Root and Groove Contact Analysis for Steam Turbine Blades*

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The reliability of the rotating blades is the most important factor in improving the reliability of steam turbines. The peak stress of the blade root and the disc groove which support the large centrifugal force of the blade should be analyzed accurately in order to assure the mechanical strength for both steady and vibratory stresses. The three-dimensional finite-element method is applied to the peak stress analysis considering slippage at the contact surface between the blade root and the disc groove. Nonlinear calculation is usually carried out to deal with the slippage effect. In this paper the appropriate methods to calculate the accurate local peak stress of the blade root and the disc groove are discussed, including nonlinear and linear analysis.

Key Words: Vibration of Rotating Body, Rotor, Contact Problem, Frictional Vibration, Numerical Analysis, Vibration

1. Introduction

The reliability of the rotating blades is the most important factor in improving the reliability of steam turbines. The peak stress of the blade root and disc groove which support the large centrifugal force of the blade should be analyzed accurately in order to assure the mechanical strength under severe operating conditions such as daily startup and shutdown operation of the recent fossil-fuel steam turbines.

The finite-element method using a three-dimensional model is applied to the stress analysis, taking into consideration the slippage at the contact surface between the blade root and the disc groove. Due to the

complicated shape of the blade root and the minute mesh size required for the peak stress calculation, a gigantic scale of modeling for the finite element method results. Although a supercomputer can deal with a gigantic model size, the maximum mesh number of the model and the convergence of the calculation should carefully studied.

Slippage of the contact surface between the root and groove should also be carefully analyzed. Nonlinear calculation is usually carried out to deal with the slippage effect. In this paper, linear calculation is also discussed, comparing with the results of nonlinear calculation.

As an example verifying the adequacy of the calculation method, a comparison between the calculated results of the root peak stress and measured stresses in the rotating tests for the actual blades is presented.

Concerning the blade root type, a side entry root in which the blades are inserted from the side surface of the disc shall be discussed in this paper.

2. Method of Contact Analysis

An explanation shall be given for an approach of

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nonlinear analysis to deal with the contact condition with friction.

2.1 Nonlinear analysis for contact condition

A problem in which the relationship between load and displacement is nonlinear, is usually calculated by means of the load incremental method. Namely, dividing load up to the final value into many increments, convergent calculations is achieved by means of Newton's methods so as to obtain the appropriate equilibrium of forces in each increment.

2.2 Definition of contact surface

In this work, we use an interface element to simulate behaviors of the contact surface with friction, on the assumption that relative slide displacements between two contact surfaces may be very small. Figure 1 shows this element, by which the contact pressure, shearing stress and relative displacement between two contact points may be presented in local coordinates, and an assumption such that the combination of the nodal points is unchanged and the local coordinate system is not rotated by mutual motion shall be applied.

For the relationship between relative motion of two surfaces and a force to be transmitted, two types of contacts, i.e., hard contact and soft contact, may be considered. In the case of hard contact, no special technique is required. On the contrary, in the case of soft contact, a deformable layer is required at the surface, of which the rigidity is rapidly increased after contact of the surfaces. The forces in three directions may be defined by applying Coulomb's friction law on

Fig. 1 Local coordinate system of three-dimensional interface element

Fig. 2 Relationship between hard pressure and amount of clearance

contact surfaces.

2.3 Relationship between contact pressure and clearance

In the case of hard contact, the value of pressure is positive and presents no limitation for the value. However, in the case where the gap between two surfaces becomes a positive value, noncontact shall be defined as shown in Fig. 2. For soft contact, the contact pressure shall be exponentially varied with the gap, as shown in Fig. 3. This relationship is determined according to numerical experiment and can simulate well the behavior of actual contact. Contact pressure P is defined as

$$\begin{cases} P=0 & (h>0) \\ P=\bar{K}h & (h\leq 0) \end{cases} \quad (1)$$

$\bar{K}$ = infinity value (negative)

$$\begin{cases} P=0 & (h>c) \\ P=\frac{P_0}{(\exp(1)-1)} \times \left[\left(1-\frac{h}{c}\right) \left(\exp\left(1-\frac{h}{c}\right)-1\right) \right] & (-6c < h < c) \\ P=\frac{P_0}{(\exp(1)-1)} \times \left[7 \exp(7) - 1 - \left(6+\frac{h}{c}\right)(8 \exp(7)-1) \right] & (h < -6c) \end{cases} \quad (2)$$

where

h ; amount of gap between two contact surfaces

C ; a value defined to be smaller than the actual clearance

P_0 ; an approximation estimated from mean pressure of the surface due to the force loaded.

2.4 Consideration for friction

A standard Coulomb's friction shall be taken as the friction, and the relative displacement and stress of the contact surfaces shall be defined as follows.

γ_1, γ_2 : Relative displacement in two orthogonal directions on the sliding surface

P_3 : Normal pressure

Q_1, Q_2 : Shearing stress in two orthogonal directions on the sliding surface

When calculated by means of the load incremental method, this relative displacement shall be taken as the increment.

Fig. 3 Relationship between soft pressure and amount of clearance

Thus, the increasing rate of the relative displacement in the sliding direction shall be represented by the following equation.

$$d\gamma_a = d\gamma_a^{el} + d\gamma_a^{sl} \quad (3)$$

$$a=1, 2$$

γ^{el} : Elastic displacement

γ^{sl} : Sliding displacement

Each increment shall be represented by

$$\Delta\gamma_a = \Delta\gamma_a^{el} + \Delta\gamma_a^{sl} \quad (4)$$

The elastic component shall be treated as linear elastic :

$$Q = G\gamma_a^{el}, \gamma_a^{el} = \sum \Delta\gamma_a^{el} \quad (5)$$

where G is a constant called adhering rigidity. Therefore the relative displacement γ_a^{el} before the generation of slide is dependent on value of G . The larger the value of G , the smaller the relative displacement γ_a^{el} becomes. Also, in this case solution approaches the actual behavior, but convergence of the solution is more difficult.

On the contrary, although convergency becomes better with smaller value of G , a large relative displacement arises before the generation of slide, resulting in significant difference from the actual behavior. Therefore, in the analysis, a method to obtain the value of G in advance by dividing the estimated value of the critical friction force (estimated value of contact pressure $\times$ coefficient of friction) with the allowable value relative displacement before generation of slide, is adopted. The condition of slide shall be in accordance with the following equations.

$$f(P_3, Q_a) = (Q_a, Q_a)^{1/2} - \bar{Q}^{sl} \quad (6)$$

$$\bar{Q}^{sl} = \mu P_3$$

μ : Coefficient of friction

If there is no slide, in other words, when $f \leq 0$ is obtained in the last increment, the following results.

$$\Delta\gamma^{sl} = 0, \Delta Q = G\Delta\gamma_a \quad (7)$$

If slide arises, namely, when $f > 0$ is obtained in the last increment, the following rule of flow and sliding limit condition shall be followed.

$$\Delta\gamma_a^{sl} = \Delta\gamma^{sl} Q_a / Q^{sl} \quad (8)$$

$$(Q_a \cdot Q_a)^{1/2} - \bar{Q}^{sl} = 0$$

Here, $\Delta\gamma^{sl}$ is an increment of sliding displacement, and Q is deduced as follows from Eq. (5).

$$Q = G(\gamma_a - \gamma_a^{sl}) = G(\gamma_a - \gamma_a^{sl}|_0 - \Delta\gamma_a^{sl}) = Q_a^{sl} - G\Delta\gamma^{sl} \quad (9)$$

Here, $\gamma^{sl}|_0$ is the sliding displacement γ_a^{sl} at the first increment, and Q_a^{sl} is a shearing stress Q at the last increment when slide has not arisen in that increment. The following equation is obtained from Eqs. (8) and (9):

$$Q_a = \bar{Q}^{sl} \cdot Q^{el} / \bar{Q}^{el} \quad (10)$$

where $\bar{Q}^{el} = (Q_a^{el} \cdot Q_a^{el})^{1/2}$.

Equation (10) is the same as in the radial return theory which is commonly used, and it represents Q_a

as being proportionally scaled on the curved surface of the sliding limit, geometrically.

3. Study on Linearization

The analyses of the blade root and the disc groove are contact problems, and treatment of the coefficient of friction intervening between the root and the groove is significant. On the other hand, if the blade root and the disc groove are considered as a continuous structure, the blade root may be regarded as an elastic body continuously supported by the elastic teeth of the disc groove. In studying to what degree this contact problem may be linearized, a basic study had been achieved by a two-dimensional FEM model before using a large-sized model. Then a verification was accomplished after studying the coefficient of friction using a three-dimensional model. In linear analysis, how to treat the elastic coefficient of the disc groove teeth, i.e., the spring constant, must be solved.

3.1 Study on two-dimensional model

As the first step, a two-dimensional model of a blade root having 3 serrations was selected, and a vertical load was applied at the center. In this case, a spring coefficient equivalent to the disc groove teeth was calculated by the following three methods for the purpose of comparison.

(1) Applying load with a load distribution specified in advance (usual distribution of three serrations is 4 : 3 : 3, stepwise from the top position downward) to the disc groove, the spring coefficient is determined from the resulting deformation (Case 1 spring).

(2) Applying a load separately by each tooth, the spring coefficient is obtained from the influence coefficients method. In this case, the assumption that the proportion of the load distribution of each tooth is the same shall be applied (Case 2 spring).

(3) Instead of a spring equivalent to the disc groove teeth supporting the blade root, the support shall be provided by the disc groove model (linearly integrated model).

Comparing the supports derived from the three methods above, the contact condition was selected for the two cases in which the friction coefficients μ were 0.0 and 0.5. The results are shown in Fig. 4. This figure shows that the spring supporting the blade root and the friction coefficient have little influence on the solution. The reason is that the load is applied vertically, no slide is generated on any tooth, and no significant difference can be found in the supporting condition.

As the second step, the solution was compared upon applying the offset loads as shown in Fig. 5 to the same model. The results are as shown in Fig. 6. A

nonlinear solution with the friction coefficient of $\mu = 0.5$, where no relative slide is generated, is almost in agreement with the linear solution obtained by the elastic support equivalent to the blade root and disc groove. However, in the case of $\mu = 0.0$, a significant slide inevitably arises, and stress distributions are quite different from the others.

3.2 Study on three-dimensional solid model

Blade roots of turbine blades are curved three-dimensionally, and in general, a load applied from a blade profile to a blade root is not uniform. Therefore, the analysis must be performed using a three-dimensional solid model. Three-dimensional solid models are shown in Fig. 7. Each model has approximately 3 500 nodal points, resulting in a large-scale analysis with more than 7 000 nodal points in total. Applying the linear analysis and coefficient between the blade root and the disc groove which are included in the method shown for the case of a two-dimensional model, the change of solution due to the value of μ was examined. A large friction coefficient results in

Fig. 4 Analytical condition of blade root analysis and displacement of blade serrations of bearing surface by uniform loads

Fig. 5 Blade root analysis by eccentric loading

worse convergence of the solution, the calculation being very difficult to achieve due to problems such as calculation execution time and computer capacity. However, utilization of a supercomputer has overcome these problems.

A load exerted by rotational centrifugal force is applied to the analysis. Figure 8 shows the result of this analysis. With progressive increase of the friction coefficient from the level of $\mu = 0$, interactive slide in gradually reduced and the convergence of the solution

Fig. 6 Results of blade root analysis by eccentric loading

Fig. 7 Three-dimensional solid model for blade root and disc groove

becomes apparent. The solution is stabilized in the range of general static friction coefficient $\mu=0.3\sim0.5$, resulting in approximate agreement with the linear solution. Figure 9 shows the result of relative displacement of the blade root and the disc groove. The fact that the relative displacement disappears at $\mu=0.3$ indicates that slippage is not generated. An examination of the extent of agreement between the calculated values and actually measured values shows that the actually measured values are approximately in agreement with those from the linear solution. This means that large slide is not generated between the blade root and the disc groove, even if the blade is long, by only rotational centrifugal force.

As a result of the above studies, the friction coefficient μ of about 0.3 shall be adopted in the nonlinear analysis. For the purpose of simplifying the analysis, it can be said that a sufficiently accurate solution may be obtained by linear analysis.

Furthermore, treatments are commonly applied to static analysis and vibration analysis. For a vibra-

Fig. 8 Analyzing method and blade root peak stress by three-dimensional FEM model

Fig. 9 Relative displacement of blade root and disc groove by centrifugal force

tion analysis, a vibratory bending load against the vibration mode to be obtained is added to the tight contact condition state formed by centrifugal force between the blade root and the disc groove.

4. Study on the Analysis Method and Modeling

There are various types of turbine blades, i.e., generally classified, a free-standing blade of which each blade is separated from others, an integral shroud blade in which movement of each blade is interlocked by the reaction force of a shroud attached at the blade tip caused by twist back of the blade, and a grouped blade in which each blade is connected with lashing wire. In the free-standing blade and integral shroud blade, the load condition of each blade is similar against the centrifugal load. In the grouped blades, however, the deformation caused by centrifugal force is quite different between the leading blade and trailing blade, as shown in Fig. 10, and the load condition of the blade root is also different. In the analysis of the disc groove, since the load conditions for the both concave and convex side teeth are different for each blade, the reaction forces from blade roots on both sides are required. A zooming method is adopted, applying the reaction force at the base of the blade profile which is obtained by analyzing the grouped blades, as shown in Fig. 11. This reaction force is added as an external load to the three-dimensional solid model of the blade root and disc groove. This analysis model may be adapted to both

Fig. 10 Blade deformation by centrifugal force

Fig. 11 Nonlinear and linear analyses flow for three-dimensional blade root and disc groove

linear and nonlinear analysis.

In the disc groove, the modeling shown in Fig. 12 is applicable. In the case of vibration analysis, vibrational reaction force is applied to this base section.

Next, the mesh size of the FEM modeling is discussed. Figure 13 shows the relationship between the number of divisions and the peak stress, using a two-dimensional model for simplification.

Fig. 12 Linear analysis flow for three-dimensional blade root and disc groove

Fig. 13 Mesh division and peak stress

The same blade root model as mentioned above is applied with a tensile load. In order to obtain the peak stress, minute divisions are required. The three-dimensional solid models used usually have 3 000 to 5 000 nodal points, and the sectional divisions are approximately 200 to 300. For the verification of the calculation using a three-dimensional solid model, the sectional divisions were increased to 400 with 10 000 nodal points in total, and the calculated result is shown in Fig. 13.

The result shows that the sectional divisions of 200 to 300 may provide satisfactory values.

5. Verification Test

A rotating test has been carried out to confirm the adequacy of the analysis. Figure 14 shows an outline of the test facility. Microgages are attached at the blade root tooth corner to measure the peak stress. The turbine disc is excessively large with long blades, and the centrifugal force applied to the blade root is as large as far over 10^6 N. Figure 15 shows the result of centrifugal stress analysis with the result of measurement in the test. Good agreement is obtained, regardless of a slight scatter in the measured values.

Fig. 14 Rotary vibrational test turbine blade

Fig. 15 Verification of peak stresses

Fig. 16 Verification of vibration stress

Fig. 17 Study of stress reduction by thinning

Figure 16 shows the measured vibration stress. The calculated vibration stress also shows a good correspondence with the measured value.

6. Example of Application of Nonlinear Analysis

As previously mentioned, a sufficiently accurate solution for practical use may be obtained by means of a linear analysis of blade roots and grooves. However, examples strictly requiring application of a nonlinear solution shall be presented. Figure 17 shows an example.

"Thinning" referred to herein is a technique to reduce peak stress by lessening the load caused by a centrifugal force in such manner that the bearing surface to be subjected to a high stress level at the edge of the root is thinly scraped off. An initial gap is provided when a load is added, at the initial start of

rotation, and with the increase of load, namely increase of rotating speed, contact begins to occur. The analysis of thinning shows that thinning markedly reduces the peak stress at the blade root, but stress at the opposite disc groove side is increased at the end of the thinning. Consequently, practical adoption of this method shall be decided taking into consideration the balance between both sides.

Nonlinear analysis is also necessary for discussing the extent of the dispersion of stress caused by the manufacturing tolerance of the blade roots and the disc grooves.

7. Conclusions

Studies of analyzing methods of the peak stress of the blade root and the disc groove in steam turbines have been summarized as follows.

(1) No large slippage due to the centrifugal force at the contact surface between the blade root and the disc groove of long rotating blades is observed with the friction coefficient of around $\mu=0.3$. Therefore, accurate results can be obtained by linear analysis, considering the appropriate elastic support of the teeth of the roots and grooves. The adequacy of calculated results has been verified by values measured in the tests.

(2) It has been confirmed that the three-dimensional finite element method with 3 000~5 000 nodal points yields the stress distribution, including the peak of both blade roots and disc grooves, with considerable accuracy.

In executing the calculations, the generalized program ABAQUS⁽¹⁾ was used for nonlinear analysis and the generalized program NASTRAN⁽²⁾ was used for linear analysis. The supercomputer CRAY Y-MP was used for the large-scale calculations.

References

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