equation⁽³⁾ substituting the corresponding volume fraction of the position, and then is expressed by a function of the multiples of gravity and the mean volume fraction of powder. Furthermore, it is of great interest to examine the possibility of exchanging the fracture site depending on the gradient distribution of corundum powder.

In the present paper, the effects of gradient distribution on the strength are studied by the ring crushing test. The relation between the gradient distribution of components and the fracture strength is measured using FGM of the corundum/plaster model system. Numerical analysis is also applied to solve the stress and strain distribution fields. The usefulness of the gradient of the strength is discussed.

2. Procedure

The materials tested in the present study were

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mixtures of plaster (density = 1.7, $E \approx 35$ GPa) assumed melt and corundum (density = 3.1, $E \approx 360$ GPa) with # 60 grain size (235–265 μm). The mixtures were cast into rings using the self-made steel mould of a horizontal centrifugal casting. Ten types of corundum distribution profiles were made. These were pure plaster, as a standard, and the combination of three different mean volume fractions of corundum powder, $R=15, 25$, and 35 vol % and three different multiples of gravity, $G=2.1, 12.4$, and 80.0. The multiples of gravity, G , is the ratio of the centrifugal force to the gravity and is computed for the following equation:

$$G=2DN^2, \quad (1)$$

where D is the diameter of the cast ring [m] and N is the rate of revolution [s^{-1}]⁽⁴⁾. The ring specimens were 30 mm in height, 90 mm in diameter, and about 15 mm in thickness (measured values are 15 ± 0.5 mm).

The rings are sectioned for the representation of the distribution of the corundum powder by a mathematical model as a function of the G - and R -values. The variation of the corundum powder is measured with the ten regions divided in the thickness direction of a radial section of the ring using the photographs. The radial compressive tests are performed by a screw-driven testing machine at a constant crosshead speed of 0.1 mm/min. All specimens were dried completely by leaving the specimens in the laboratory for 3 months, then drying the specimens in an oven at 40°C for 24 hours prior to testing.

3. Analysis

3.1 A mathematical model of corundum distribution

The distribution curve of components is represented by the equation outlined in Ref. (2). The expression for distribution is a function of normalized distance with thickness, X , where $X=0.0$ and $X=1.0$ are the outer and inner surfaces of the ring, respectively:

$$f(X) = (a/3)X^3 - (a/2)X^2 + b, \quad (2)$$

where a and b are coefficients of distribution. The coefficient of the a -value is expressed by means of a simple function of G - and R -values which contains fewer coefficients and has good fitness with experiment. If the a -value is given, the coefficient of the b -value is calculated by solving

$$R = \int_0^{X_0} f(X) dX,$$

where X_0 is the solution of $f(X)=0$ if $f(1)<0$ and $X_0=1$ if $f(1)\geq 0$.

3.2 Distribution of elastic modulus

The FGM treated in this study is a kind of particulate-reinforced composite where the particles are distributed gradiently towards the radial direction.

Therefore, the distribution of the elastic modulus, $E(X)$, is easily obtained with Wakashima's equation⁽³⁾ approximating the Poisson's ratio $\nu=1/3$:

$$E(X) = (1-f)E_m + fE_p - f(1-f)(E_p - E_m)^2 / \{fE_m + (1-f)E_p + E_m/2\}, \quad (4)$$

where E_m is the elastic modulus of the matrix, E_p is that of the particle, and f is the volume ratio of particles at the position X calculated by Eq. (2), i.e., $f=f(X)/100$.

3.3 Stress and strain yielded in the ring

The distribution patterns of stress and strain in the ring specimens are analyzed numerically by the conventional curved beam theory. To ascertain the distribution of stress and strain within a curved beam subjected to a radial load, one must consider a number of fundamental assumptions⁽⁵⁾: (1) the beam is of constant cross section and the locus of the centroids is a plane curve, (2) the cross section of the beam has an axis of symmetry which lies in a plane curve, (3) applied load must be in the direction of an axis of symmetry plane cross section, and (4) the plane radial section remains a plane after loading.

For a curved beam with the radius of the neutral surface, r , the stress and strain at a distance of e from the neutral surface give

$$\varepsilon = (r\varepsilon_0 + e\omega_0)/(r+e) \quad (5)$$

$$\sigma = E(X) \cdot \varepsilon, \quad (6)$$

where ε_0 , longitudinal strain, and ω_0 , angular strain, are independent of e and the relation between e and X is

$$X = (t_0/2 - e)/t_0, \quad (7)$$

where t_0 is the initial thickness. Under the action of external forces, the forces at any cross section can be reduced to a longitudinal force, T , and a bending moment, M . The sum of all stresses over a cross section is equal to T , and the moment about the neutral axis is equal to M . The equations are

$$T = \int \sigma dA = \varepsilon_0 \int \{r/(r+e)\} E(X) dA + \omega_0 \int \{e/(r+e)\} E(X) dA \quad (8)$$

$$M = \int \sigma e dA = \varepsilon_0 \int \{re/(r+e)\} E(X) dA + \omega_0 \int \{e^2/(r+e)\} E(X) dA. \quad (9)$$

If T and M for any cross section are given, the two unknown quantities ε_0 and ω_0 in Eq. (5) are obtained by solving the simultaneous equations of Eqs. (8) and (9). The profiles of strain and stress distribution are calculated from Eqs. (5) and (6), respectively.

4. Results of Analysis

4.1 Establishment of gradient composition

The corundum distribution profiles are represented by a continuum function of normalized distance with

thickness. The measured distribution patterns are arranged as histograms. As examples, those for 35 vol% specimens are shown in Fig. 1 by a dotted line. Figure 1 expresses the relation between the volume fraction of powder and the position in the thickness direction normalized by the thickness; i.e., 0.0 is the outer surface and 1.0 is the inner surface of the ring. The distributions are arranged to be expressed uniformly by Eq.(2). The coefficient of a in Eq.(2) is expressed as functions of the G - and R -values. One of the simple functions is

$$a = m(\log G)^n, \quad (10)$$

where m and n are

$$m = 16.14 + 2.957 R \quad (11)$$

$$n = 2.469/R^{0.260}. \quad (12)$$

The calculated profiles are also shown by solid curves in Fig. 1 for comparison and are suitable for the histograms.

4.2 Gradient distribution of elastic modulus

The distributions of elastic modulus corresponding to the gradient composition expressed uniformly by mathematics are calculated from Eq.(4) and shown in Fig. 2. The elastic modulus of corundum is greater than that of plaster. Then, the elastic modulus of the outer surface of the ring for each G -value becomes greater in the order of $R=35, 25, 15$ vol% and the effects of gradient composition on the elastic modulus are obvious in this outer region. The characteristics of the elastic modulus variation have the same tendency as those of the powder distribution profiles shown in Fig. 1.

Fig. 1 Corundum distribution profiles of measured (histograms) and modeled (lines) as a function of normalized distance with thickness for $R=35$ vol% specimens

4.3 Stress and strain field yielded in the ring

The stress and strain using linear mechanics are analyzed under the action of a unit load. The results of three cases are shown in Fig. 3. Those are specimens of 15 vol% · 2.1 g, 25 vol% · 12.4 g, and 35 vol% · 80.0 g, for which the elastic modulus variation is the least, intermediate, and the largest, respectively. In Fig. 3, the x and y axes are the axes of symmetry and the y axis coincides with the load action line. The stresses are normalized with the maximum of an absolute stress, and tensile stress is acting in an inner region of the ring along the y axis and an outer region of the ring along the x axis. The maximum values of tensile stress and strain are always generated at the inner surface on the load action line. However, the positions of the maximum values of compressive stress and strain change depending on the elastic modulus distribution. The effects of the elastic modulus gradient are clearly observed on the change from linear stress distribution to nonlinear distribution having a point of inflection.

A brittle fracture is caused under the action of a tensile stress. Then, the maximum values of tensile stress and strain under unit load are normalized with those of a plaster specimen for the comparison of relative value, and those relations are plotted in Fig. 4. The results of $G=1$ for the case of homogeneous particulate-reinforced composites are also plotted in Fig. 4 as a reference. The maximum tensile strain decreases corresponding to the increase of a macroscopic elastic modulus under constant load. The R -value for constant G -value increases with decrease of stress and strain. The G -value for constant R -value

Fig. 2 Influence of corundum volume fractions and the multiples of gravity on the elastic modulus variations

Fig. 3 Distributions of strain and stress for radial compressive specimens determined by the curved beam theory. Tensile and compressive stress or strain regions are denoted by + and - marks, respectively

Fig. 4 Influence of the composition gradient on the maximum tensile strain and corresponding stress yielded under the fixed radial compressive load

increases with decrease of stress and increase of strain. Therefore, if the fracture obeys the strain criterion, the maximum fracture load may be observed for the specimen of 35 vol % · 2.1 g, and if the

fracture obeys the stress criterion, the maximum fracture load may be observed for the specimen of 35 vol % · 80.0 g.

5. Results of Ring Crushing Test

5.1 Crushing of ring specimen

More than 10 pieces for each combination of R - and G -value are tested. The fracture profile and crushing load are observed. Cracks are always initiated symmetrically from the inner surface on the load action line where maximum tensile strain is generated. Secondary cracks are observed along the outer surface of the horizontal cross section to the applied load, and the ring is divided into four parts. As one of the examples, Fig. 5 shows the cracks, indicated by arrows, in a specimen of 25 vol % · 80.0 g where white cracks whose length is about 75% their thickness propagate in the colored region. Against the expectation as mentioned in the introduction, cracks are initiated in the same manner even if the gradient of composition is larger and many corresponding defects seem to exist. The extreme is the case of the 35 vol % · 80.0 g specimen where the maximum volume fraction of the corundum at the outer surface is about 70 % and

the corresponding elastic modulus is the largest of all as shown in Fig. 2. As a result, the strain or stress value of the outer surface at the horizontal cross section is less than the critical values to initiate the fracture.

5.2 Results of crushing load

The relation between the ring crushing load and the fracture strain for each R -value is shown in Fig. 6. The broken, solid, dotted, and single-dotted straight lines indicate the results for plaster, 2.1 g, 12.4 g, and 80.0 g specimens of 15 mm thickness, respectively. As a result of the nonuniformity of the ring thickness, plotted points are not on but around these lines. The statistical evaluation of the ring crushing load uses the Weibull distributions because the fracture of brittle materials is assumed to be initiated from the weakest flaw in the specimen and the scatter of strength seems to be relatively large. The ranked values of crushing load for each distribution profile fall on a straight line on the Weibull probability paper and are then confirmed to be the two-parameter Weibull distribution. Two Weibull parameters, a shape parameter and a scale parameter, the mean, the variance, and the coefficient of variation are listed in Table 1. The smallest shape parameter ($\alpha=4.2$) was obtained in the case of the plaster specimen.

5.3 Effect of gradient composition on crushing strength

The profiles of distribution are controlled by G - and R -values in this study, and the effects of the gradient composition on the strength are evaluated using the fracture strain, ϵ_f , and fracture stress, σ_f , calculated from the mean crushing load, p_f . The fracture stress equals the fracture strain multiplied by the elastic modulus at the site as indicated by Eq. (6). Then the relation between the crushing load and the fracture strain is shown in Fig. 7. For constant R -values, both

Fig. 5 Development of the cracks (denoted by arrows) in the radial compressive test specimen for 25 vol % · 80.0 g

(a) 15 vol %

(b) 25 vol %

(c) 35 vol %

Fig. 6 Radial crushing load as a function of crack initiation strain

crushing load and fracture strain increase with increasing of the gradient of distribution, i.e., the G -value. The maximum values are $P_f=771$ N, $\varepsilon_f=0.0215\%$, and $\sigma_f=7.53$ MPa for the specimen of 25 vol % · 80.0 g. The minimum values are $P_f=395$ N, $\varepsilon_f=0.0071\%$, and $\sigma_f=3.62$ MPa for the specimen of 35 vol % · 2.1 g.

The effect of the R -value on the strength is discussed while we compare the results of the constant G -value. For the materials of $G=2.1$ which are nearly equal to homogeneous particulate-reinforced composite materials, the fracture strain is decreased but the crushing load remains constant as the R -value increases, and these results are comparable with the results of Fig. 4. The fracture strain seems to decrease in proportion to the amount of the internal flaw, which seems to originate from the interface between plaster and corundum, while the mean elastic modulus of the specimen is increased. Then it is explained that the crushing load is not varied apparently as a result.

For the case of $G=12.4$, the crushing load is independent of the R -value. In comparison with the case of $G=2.1$, the gradient of distribution is greater and the internal flaw near the inner surface decreases. Therefore, the fracture strain seems to relatively increase and a greater crushing load is obtained. It is also observed that the fracture strains for the case of $G=2.1$ and 12.4 are increased as the volume fractions of corundum at the inner surface of the ring, which are the value of $X=1.0$ in Fig. 2 and in proportion to the elastic modulus, are increased.

Corundum powder is not observed at the inner surface of the ring for the case of $G=80.0$, and the

structure is different from that in the case of $G=2.1$. The cracks for the $G=80.0$ specimens are also initiated from the inner surface of the ring as mentioned in section 5.1. The p_f , ε_f , and σ_f are larger in the order of $R=25, 15, 35$ specimens, and the tendency is different from that of $G=2.1$ and 12.4. The fracture strain and stress do not satisfy the simple forecast those the fracture stress and strain are coincident with that of the plaster specimen if the fracture in the plaster is controlled.

5.4 Evaluation of gradient composition on strength

The effect of the gradient composition on the function of strength is discussed using the relation between the fracture strain and the volume fraction of powder at the inner surface of the ring shown in Fig. 8. The solid line indicates the fracture criterion of the composition such as composite materials, and the dotted line corresponds to the fracture initiated from the 100 vol % plaster region. The fracture modes of both cases seem to be different from each other. In spite of the 15 vol % difference of the crushing load between 378 N of the 15 vol % · 2.1 g specimen and 436 N of the 35 vol % · 12.4 g one, both plot points along the solid line show nearly equal fracture strain on account of the gradient composition. According to these results, it seems to be possible to estimate the necessary gradient composition to endure the required crushing load if the required fracture strain is in the range shown by the solid line in Fig. 8.

The fracture criterion shown by the dotted line in Fig. 8 may depend on the stress or strain gradient in the ring. The tendency seems to coincide with the

Table 1 Summary of the Weibull parameters for the radial crushing tests

R	G	α	β	μ	v	η
0	—	4.21	434	394	11181	0.155
15	2.1	11.83	394	378	1456	0.102
15	12.4	6.04	510	473	8307	0.192
15	80.0	7.39	685	642	10510	0.159
25	2.1	9.77	411	391	2296	0.122
25	12.4	9.68	491	467	3332	0.124
25	80.0	17.62	794	771	2863	0.069
35	2.1	13.57	373	359	1033	0.089
35	12.4	10.16	458	436	2648	0.118
35	80.0	7.11	557	521	7451	0.166

R: vol % of powder, G: Multiples of gravity
 α : Shape parameter, β : Scale parameter
 μ : Mean load /N, v : Variance
 η : Coefficient of variation

Fig. 7 Effect of composition gradient on the mean crushing load and the mean fracture strain

Fig. 8 Effect of volume fraction of corundum at the crack initiated on the fracture strain

order of the shape parameter given in Table 1. It is reported that the larger shape parameter gives smaller effective volume and the smaller effective volume gives the larger fracture strength⁽⁶⁾. The dotted line seems to indicate, at the same time, that the function of strength does not always improve with the larger gradient of components. However, the minimum fracture strain of corundum/plaster FGM observed in this study is greater or equal to that of pure plaster specimen, and the function against radial compressive load seems to be improved with the introduction of a gradient composition.

6. Conclusions

The effects of the gradient composition on a function of the strength of a brittle material were tested and analyzed for the ring crushing test of plaster/corundum FGM model materials having nine kinds of gradient compositions which are the combination of three kinds of multiples of gravity and three kinds of mean volume fraction of corundum powder, and the results written below were obtained.

(1) The maximum tensile stress and strain are

always yielded at the inner surface of the ring on the load action line, and the positions of the maximum compression stress and strain vary depending on the gradient composition.

(2) The initial cracks always start from the inner surface of the ring on the load action line where the maximum tensile stress and strain are yielded, and the secondary cracks start from the outer surface of the ring normal to the load axis.

(3) The maximum ring crushing load, fracture stress, and fracture strain observed for the specimen of 25 vol % · 80.0 g were $P_f = 771$ N, $\epsilon_f = 0.0215\%$, and $\sigma_f = 7.53$ MPa, respectively, and the minimum values observed for the 35 vol % · 2.1 g specimen were $P_f = 359$ N, $\epsilon_f = 0.0071\%$.

(4) When corundum powder is observed in the region of crack initiation, the fracture strain has volume fraction dependence such that the smaller corundum volume fraction at the inner surface of the ring corresponds to the greater fracture strain.

(5) When there is no corundum powder at the inner surface of the ring, the mean volume fraction dependence of the fracture strain is not observed and the function of the ring crushing strength is increased to introduce the gradient of composition.

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