Determination of the Complex Shear Modulus of Damping Materials by the Composite Beam Test Method*

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This investigation presents a method for determining the complex shear modulus of damping materials from a torsional resonance test of a rectangular covered specimen with damping layer materials on bonded both sides of the elastic layer. In order to utilize this method, an expression of the torsional system is provided in this paper, which is based on an exact solution of torsional rigidity of the three-layered rod previously presented. The complex shear modulus for a damping rubber is determined by means of this method. The determined values are reliable up to the transition temperature range of the rubber, but they scatter above that temperature range. An error analysis for the covered specimen clarifies that the resultant errors of the complex shear modulus mainly depend on the accuracy of the elastic layer thickness and its measured shear modulus. The sandwich specimen is superior to the covered one due to the sufficient reduction of the resultant error over a wide range of damping materials.

Key Words: Material Testing, Sandwich Rod, Covered Rod, Determination, Viscoelastic Material, Torsional Test, Composite Beam Test Method, Complex Shear Modulus, Error Analysis

1. Introduction

The laminated damping steel sheet with a high polymer core is increasingly demanded in many industrial fields recently, especially in the domestic automobile industry^{(1),(2)}. The damping characteristic of such a steel sheet depends mainly on the complex shear modulus (complex representation of shear modulus and loss factor) of the core material. Thus its value must be determined accurately in order to perform the optimal laminated material design.

The inverse methods for determining the complex modulus of the damping material in the core or on the covered layers of laminated beams, by inversely utilizing the analytical results of the bending resonant vibration test, have been presented^{(3)~(6)}. In these methods (composite beam test method or coated beam method), the complex shear modulus and Young's modulus are determined approximately using the sandwich beam and the covered two- or three-

layered beam, respectively. These methods have been standardized to the DIN specifications⁽⁷⁾ and the ISO standards committee has specified them for the determination of the dynamic mechanical properties of damping materials, and they are also undergoing the last step of the specification.

However, both axial and shear stresses in the damping layer are produced in the bending test. Therefore, the exact theoretical relationship which determines either Young's modulus or the shear modulus cannot be obtained from the bending test alone. The basic analyses usually used for the sandwich beams derived by Kerwin⁽⁸⁾ and Unger⁽⁹⁾ assume that the axial force in the damping layer can be ignored, in comparison with the axial force in the elastic layer. Therefore, the limitation of these analyses applying to the inverse bending method should be thoroughly examined. However, discussions of the limitation have not been conducted so far, except for the experimental evaluation of extremely soft damping materials⁽¹⁰⁾ and a theoretical discussion of the limitation for the Kerwin's theory itself in the application of the two-dimensional theory⁽¹¹⁾ of elasticity.

With consideration of the above unsettled prob-

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lems in the bending inverse method, the theoretical results of the laminated rod in torsion, which had been precisely analyzed in the closed form⁽¹²⁾, should be applied to the inverse method, especially in the determination of the complex shear modulus.

This method has been applied previously to the determination of the complex shear modulus of the thin epoxy resin adhesive layer⁽¹³⁾, using the sandwich specimen. We apply it to the determination of the complex shear modulus of a damping material using the covered specimen shown in Fig. 1.

Error analysis is essential for reliable determination with the inverse method, and it is carried out for the covered and the sandwich specimens.

2. Theory of Determination of the Complex Shear Modulus

2.1 Torsional rigidity of the rectangular covered rod

The exact torsional rigidity of the covered rod shown in Fig.1, where each layer thickness is a_i , width b and shear modulus G_i , is given by⁽¹²⁾

$$C/G_2b^4 = \varepsilon_1 \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64t_1}{k_n^5} \right) + \left(\frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64t_2}{k_n^5} \right) + \varepsilon_3 \left(\frac{R_3}{3} - \sum_{n=0}^{\infty} \frac{64t_3}{k_n^5} \right) + \sum_{n=0}^{\infty} \frac{32Nv}{k_n^5 Dv}, \quad (1)$$

where

$$R_i = a_i/b, \quad t_i = \tanh(k_n R_i/2), \quad T_i = \tanh(k_n R_i),$$

$$k_n = (2n+1)\pi, \quad \varepsilon_1 = G_1/G_2, \quad \varepsilon_3 = G_3/G_2$$

$$Nv = 2\varepsilon_1 T_1(t_1+t_2)^2(\varepsilon_3 T_3+t_2) + 2\varepsilon_3 T_3(t_2+t_3)^2$$

$$\times (\varepsilon_1 T_1+t_2) - \varepsilon_1 \varepsilon_3 T_1 T_3(1-t_2^2)(t_1-t_3)^2$$

$$Dv = (1+\varepsilon_1 t_2 T_1)(\varepsilon_3 T_3+t_2)(1+\varepsilon_3 t_2 T_3)(\varepsilon_1 T_1+t_2).$$

If we take a symmetrical covered rod such as $R_1=R_3=R$ and $\varepsilon_1=\varepsilon_3=\varepsilon$, the torsional rigidity is simplified to

$$C/G_2b^4 = 2\varepsilon \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64t_1}{k_n^5} \right) + \left(\frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64t_2}{k_n^5} \right) + \varepsilon \sum_{n=0}^{\infty} \frac{64T_1(t_1+t_2)^2}{k_n^5(1+\varepsilon t_2 T_1)}. \quad (2)$$

Furthermore, since the value of $\varepsilon t_2 T_1$ in the third term of Eq.(2) can be ignored as an extremely small value in comparison to a unity, we obtain an approximate expression of Eq.(2) as

Fig. 1 Cross section of the symmetrical covered specimen

$$C/G_2b^4 = 2\varepsilon \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64t_1}{k_n^5} \right) + \left(\frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64t_2}{k_n^5} \right) + \varepsilon \sum_{n=0}^{\infty} \frac{64T_1(t_1+t_2)^2}{k_n^5} \quad (3)$$

A comparison between the calculated results of Eq.(2) and Eq.(3) is shown in Table 1. These calculated values coincide well under the condition of shear modulus ratio of $\varepsilon=10^{-2}$, although the approximate result takes a slightly higher value at $\varepsilon=10^{-1}$ than the exact one. We derive the basic equations of the determination from Eq.(3) in the following, since ordinary damping materials seem to have lower ratio than $\varepsilon=10^{-1}$, and also, the properties of rigid damping materials ($\varepsilon=10^{-1}$) can be easily determined by using the specimen made of itself.

2.2 Basic equations of determination

The torsional frequency equation of a rod with homogeneous and uniform cross section, which has masses with the polar moment of inertias (J_1 and J_2) at both free ends, is represented by⁽¹⁴⁾

$$(mn\beta^2-1)\tan\beta-(m+n)\beta=0, \quad (4)$$

where $m=J_1/lJ_b$, $n=J_2/lJ_b$, $\beta=\omega l\sqrt{J_b/C}$, l is rod length and J_b is polar moment of inertia per unit rod length.

For a low-frequency range, Eq.(4) is applicable to the covered rods with various end conditions, in terms of the torsional rigidity of Eq.(3) and the inertia moment ratios m and n . For example, the free-free and the fixed-fixed end conditions become $m=n=0$ and $m=n=\infty$, respectively.

In this paper, the cantilever specimen is used because of its easy treatment. The cantilever specimen has an inertia bar at its free end in order to excite electromagnetically and detect the resonance amplitude of the inertia bar as shown in Fig. 2 (a). Then, the basic equation of the determination is given from Eq.(4) as

$$\beta \tan \beta = J, \quad (5)$$

where $J=1/n=lJ_b/J_2$.

The approximate solutions of Eq.(5) are given as Eqs.(6), and a comparison between Eqs.(6) and the exact solution is shown in Table 2.

Table 1 Comparison between exact and approximate values for torsional rigidity of the covered rod (%) (Approximate /Exact -1) $\times 100\%$: $n=50$

		$\varepsilon = 10^{-1}$			$\varepsilon = 10^{-2}$		
R_2		0.05	0.1	0.2	0.05	0.1	0.2
R_1/R_2	0.1	0.01	0.24	0.45	—	0.01	0.04
	1.0	0.01	0.52	1.00	—	0.01	0.09
	5.0	0.02	1.19	1.54	—	0.01	0.12

$$\left. \begin{aligned} \beta_1 &= \sqrt{J(1-J/3)} \\ \beta_k &= (k-1)\pi + \frac{J(1-J/3)}{(k-1)\pi} \quad k=2,3,\dots \end{aligned} \right\} \quad (6)$$

The difference between the approximate and the exact values becomes small if the inertia ratio (J) is restricted to be $J < 0.5$. Then, the resonance angular frequencies are given, from Eq. (6), as

$$\omega_1^2 = \frac{CJ}{J_b l^2} \left(1 - \frac{J}{3}\right), \quad (7)$$

$$\omega_k^2 = \frac{C}{J_b l^2} \left\{ (k-1)\pi + \frac{J(1-J/3)}{(k-1)\pi} \right\}^2 \quad k=2,3,\dots \quad (8)$$

The loss factor η_c of the covered specimen (designated as the composite loss factor in the following) is given by the complex resonant angular frequency ω^* or the complex torsional rigidity C^* obtained by replacing the shear modulus G_1 of the covered layers with the complex one $G_1^* = G_1(1 + j\eta)$

Table 2 Comparison between approximate and exact values of Eq. (5) (%)
(1-Approximate /Exact) $\times 100\%$

k	J = 0.5	J = 0.4	J = 0.3	J = 0.2	J = 0.1
1	1.190	0.752	0.422	0.173	0.045
2	0.550	0.357	0.204	0.091	0.023
3	0.190	0.121	0.068	0.030	0.008

in Eq. (7) or Eq. (8). The imaginary part η in the parentheses is referred to as the loss factor of the damping material and j is the complex unit. Then, η_c is given as

$$\eta_c = \frac{\text{Im}(\omega_k^{*2})}{\text{Re}(\omega_k^{*2})} = \frac{\text{Im}(C^*)}{\text{Re}(C^*)}. \quad (9)$$

Consequently, the basic equations of the determination of G_1 and η of the damping material are given from Eqs. (7) ~ (9) as

$$G_1 = \left\{ \frac{J_b l^2 \omega_1^2}{b^4 J(1-J/3)} - G_2 S_2 \right\} \frac{1}{S_1}, \quad (10)$$

$$G_1 = \left\{ \frac{J_b l^2 \omega_k^2}{b^4 [(k-1)\pi + J(1-J/3)/(k-1)\pi]^2} - G_2 S_2 \right\} \frac{1}{S_1} \quad k=2,3,\dots, \quad (11)$$

$$\eta = \left(1 + \frac{G_2 S_2}{G_1 S_1}\right) \eta_c, \quad (12)$$

where

$$S_1 = 2 \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64 t_1}{k_n^5} \right) + \sum_{n=0}^{\infty} \frac{64 T_1 (t_1 + t_2)^2}{k_n^5}, \quad (13)$$

$$S_2 = \frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^5}. \quad (14)$$

In Eq. (10) (or Eq. (11) for the higher modes), since the values of shear modulus (G_2) of the elastic layer, other dimensions (R_1 , R_2 , b and l) of the specimen, the polar moment of inertia (J_b) and its ratio (J) are known previously, the values of G_1 and η are determined by the torsional resonant test utilizing Eq. (10) through Eq. (14).

3. Experimental Procedures and Results

3.1 Specimens

The specimens are prepared by bonding a commercially available damping rubber (Yokohama Gum Co. C-1, 1 mm³) to an elastic layer material (SUS 420: 160 mm² $\times$ 10 mm³, 1 mm³ and 0.5 mm³) by means of two-component epoxy resin adhesive. In order to control the total thickness of the specimens, they are secured with block gage settings between two ground steel plates, and cured for 24 hours at room temperature. The thickness of each layer is determined as the average value of ten points along the longitudinal direction before bonding. The upper parts of the damping rubbers are constrained by steel pieces which maintain the fixed end condition as shown in Fig. 2 (b), and the free end of the elastic layer is welded to the inertia bar.

3.2 Experimental setup

The experimental setup and the flow diagram is shown in Fig. 2 (a). The fixed end of the specimen is mounted to the support after clamping it with the supporting device, and the center of the inertia bar is slightly supported by a needle support. One end of the inertia bar is excited at a constant amplitude (100

(a) Experimental setup and flow diagram

(b) Detail of the clamping part

Fig. 2 Experimental setup

μm), and the decay curves at the resonant frequency of the other end detected by an eddy current pickup are memorized in the digital memory and stored the floppy disk of the computer. The composite loss factor is calculated from the logarithmic decrement of each decay curve at the various temperatures ranging from -20°C to 80°C .

3.3 Determined results

Figures 3 (a) and (b) show the resonant frequencies of the fundamental mode and the composite loss factors, and Fig. 4 shows the determined shear modulus and loss factor of the damping rubber based on the results of Fig. 3. The symbol V-E-V in these figures shows the thickness components of each specimen.

Both values of G_1 and η in Fig. 4 can be determined without scatter up to the glass transition temperature T_g of the damping rubber (T_g is about 20°C), but the determined values of G_1 scatter at the temperature range over T_g , and the values of η have a peculiar tendency in the elastic layer thickness of the specimen. The cause of the scatter is related to the resultant error accompanied by individual error components and also deformation of the adhesive layers in the high-temperature range.

The deformation of the adhesive layers can be presumed from the fact that the rate of decrease of the resonant frequency in Fig. 3(a) becomes extremely slow at the temperatures over T_g and the values of η in Fig. 4 rise again after having reached the peak values. Such a rise of η is caused by the lower value of G_1 due to the softening of the adhesive layers in the high-temperature range, that is, a lower value of G_1 results in a higher value of η , as described in the later error analysis. Although the effects of the adhesive layer appear to be avoided by means of utilizing a heat-proof adhesive or making the adhesive layer

thinner, such effects must be estimated more precisely based on the five-layer theory⁽¹⁵⁾ taking into account the adhesive layer deformation. Therefore, we next discuss the resultant error of the determined values and their components.

4. Error Analysis and Discussions

The accuracy of the determined values depends on various error factors because this method involves an inversion process. In order to obtain reliable results, an error analysis is carried out for both the covered and also sandwich specimens in the following section.

4.1 Error analysis of the covered specimen

From Eq.(10) and Eq.(12), values of both G_1 and η are represented by functions $f(\lambda_i)$ and $g(\mu_i)$ related to error factors λ_i and μ_i as

$$G_1 = f(R_1, R_2, G_2, \omega) = f(\lambda_i), \quad (15)$$

$$\eta = g(R_1, R_2, G_2, G_1, \eta_c) = g(\mu_i). \quad (16)$$

The error magnification factors p_i and q_i (abbreviated EM factors, and suffix i corresponds to each error factor in the order of Eq.(15) and Eq.(16)) with respect to G_1 and η , respectively, are given as

$$\frac{\Delta G_1}{G_1} = \left\{ \sum_{i=1}^4 \left(p_i \frac{\Delta \lambda_i}{\lambda_i} \right)^2 \right\}^{1/2}, \quad (17)$$

$$\frac{\Delta \eta}{\eta} = \left\{ \sum_{i=1}^5 \left(q_i \frac{\Delta \mu_i}{\mu_i} \right)^2 \right\}^{1/2}. \quad (18)$$

The EM factors p_i and q_i in Eq.(17) and Eq.(18) are given as

$$p_i = \frac{\lambda_i}{f(\lambda_i)} \cdot \frac{\partial f(\lambda_i)}{\partial \lambda_i}, \quad q_i = \frac{\mu_i}{g(\mu_i)} \cdot \frac{\partial g(\mu_i)}{\partial \mu_i}, \quad (19)$$

where

Fig. 3 Resonant frequency and composite loss factor of the covered specimen (Fundamental mode)

Fig. 4 Determined values of G_1 and η of the damping rubber

$$\left. \begin{aligned}
 p_1 &= -\frac{R_1 S_4}{S_1}, \quad p_2 = -\frac{R_2(\epsilon S_3 + S_5)}{\epsilon S_1}, \\
 p_3 &= -\frac{S_2}{\epsilon S_1}, \quad p_4 = 2\left(1 + \frac{S_2}{\epsilon S_1}\right), \\
 q_1 &= -\frac{R_1 S_2 S_4}{(\epsilon S_1 + S_2)}, \quad q_2 = \frac{R_2(S_1 S_5 - S_2 S_3)}{(\epsilon S_1 + S_2) S_1}, \\
 q_3 &= -q_4 = -\frac{S_2}{\epsilon S_1 + S_2}, \quad q_5 = 1, \\
 S_3 &= \sum_{n=0}^{\infty} \frac{64 t_1(t_1 + t_2)(1 - t_2^2)}{k_n^4}, \\
 S_4 &= \sum_{n=0}^{\infty} \frac{64 \{T_1 + t_2(1 - T_1^2)\}^2}{k_n^4}, \quad S_5 = \sum_{n=0}^{\infty} \frac{32 t_2^2}{k_n^4},
 \end{aligned} \right\} \quad (20)$$

and S_1 and S_2 are given in Eq.(13) and Eq.(14). We should take note of the minus sign of q_4 (EM factor of η with respect to G_1) in Eq.(20). This means that the error of G_1 is transformed to the error of η and the value of η becomes higher if the value of G_1 is determined to be lower, or vice versa.

4.2 Numerical results for the covered specimen

In order to calculate the resultant error of G_1 and η from Eq.(17) and Eq.(18), we must assume the average errors $\Delta\lambda_i/\lambda_i$ and $\Delta\mu_i/\mu_i$ of the error factors (R_1, R_2, G_2, ω and η_c), except for the transformed error $\Delta G_1/G_1$. These average errors are estimated empirically, as shown in Table 3, based on the measured values in this determining process.

The resultant errors of G_1 and η are shown in Fig. 5 for the covered specimens ($R_2=10^{-1}$ and $R_2=5 \times 10^{-2}$) applying the estimated values in Table 3. In the case of a relatively hard damping material ($\epsilon > 10^{-2}$; this case corresponds to the temperature range under

Table 3 Measured average error and its estimated value (%)

$\Delta R_1/R_1$	$\Delta R_2/R_2$	$\Delta G_2/G_2$	$\Delta \omega/\omega$	$\Delta \eta_c/\eta_c$
Measured Value / Estimated Value				
1.56/2.0	0.57/1.0	1.48/2.0	0.19/0.2	8.08/10.0

Fig. 5 Resultant errors of G_1 and η (Covered specimen)

15°C in Fig. 4), the resultant errors of both G_1 and η take low values (under 20%) for $R_1=10^{-1}$; then a stable determination can be expected. In the case of a relatively soft damping material ($\epsilon=10^{-3}$; this case corresponds to near 80°C in Fig. 4), both resultant errors take relatively low values (about 30%) for a specimen with a thin elastic layer ($R_2=5 \times 10^{-2}$), but take high values (over 100%) for a specimen with a thick elastic layer ($R_2=10^{-1}$).

These numerical results of the resultant errors approximately correspond to the scatter of the determined values in Fig. 4, except for increase in η after the attainment of peak values.

Subsequently, in the case of the specimen with a thin elastic layer ($R_2=5 \times 10^{-2}$), we discuss controlling factors of the error of the determined values. The calculated results for the EM factors with respect to each error factor are shown in Fig. 6.

Figure 6 shows that the resultant error of G_1 of an extremely soft damping material ($\epsilon=10^{-4}$) depends on the EM factors p_2, p_4 and p_3 (R_2, ω and G_2 , respectively). In order to reduce the resultant error, high

Fig. 6 Error magnification factors of G_1 and η (Covered specimen)

Fig. 7 Resultant error components (Covered specimen)

accuracy of the measured values with such high EM factors should be attained. However, each limitation of attainable accuracy for these values generally exists. If we consider the estimated values of these limitation as shown in Table 3, the resultant error components ($\bar{p}_i = p_i \Delta \lambda_i / \lambda_i$, $\bar{q}_i = q_i \Delta \mu_i / \mu_i$) and the resultant errors of G_1 and η can be calculated as shown in Fig. 7. From the soft damping material ($\varepsilon = 10^{-4}$, shown as solid lines in Fig. 7), the resultant error of G_1 is controlled mainly by $\bar{p}_2$ and $\bar{p}_3$, that is, the accuracy of the elastic layer thickness and its measured shear modulus, and does not depend strongly on the accuracy of the damping layer thickness or of the resonant frequency.

The resultant error of η is controlled by $\bar{q}_4$, that is, the transmitted error of G_1 . This indicates that resultant errors of both G_1 and η consequently depend on the accuracy of the thickness and the measured shear modulus of the elastic layer.

On the other hand, the resultant error components of G_1 for a relatively hard damping material ($\varepsilon = 10^{-2}$, shown as dotted lines in Fig. 7) show the same features as the above case when the damping layer is relatively thin, and also the resultant error is reduced markedly. But with the increase of the damping layer thickness, the resultant error of G_1 comes to depend on the error of the damping layer thickness ($\bar{p}_1$), and that of η depends on the error of the composite loss factor ($\bar{q}_5$).

From the above discussion of the covered rod specimen, the values of G_1 and η of relatively hard damping materials can be reliably determined. However, these values for the soft damping materials become difficult to obtain if the determination is carried out without careful attention to the accuracy of above mentioned controlling factors.

4.3 Error analysis for the sandwich specimen

We now carry out the error analysis for the symmetrical sandwich specimen in order to discuss the possibility of reliable determination of G_1 and η for soft damping materials. The sandwich specimen is of an inverse composition of the covered specimen in Fig. 1, and suffix i corresponds to each layer.

Determining equations for both G_2 and η of the sandwich specimen can be given for the fundamental mode⁽¹³⁾ as

$$G_2 = \left\{ \frac{J_b l^2 \omega_1^2}{b^4 J (1 - J/3)} - G_1 S_1 \right\} \frac{1}{S_2}, \quad (22)$$

$$\eta = \frac{S_1 + \varepsilon S_2}{\varepsilon S_3} \eta_c, \quad (23)$$

where

$$\left. \begin{aligned} S_1 &= 2 \left(\frac{R_1}{3} - \sum_{n=0}^{\infty} \frac{64 t_1}{k_n^5} \right) \\ S_2 &= \frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^5} + \sum_{n=0}^{\infty} \frac{64 T_1 (t_1 + t_2)^2}{k_n^5 (\varepsilon + t_2 T_1)} \\ S_3 &= \frac{R_2}{3} - \sum_{n=0}^{\infty} \frac{64 t_2}{k_n^5} + \sum_{n=0}^{\infty} \frac{64 t_2 T_1^2 (t_1 + t_2)^2}{k_n^5 (\varepsilon + t_2 T_1)^2} \end{aligned} \right\} \quad (24)$$

All variables in Eq. (24) are previously shown in Eq. (1), except for $\varepsilon = G_2/G_1$.

Values of both G_2 and η are represented as the functions related to each error factor as

$$G_2 = f(R_1, R_2, G_1, \omega), \quad (25)$$

$$\eta = g(R_1, R_2, G_1, G_2, \eta_c). \quad (26)$$

The resultant errors of G_2 and η can be calculated in the same manner as for the covered rod specimen if the estimated error values of each error factor is assumed. In this case, estimated values of $\Delta R_1/R_1 = 1\%$, $\Delta R_2/R_2 = 2\%$ and $\Delta G_1/G_1 = 2\%$ are assumed and other values are the same as shown in Table 3. Suffix i of the EM factors p_i and q_i follows the order of the error factors in Eq. (25) and Eq. (26), respectively.

Resultant errors of both G_2 and η , and their components ($\bar{p}_i$, $\bar{q}_i$) are shown in Fig. 8 and Fig. 9, respectively. In accordance with a decrease of the damping core layer thickness for the soft damping material ($\varepsilon = 10^{-4}$), both resultant errors are markedly reduced. From the resultant error components in Fig. 9, the resultant error of G_2 is found to be controlled by the accuracy of the outer elastic layer thicknesses

Fig. 8 Resultant errors of G_2 and η (Sandwich specimen)

Fig. 9 Resultant error components (Sandwich specimen)

($\bar{\rho}_1$) and the measured shear modulus ($\bar{\rho}_3$). Also, that of η is controlled by the transmitted error of G_2 ($\bar{q}_4$) in the case of soft core layer ($\varepsilon=10^{-4}$). Therefore, the resultant error of the determined values for the sandwich specimen mainly depends on the accuracy of the elastic layer properties as the same manner of the covered rod specimen, if the core layer is thin.

It is clear from the error analyses for the two types of specimens that the sandwich specimen provides much higher accuracy of the determined values for soft damping materials than does the covered specimen.

5. Conclusions

In this paper, a method for determining the complex shear modulus of damping materials (the composite beam test method) is proposed, by means of the torsional resonant test of a covered and sandwich specimen. The dynamic shear modulus and the loss factor of the damping rubber were determined by this method, applying the covered specimen with changing temperature. Reliable determined values are obtained up to the transition temperature range of the damping rubber, but errors of these values increase in the high-temperature range due to softening of the rubber.

From an error analysis of the covered rod specimen, it was found that these errors depend mainly on the measured accuracy of the thickness and the dynamic shear modulus of the elastic layer. These errors may be reduced by providing the measured values of the elastic layer with higher accuracy. However, as it is difficult in general to attain extremely high accuracy, then, application of the covered specimen in the determination inherently results in unreliable determination in the high-temperature range or for relatively soft damping materials. The error analysis for the sandwich specimen provides sufficiently reliable determination for relatively soft damping materials in comparison with the covered specimen.

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