Active Cooling and Thermal Stress Reduction by Use of Porous Materials* (Hollow Cylinder Model)

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By using the thermoporoelasticity theory proposed previously by the author, thermal stresses induced in a fluid-saturated porous hollow cylinder whose inner surface is heated by burning gas, and the wall of which is cooled by the fluid injection from its outer surface, are analyzed. Two situations of loading conditions are considered: (A) the cylinder is subjected to a sudden rise in temperature of the gas and is simultaneously pressurized at its outer surface to cool the wall, and (B) steady-state cooling is abruptly disturbed by a sudden loss of pressurization, and after a while it is recovered. The main focus is placed on the effect of heat advection due to active fluid pressurization and injection on the reduction of temperature and thermal stresses. Since the formulated problem is an axisymmetric one, the displacement field is decoupled from the temperature and pore-pressure fields which are still coupled to each other. The coupled nonlinear diffusion equations are solved by the implicit Crank-Nicolson method. It has been shown that the active fluid pressurization and injection is effective in suppressing the maximum thermal hoop stress at the outer surface without the occurrence of the excess compressional stress at the inner surface.

Key Words: Elasticity, Thermal Stress, Porous Material, Heat Advection, Active Cooling, Fluid Injection, Hollow Cylinder, Thermal Stress Reduction, Coupling, Nonlinearity

1. Introduction

Reduction of thermal stresses by some means may have a great hold on success in the development and design of structures under extremely severe thermal conditions. As one of the thermal stress reduction measures, great attention is recently paid to the so-called functionally gradient materials (FGM), while another method of more active control of heat flux and temperature by means of heat advection has been proposed and closely studied⁽¹⁾ regarding thermal protection of, for example, rocket nozzles, reentry vehicle noses, and gas turbine blades.

The basic idea of the present paper is to reduce thermal stresses by active control of heat transfer or thermal insulation by heat advection, that is, heat transportation by pore-fluid flow through a permeable porous material, rather than by the conventional passive thermal insulation through the use of impermeable porous material as well as the FGM. Previous

work regarding the heat advection was mainly concerned with the heat transfer between a porous wall surface and mainstream gas and paid only a little attention to the heat conduction within the porous materials; No work has been done on thermal stress reduction by heat advection.

Regarding hydraulic fracturing and water flooding as an oil and gas technology, we previously proposed a thermal stress theory of fluid-saturated porous elastic materials, which took into account the heat transportation by pore-fluid flow through permeable porous materials (heat advection)⁽²⁾. Here, on the basis of that theory, we have analyzed the thermal stresses within the wall of a porous elastic hollow cylinder in order to elucidate the influence of fluid pressurization and injection on the thermal stress reduction. Considered here is a case of cooling the wall, which is heated by burning gas in the cylinder bore, by injecting fluid into the permeable porous wall from its outer surface. Two situations of loading conditions are given as follows: (A) the cylinder is subjected to a sudden rise in temperature of the gas and is simultaneously pressurized at its outer surface to cool the wall, and (B) steady state cooling is

* Received 10th June, 1992. Paper No. 91-1048 A

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abruptly disturbed by a sudden loss of pressurization, and after a while it is recovered.

Since the formulated problem is an axisymmetric one, the displacement field is decoupled from the temperature and pore fluid pressure fields which are still coupled to each other. It may be difficult to analytically solve these coupled equations because the temperature equation contains a nonlinear term associated with the heat advection and, furthermore, the pressure equation has integral terms of the temperature and pressure. Thus, we have adopted the implicit Crank-Nicolson method to solve them. In addition to the heat advection, we have taken into account the effect of the blowing rate at the inner surface on the coefficient of surface heat transfer (boundary conductance). For the sake of simplicity, we have neglected the radiation from the elevated temperature gas, although it may play an important role in such cases as the above-mentioned examples.

2. Governing Equations

Let us cite governing equations of thermoporoelasticity for the case of the irrotational displacement field⁽²⁾. Modified Duhamel-Neumann equations for fluid-saturated porous thermoelastic media are as follows:

$$\sigma_{ij} = 2G \left[\varepsilon_{ij} + \frac{\nu}{1-2\nu} \varepsilon_{kk} \delta_{ij} \right] - \frac{3(\nu_u - \nu)}{B(1+\nu_u)(1-2\nu)} p \delta_{ij} - \frac{2G\alpha(1+\nu)}{1-2\nu} \Delta\theta \delta_{ij}, \quad (1)$$

where σ_{ij} and ε_{ij} are total stress and average strain tensors, respectively; p and $\Delta\theta$ are pore fluid pressure and temperature changes from their reference ones, respectively. It should be added that we use the same temperature for the matrix solid and pore fluid, because local heat exchange between both the components may be rapid enough in comparison with global heat and fluid diffusions. Shear modulus, Poisson's ratios for the drained and undrained states⁽³⁾, Skempton's constant, and the linear thermal expansion coefficient of the matrix are denoted by G , ν , ν_u , B , and α , respectively. The symbol δ_{ij} is the Kronecker delta.

For the thermoporoelastic media, the modified Navier equation is given by

$$u_{i,jj} + \frac{1}{1-2\nu} u_{j,ji} - \frac{3(\nu_u - \nu)}{GB(1+\nu_u)(1-2\nu)} p_{,i} - \frac{2\alpha(1+\nu)}{1-2\nu} \Delta\theta_{,i} = 0, \quad (2)$$

where u_i is an average displacement vector of the porous matrix, and is related to the strain tensor by the following equation:

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2. \quad (3)$$

The diffusion equations governing the temperature and pore fluid pressure in the fluid-filled porous medium are given by

$$\frac{\partial \Delta\theta}{\partial t} = C_0 \Delta\theta_{,jj} + \frac{\kappa}{v_0} (\Delta\theta p_{,j})_{,j}, \quad (4)$$

$$\frac{\partial p}{\partial t} = C p_{,jj} + C' \frac{\partial \Delta\theta}{\partial t} - C'' \frac{d}{dt} g(t), \quad (5)$$

where C_0 and C are thermal and fluid diffusivities. Permeability of the pore fluid and porosity are denoted by κ and v_0 . Constants C_0 , C , C' , and C'' are given by

$$C_0 = \frac{\bar{\kappa}}{\rho^T C^T}, \quad (6)$$

$$C = \frac{2\kappa GB^2(1+\nu_u)^2(1-\nu)}{9(1-\nu_u)(\nu_u-\nu)}, \quad (7)$$

$$C' = \frac{2GB^2(1+\nu_u)^2(1-\nu)}{9(\nu_u-\nu)(1-\nu_u)} \left[\frac{6\alpha(\nu_u-\nu)}{B(1-\nu)(1+\nu_u)} + v_0(\bar{\alpha}-3\alpha) \right], \quad (8)$$

$$C'' = \frac{2GB(1-\nu)(1+\nu_u)}{3(1-2\nu)(1-\nu_u)}, \quad (9)$$

where $\bar{\kappa}$, ρ^T , and C^T are the thermal conductivity, mass density, and specific heat of the fluid-saturated porous medium; $\bar{\alpha}$ is the thermal expansion coefficient of the pore fluid. The function $g(t)$ appearing in Eq. (5) is related to the volume strain, pore fluid pressure change, and temperature change by

$$\varepsilon_{ii} = \frac{3(\nu_u - \nu)}{2GB(1-\nu)(1+\nu_u)} p + \frac{\alpha(1+\nu)}{1-\nu} \Delta\theta + g(t). \quad (10)$$

3. Problem Formulation and Boundary Conductance

Let us consider a fluid-saturated poroelastic hollow cylinder with inner radius a and outer radius b , whose inner and outer surfaces are completely permeable. We adopt the cylindrical polar coordinate system (r, θ, z) , taking the cylinder axis as the z -coordinate.

We consider the case of cooling of the cylinder wall, which is heated by high temperature gas, by pressurizing and injecting fluid from its outer surface. Two situations of loading conditions are assumed: (A) the cylinder is subjected to a sudden rise in temperature of the gas and simultaneously pressurized at its outer surface to cool the wall and suppress the thermal stresses induced in it, and (B) steady-state cooling is abruptly disturbed by a sudden loss of pressurization, and its recovery is made after a while (e.g., t_0 minutes) and starts again to cool the wall and suppress the thermal stresses. For both these cases, mechanical boundary conditions are given by

$$\sigma_r = 0, p = 0 \quad \text{at} \quad r = a, \quad (11)$$

$$\sigma_r = -p_0 f_m(t), p = p_0 f_m(t) \quad \text{at} \quad r = b, \quad (12)$$

with p_0 being a reference pressure rise. The function

$f_m(t)$ presents the mechanical loading history, which will be given later for each case. Thermal boundary conditions are written as

$$\partial \Delta \theta / \partial r = h \cdot \{ \Delta \theta - \Delta \theta_0 f_i(t) \} \quad \text{at } r = a, \quad (13)$$

$$\Delta \theta = 0 \quad \text{at } r = b, \quad (14)$$

with $\Delta \theta_0$ denoting a reference rise in gas temperature. The function $f_i(t)$ expresses a thermal loading history, also given later. The symbol h denotes the relative coefficient of surface heat transfer (the relative boundary conductance), which depends on the blowing velocity, and consequently on the pore fluid pressure gradient at the inner surface. This dependence will be discussed later in this section. For cases (A) and (B), the loading history functions $f_m(t)$ and $f_i(t)$ are assumed to be given by

$$(A) \quad f_m(t) = H(t), f_i(t) = H(t), \quad (15)$$

$$(B) \quad f_m(t) = H(-t) + H(t - t_0), f_i(t) = 1, \quad (16)$$

where $H(t)$ represents the Heaviside unit step function. For the sake of simplicity, we have assumed that the temperature at the outer surface is equal to that of the injected fluid for both cases.

Now, we shall discuss the dependence of h on the blowing velocity V_w . For the case of turbulent boundary layers on a flat plate having a uniform blowing rate, the relative heat transfer coefficient, independent of the Reynolds number, is approximated by⁽³⁾

$$\frac{h}{h_0} = \exp \left\{ -\frac{3}{5} \frac{1}{St_0} \frac{\rho_w V_w}{\rho_\infty u_\infty} \right\}, \quad (17)$$

where h_0 and St_0 are the relative heat transfer coefficient and the Stanton number where there is no blowingout, respectively. The quantities $\rho_w V_w$ and $\rho_\infty u_\infty$ are the blowing and mainstream mass fluxes. Here, assuming the pore fluid is the same as the injected fluid and that it has a very small compressibility, we have $\rho_w = \rho_\infty = \rho_0$. In addition, we assume for simplicity that h_0 contains all effects of various complex phenomena such as the contribution of the latent heat associated with evaporation of the blown-out fluid to the heat transfer. Furthermore, we consider that Eq.(17) is valid for the curved inner surface of the cylinder, although it is an evaluation for a flat plate. Denoting the specific heat at constant pressure of the pore fluid by C_p , the Stanton number where there is no blowingout is $St_0 = (\chi h_0) / (\rho_0 u_\infty C_p)$. Let us define the Biot number for such a case as $Bi = a h_0$. Taking it into account that the blowing velocity is given by

$$V_w = \chi \partial p / \partial r, \quad (18)$$

we can rewrite Eq.(17) into

$$\frac{h}{h_0} = \exp \left\{ -\gamma \frac{\partial(p/\sigma_0)}{\partial(r/a)} \right\}, \quad (19)$$

where σ_0 (a reference thermal stress) and γ are expressed in the form

$$\sigma_0 = \frac{2G(1+\nu)\alpha\Delta\theta_0}{(1-\nu)}, \quad (20)$$

$$\gamma = \frac{3}{5} \frac{1}{Bi} \frac{\sigma_0 \rho_0 C_p \chi}{\bar{\kappa}}. \quad (21)$$

With the aid of Eqs.(6) and (7), this last equation can be rewritten as

$$\gamma = \frac{3}{5} \frac{1}{Bi} \frac{\sigma_0}{G} \frac{9(1-\nu_u)(\nu_u-\nu)\rho_0 C_p C}{2B^2(1+\nu_u)^2(1-\nu)\rho^T C^T C_0}. \quad (22)$$

4. Solution

If the cylinder under consideration is long enough, the only nonzero component of the displacement vector is $u_r = u(r, t)$; the displacement field is irrotational. Thus, the governing equations shown in section 2 are valid for this problem formulated in the previous section. First, expressing the modified Navier equation in terms of the cylindrical polar coordinates and integrating twice the resulting equation with respect to r yields

$$u = C_1(t)r + C_2(t)\frac{1}{r} + \frac{\lambda}{2G} \frac{1}{r} \int_a^r \xi p(\xi) d\xi + \frac{(1+\nu)\alpha}{1-\nu} \frac{1}{r} \int_a^r \xi \Delta \theta(\xi) d\xi \quad (23)$$

with $C_1(t)$ and $C_2(t)$ being integral constants to be determined later. Material constant λ is defined as

$$\lambda = \frac{3(\nu_u - \nu)}{B(1+\nu_u)(1-\nu)}. \quad (24)$$

Obtaining strain components ε_r and ε_θ ($\varepsilon_z = 0$) from Eq.(23) with the help of Eq.(3), and substituting them into Eq.(10), we have

$$g(t) = 2C_1(t). \quad (25)$$

[Note that, although the thermal and fluid diffusion equations, (4) and (5), are not coupled to the displacement field, they are indirectly coupled to Eq.(23) by Eq.(25) through the associated boundary conditions.] Using the stress boundary conditions (11) and (12) in addition to Eq.(1) to determine $C_1(t)$ and $C_2(t)$, and expressing the stresses in terms of the pore fluid pressure $p(r, t)$ and temperature $\Delta \theta(r, t)$ yields

$$\frac{\sigma_r}{\sigma_0} = -\frac{a^2}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right) \left\{ -\frac{p_0}{\sigma_0} H(t) + F(b, t) \right\} + F(r, t), \quad (26)$$

$$\frac{\sigma_\theta}{\sigma_0} = -\frac{a^2}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right) \left\{ -\frac{p_0}{\sigma_0} H(t) + F(b, t) \right\} + F(r, t) - E(r, t), \quad (27)$$

$$E(r, t) = \lambda \frac{p(r, t)}{\sigma_0} + \frac{\Delta \theta(r, t)}{\Delta \theta_0}, \quad (28)$$

$$F(r, t) = \lambda \frac{1}{r^2} \int_a^r \xi \frac{p(\xi, t)}{\sigma_0} d\xi + \frac{1}{r^2} \int_a^r \xi \frac{\Delta \theta(\xi, t)}{\Delta \theta_0} d\xi. \quad (29)$$

Substituting into Eq.(5) the function $g(t)$ given by Eq.(25) in which $C_1(t)$ has already been determined, and expressing the thermal and fluid diffusion equations, (4) and (5), in terms of the cylindrical coordinates and nondimensionalized quantities, we

obtain

$$\frac{\partial \Delta \bar{\theta}}{\partial \bar{t}} = \frac{\partial^2 \Delta \bar{\theta}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \Delta \bar{\theta}}{\partial \bar{r}} + \frac{\sigma_0}{G A_0} \left\{ \frac{\partial \Delta \bar{\theta}}{\partial \bar{r}} \frac{\partial \bar{p}}{\partial \bar{r}} + \Delta \bar{\theta} \left(\frac{\partial^2 \bar{p}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{p}}{\partial \bar{r}} \right) \right\}, \quad (30)$$

$$\begin{aligned} \frac{\partial \bar{p}}{\partial \bar{t}} = & \frac{1}{A_0} \left(\frac{\partial^2 \bar{p}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{p}}{\partial \bar{r}} \right) + A_2 \frac{\partial \Delta \bar{\theta}}{\partial \bar{t}} \\ & - A_3 \frac{1}{\bar{r}_0^2 - 1} \left\{ \lambda \frac{d}{d\bar{t}} \int_1^{\bar{r}_0} \bar{\xi} \bar{p}(\bar{\xi}, \bar{t}) d\bar{\xi} \right. \\ & \left. + \frac{d}{d\bar{t}} \int_1^{\bar{r}_0} \bar{\xi} \bar{p}(\bar{\xi}, \bar{t}) d\bar{\xi} \right\}, \end{aligned} \quad (31)$$

where $\Delta \bar{\theta}$ and $\bar{p}$ are quantities nondimensionalized by $\Delta \theta_0$ and σ_0 , respectively, and $r_0 = b/a$. The other nondimensionalized quantities are defined by

$$\bar{t} = t/(a^2/C_0), \quad \bar{r} = r/a, \quad (32)$$

$$A_0 = C_0/C, \quad (33)$$

$$A_1 = 1/\nu_0 \left[\frac{2B^2(1+\nu_u)^2(1-\nu)}{9(1-\nu_u)(\nu_u-\nu)} \right], \quad (34)$$

$$\begin{aligned} A_2 = & \frac{1-\nu}{2(1+\nu)} \left\{ 2\lambda + \nu_0 \left(\frac{\bar{a}}{a} - 3 \right) \right\} \\ & \times \left[\frac{2B^2(1+\nu_u)^2(1-\nu)}{9(1-\nu_u)(\nu_u-\nu)} \right], \end{aligned} \quad (35)$$

$$A_3 = \frac{2B(1-\nu)(1+\nu_u)}{3(1-\nu_u)}. \quad (36)$$

It is seen that we can obtain stresses $\sigma_r(r, t)$ and $\sigma_\theta(r, t)$ with the help of Eqs. (26) - (29) if we solve Eqs. (30) and (31) with respect to $\Delta \theta(r, t)$ and $p(r, t)$ by some method.

5. Numerical Calculation and Results

Since Eqs. (30) and (31) comprise a system of simultaneous nonlinear integrodifferential equations with respect to $\Delta \theta$ and p , it is difficult in general to analytically solve them, although we can obtain the analytical solutions for the steady state. We have resorted to the finite difference method (the Crank-Nicolson scheme), taking $\Delta \bar{r} = 1/50$, $\Delta \bar{t} = 1/50^2$ or $2/50^2$. The Simpson rule has been applied to the integrals in Eq. (31).

For the numerical calculation, we have taken $\nu_0 = 0.1$, $\nu = 0.2$, $\nu_u = 0.3$, and $B = 0.75$ as their typical values and chosen $\bar{a}/a = 25$ and $\bar{r}_0 = 2$. If we choose sintered alumina as a porous material and liquid hydrogen or water as a pore fluid, $\rho_0 C_p / \rho^T C^T = 0.245$ or 1.334 . Thus, we have assumed that it is unity. Since we know from the previous paper⁽⁴⁾ that we can obtain a larger effect of pressurization and injection for larger A_0 , we have supposed for most cases that $A_0 = 0.0001$.

Figures 1 to 5 show results of the numerical calculation, the first four figures corresponding to case (A), the last one, to case (B).

Figure 1 shows the distribution of hoop stress σ_θ with nondimensional time as a parameter. It can be seen that the hoop stress is compressive over the wall thickness immediately after the abrupt rise in temper-

ature of the gas and the simultaneous intentional pressurization at the outer surface; with the passage of time, the compressive hoop stress increases near the inner surface, whereas the hoop stress near the outer surface changes from compression to tension. The maximum tensile and compressive hoop stresses occur at the inner and outer surfaces, respectively, in the steady state.

Fig. 1 Hoop stress distribution for various times

Fig. 2(a) Temperature distribution at the steady state for various pressurizations

Fig. 2(b) Hoop stress distribution at the steady state for various pressurizations

Most porous materials, especially sintered ones, are brittle and weak in tension. Thus we shall consider only the steady state in what follows.

Figure 2(a) shows the radial distribution of temperature change, Fig. 2(b), that of hoop stress, both with the pressurized pressure as a parameter. These figures reveal that applied pressure as slight as 2% or 4% of the reference thermal stress σ_0 markedly lowers the temperature near the inner surface and consequently decreases the tensile stress near the outer surface. Note that the maximum stress at the outer surface declines to compression even for the 4%

Fig. 3(a) Temperature distribution for the various Biot numbers

Fig. 3(b) Hoop stress distribution for the various Biot numbers

Fig. 4 Critical external pressure as a function of the Biot number

pressure of the reference. In contrast, the compressive stress near the inner surface is little influenced by the pressurization. Thus, we can say that the effect of the heat transportation by pore fluid flow on the reduction of tensile thermal stress is considerable. The slight pressurization and injection of fluid is effective in lowering the maximum tensile stress at the outer surface, keeping the increase in compressive stress slight at the inner surface.

The dependence of the temperature and stress distributions on the Biot number is illustrated in Figs. 3(a) and 3(b), similar to Figs. 2(a) and 2(b). The Biot number is that for where there is no blowingout, and the effect of blowingout is taken into account by Eq.(18). For small Biot number, the temperature decreases sharply, and, corresponding to this, both the tensile stress near the outer surface and compressive one near the inner surface decrease; they decline to a weak compression over the wall thickness for $Bi = 0.1$.

Fig. 5(a) History of the maximum temperature for various pressurizations

Fig. 5(b) History of the maximum hoop stress for various pressurizations

From this, we may say that it is easier to control the temperature and stress by fluid pressurization and injection for small Biot number.

As explained regarding Figs. 2(a) and 2(b), with higher pressurizing pressure, the tensile stress at the outer surface decreases and finally changes into compression. Figure 4 shows the critical pressure at which the hoop stress changes its sign for decreasing pressure, that is, the applied pressure p_0 for $\sigma_{\theta\max}=0$, as a function of the Biot number for $A_0=0.001, 0, 0.0005$, and 0.0001 . From this figure, we can understand that, for instance, when $Bi=0.1$, we should apply the external pressure of $p_0/\sigma_0 \doteq 0.4 \times 10^{-2}$ in order to avoid the occurrence of tensile stress in the wall.

Figures 5(a) and 5(b) are for case (B). The former depicts the history of temperature at the inner surface, while the latter, that of the hoop stress (maximum stress) at the outer surface. In these figures, we have assumed that the pressurization is lost at the nondimensional time $t=0$ and recovered at $t=t_0=0.4$, and have taken the applied pressure as a parameter. Before the loss of pressurization ($t<0$), the steady state holds; both the maximum temperature and stress are lower for higher pressurization. Loss of pressurization elevates the maximum temperature and stress sharply, and these asymptotically approach values of the steady state without pressurization. Recovery of pressurization at $t=t_0$ allows the maximum temperature and stress to sharply relax and return to the original steady states. Although only the case of $t=0.4$ is shown in the figures, we can see that shorter duration of pressurization loss suppresses the rise in the maximum temperature and stress more.

6. Conclusions

For the two cases of thermal and mechanical loading histories, we have analyzed the temperature

and thermal stresses induced in a fluid-saturated porous elastic hollow cylinder and clarified some effects of the fluid pressurization and injection from the outer surface on the relief of maximum temperature and thermal stresses. The results may be summarized as follows.

(1) The slight fluid pressurization and injection from the outer surface is effective in order to suppress the maximum temperature at the inner surface and the maximum hoop stress at the outer surface, as well as to avoid the occurrence of excess compressional stress at the inner surface.

(2) For small Biot number, it is rather easy to actively control the maximum temperature and stress.

(3) The critical external pressure to avoid the occurrence of tensile stress in the wall are shown in the form of curves as a function of the Biot number.

(4) The sharp rise in the maximum temperature and stress due to the pressurization loss is lowered by its faster recovery.

The author is grateful to Mr. Yoshiyuki Goto and Tatsuo Imagawa for their assistance. The present study was partially supported by a Grant-in-Aid for Scientific Research in 1992-1993 by the Ministry of Education, Science and Culture.

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