Ultimate Strength of Square Tubes Under Bending Loads*

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Buckling behavior of square tubes under bending load depends on the direction of the bending moment. This paper considers two directions. The first has an axis which crosses at right angles to a side of a cross section; the second has an axis which crosses at right angles to a diagonal line of a cross section. Four-point bending tests are conducted at room temperature and at 300 °C, using square tubes of three thicknesses (2.0, 2.5, and 3.0 mm), but having the same inside width (115 mm, excluding corner radius) and length (4 000 mm). Local deformations and ultimate bending loads are measured. An empirical formula of the relationship between buckling wave growth and bending load is obtained by analogy with an equation derived from the results of bending theory for plates with an initial curvature. An empirical formula of the ultimate bending moment is also obtained by applying Kármán's effective width theorem for critical loads in plates to square tubes. These formulas are proven to represent the behavior of test and analysis values very well.

Key Words: Ultimate Strength, Postbuckling Behavior, Square Tubes, Bending Load, Structural Analysis, Experimental Study

1. Introduction

Square tubes are characterized by their ability to support loads exceeding buckling loads without a rapid decrease in their strength against such overload. Thus, square tubes can be used under loads greater than buckling loads if local deformations due to buckling are negligible. However, it is necessary to understand their post-buckling behavior. As an example, square tubes are used at 300 °C in a boiling water reactor as fuel channels, which surround fuel rods, function as channels for refrigerant and also give rigidity to fuel bundles. Under seismic excitation, fuel channels are subjected to bending load.

In this paper, we consider fuel channels to be one example of square tubes and investigate the propaga-

tion behavior of local deformations after buckling and the ultimate bending loads of the square tubes through analysis of 4-point bending test results. Buckling behavior of square tubes under bending load depends on the direction of the bending moment. Bending tests treat two directions. The first has an axis which crosses at right angles to a side of a cross section; the second has an axis which crosses at right angles to a diagonal line of a cross section.

Furthermore, the empirical formulas of the buckling wave growth and the ultimate bending moment are discussed.

Nomenclature

E : modulus of elasticity
 ν : Poisson ratio
 σ_y : yield stress
 H' : strain-hardening rate
 t : thickness of a square tube
 b : inner width of a square tube
 I : moment of inertia
 M : bending moment
 R : reaction force per unit length
 w : deflection

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- δ_i : initial imperfection
 ρ : radius of curvature
 θ : temperature, angle of rotation

Subscripts

- m : function of loading mode
 θ : function of temperature
 1, 2: loading modes
 h, r : elevated and room temperatures
 c, k : ultimate strength by test and by calculation

2. Buckling Tests

2.1 Geometry and material properties

Square tubes of three thicknesses (2.0, 2.5 and 3.0 mm) which have the same inside width (115 mm, excluding corner radius) and length (4 000 mm) are tested. The material properties of the test specimens at (20 °C and 300 °C) are shown in Table 1.

2.2 Buckling test apparatus

The test apparatus consists of a loading machine, measuring instruments and heaters. The loading

Table 1 Material properties

Temperature θ °C	Modulus of elasticity E GPa	Yield stress σ_y MPa	Poisson ratio ν
20	93.8	373	0.35
300	77.9	147	0.42

- ① Load cell ② Test specimen
 ③ Pinned support ④ Crosshead

Fig. 1 Test apparatus

Fig. 2 Sketch of loading modes

machine applies a 4-point bending load to the test specimen (Fig. 1). To prevent local deformations, blocks are inserted into the square tube at the center and at the points to which loads are applied directly.

2.3 Test conditions and measured items

Two loading directions, called loading modes 1 and 2, are used (Fig. 2). Deflection, load, and deformation of cross section due to buckling wave are measured.

2.4 Results of buckling tests

2.4.1 Temperature distribution Four-point bending tests were conducted at room temperature and at 300 °C. The temperatures were measured only at several points during the test, because temperature did not change much with the test specimen. Table 2 shows an example of the measured temperatures and the points where temperature was measured. Here, the temperatures at the central part, where buckling occurs, are generally within 300 °C ± 20 °C.

2.4.2 Relationship between bending moment and crosshead displacement Figures 3 and 4 show some examples of the relationship between crosshead displacement and bending moment obtained in the bending test. Figure 3 shows the relationship in Mode 1 and Fig. 4 shows the relationship in Mode 2. The moment was calculated as the product of the load measured by the load cell and the distance between loading and supporting points. As the crosshead displacement increases in Fig. 3, the bending moment increases nearly linearly, but for higher crosshead

Fig. 3 Relationship between crosshead displacement and bending moment (loading mode 1)

Table 2 Measured temperatures

No.	Temperature (°C)		
	Before Test	Approximately halfway through test	After Test
1	326	320	299
*2	300	304	296
*3	299	298	298
*4	297	298	298
5	274	276	280
6	319	311	309
7	318	314	296
*8	295	302	293
*9	291	292	293
10	315	307	303
11	319	315	298
*12	298	305	297
*13	294	295	293
14	319	311	304
15	310	308	292
*16	287	288	286
17	300	293	287
18	314	304	305
19	310	301	303
20	303	295	302

* central part subjected to uniform bending moment

displacements, the bending moment increase slows, reaches a peak, and then begins to decline. Here, the buckling mode is symmetric. In Fig. 4, as the crosshead displacement increases, the bending moment first also increases almost linearly, then declines rapidly at a crosshead displacement of 35 mm. Here, the buckling mode is asymmetric. In Figs. 3 and 4, directly after the start of loading, the crosshead displacement is not proportional to the bending moment. This is due to the weight of the specimen and the jigs, and the gaps between jigs. In the elevated temperature tests, the crosshead displacement increases (○), and some time later, the load decreases (●). This behavior is particularly prominent in nonlinear conditions, and seems to be due to the strain rate dependency of yield stress. In detail, it seems that when the crosshead displacement is increasing, the strain rate is so large that the relationship between load and crosshead displacement behaves as if there is no yield. When the displacement is fixed or as the displacement growth

slows, the strain rate decreases, causing the load to decrease.

2. 4. 3 Local deformation behavior after buckling

Local deformation behavior after buckling of Modes 1 and 2 is shown in Figs. 5 and 6. Figure 5 (a) shows, for Mode 1, deformation with the load held, and Fig. 5 (b) shows residual deformation with the load removed. Figures 6 (a) and (b) show the same (for Mode 2). While residual deformations in Mode 1 are widespread, those in Mode 2 are concentrated locally.

3. Buckling Analyses

As bending deformation after buckling increases, the local buckling wave grows and the test specimen collapses upon reaching to ultimate bending load. To understand the large-deformation behavior in this process, we carried out large-deformation and elastoplastic analyses using the general-purpose structural analysis program MARC.

Fig. 4 Relationship between crosshead displacement and bending moment (loading mode 2)

3.1 Analysis model and method

The analysis model is shown in Fig. 7. Only half of the square tube is treated under symmetrical conditions. The bending moment in Mode 1 was considered. Assuming a buckling mode so as to understand deformation behavior after local buckling, and giving a slight initial deformation to the compressive side of the analysis model, we investigated how the deformation would increase with increases in the angle of end rotation and how flexural rigidity would change as deformation proceeded. The initial deformation of a cosine curve figure with a half-wave of the same length as the width was given as the buckling mode. Amplitudes of ± 1.0 mm and ± 0.25 mm were chosen. An isoparametric 4-node double-curved quadrilateral thin-shell element which uses a bi-cubic interpolative function having 12 degrees of freedom per node was used. This element is based on the Koiter-Sanders shell theory, fulfills continuity requirements, and represents rigid body modes exactly. We used modulus of elasticity $E=77.9$ GPa, Poisson ratio $\nu=0.42$, Mises' yield conditions, and isotropic hardening rules. We approximated the stress-strain curve with two

(a) Deformation in loading

(b) Permanent deformation after unloading

Fig. 5 Local deformation distribution (loading mode 1)

(a) Deformation in loading

(b) Permanent deformation after unloading

Fig. 6 Local deformation distribution (loading mode 2)

straight lines, and used the yield stress $\sigma_y = 147$ MPa and the strain hardening rate $H' = 8.32$ GPa. Although the present large-deformation analysis is based on the total Lagrange method, where Kirchhoff stresses and Green strains are used, we used approximately nominal stresses (Lagrange stresses) and nominal strains.

Next, we explain analysis results on the effect of

Fig. 7 Analysis model (loading mode 1)

Fig. 8 Effects of initial deformation on postbuckling behavior

initial imperfection, bending moment and local deformation behavior, and the comparison between tests and analyses.

3.2 Analysis results

3.2.1 Effect of initial imperfection

The results of computations for initial deformation amplitudes of ± 1.0 mm and ± 0.25 mm for investigating the effect of initial deformation are shown in Fig. 8. In the low angle-of-rotation θ range, there are some differences in reaction force and bending moment between the two initial deformations. As the angle of rotation increases, these differences decrease. This suggests that there will be little difference in the results of computation near the ultimate collapse between the amplitudes ± 1.0 mm and ± 0.25 mm given as initial deformations. The moment M in the figure was calculated from the reaction force R_i at node i shown in Fig. 7 and distance d_i from the neutral axis,

$$M = \sum_i R_i \times d_i. \quad (1)$$

When the local deformation of the top center increases, the total bending moment M also increases up to ultimate strength but the reaction force of the top center R begins to decrease before ultimate strength due to effective-width phenomena. Thus, the reaction force demonstrates how the bearing force of the top center (compressive side) decreases as the buckling wave grows.

3.2.2 Relationship between bending moment and angle of end rotation

Variations in reaction force and bending moment due to angle of rotation are shown in Fig. 9. With a plate thickness of 2.0 mm, ultimate collapse occurs at an angle of rotation of about 0.010 rad, at which the maximum bending moment is 3.0 MNmm. For an initial deformation of ± 1.0 mm and plate thickness of 2.5 mm, the ultimate collapse occurs at an angle of rotation of about 0.014 rad, at which the maximum bending moment is

Fig. 9 Relationship between bending moment M or reaction force R and angle of rotation θ

4.3 MNmm. For a plate thickness of 3.0 mm, we gave an initial deformation ± 0.25 mm, but did not continue the computation up to the ultimate collapse. The above analysis is carried out using the half model, so the bending moment is for half of the entire square tube.

3.2.3 Relationship between local deformation amplitude and angle of end rotation Figure 10 shows how the cross section at the center of the model varies as the angle of rotation varies. It shows that for plate thicknesses of 2.5 mm and 3.0 mm, the increasing rate of local deformation amplitude decreases at an angle of rotation of about 0.010 rad.

3.2.4 Comparison between tests and analyses A comparison between test and analysis values to examine the validity of the analysis is shown in Fig. 11. Figure 11 shows the relationship between bending moment and displacement at the center of the square

Fig. 10 Relationship between local deformation U or V and angle of rotation θ

Fig. 11 Comparison of test and analytical results on bending moment and center displacement

tube. According to the buckling computation of box beams, initial buckling occurs at about 22 mm. The test results agree with the results of linear analysis up to about 35 mm. They also show two measured moments directly before and some minutes after crosshead displacement was held. The material constants used in the analysis are suitable for sufficiently low strain rates. Thus, the value to be compared with the analysis value is the one measured some minutes after crosshead displacement was held. In this comparison, the analysis value agrees with the test value with high accuracy.

A comparison between test and analysis values to examine the validity of the analysis is also shown in Fig. 12. Figure 12 shows measured and computed strain distribution. The strain gage is pasted onto the axis of symmetry on the top (compressive side) of the square tubes. The gage positions did not exactly coincide with the mesh of the analysis model, and we chose positions near the gage positions. In the moment range up to about 5.4 MNmm, buckling waves occur, and the tensile strains due to bending increase rapidly. In a nonlinear analysis, on the other hand, Mode 1 has a larger initial deformation (± 1.0 mm) than the actual value. Thus, a tensile strain due to bending occurs from the beginning, and the compressive strain is smaller than in the test and elastic analysis. As the buckling wave grows, the tensile strain starts rising at around 3.6 MNmm. There are considerable differences between test and analysis values because of the difference in initial deformation between analysis model and test specimen, and, as described above, the delay of initial buckling in the test. However, there are qualitative agreements, which assure that analyses can represent the buckling behavior correctly. The deformation behavior and the maximum deformations obtained by computation agree well with test values.

Fig. 12 Comparison of measured and calculated strains

4. Discussion

Here, we discuss the empirical formulas of the deformation after buckling and the ultimate strength.

4.1 Deformation after buckling

The tubes can be assumed to consist of two flanges and two webs in loading mode 1. If a bending moment is applied as shown in Fig. 13, a uniform tensile or compressive load acts on the flanges, and an in-plane bending load acts on the webs. We consider a rectangular plate under a compressive load of $-N_x$ per unit of length and assume that in this plate there exists an initial deflection in the form of a sinusoidal wave with an amplitude α relatively small compared to the plate thickness, and an additional deflection caused by the compressive load. The maximum deflection $w_{\max}$ at the center of the plate is given by⁽¹⁾

$$w_{\max} = \frac{\alpha}{1 - \beta N_x} \quad \text{where } \beta = \text{constant.} \quad (2)$$

We assume, by analogy with Eq. (2), the following relationship between local deformation u generated at the center of a wall surface of a tube under bending

Fig. 13 Sketch of a square tube under bending load

(a) Loading mode 1 at 300°C

load and bending moment M ,

$$u = \frac{A}{1 - BM} \quad \text{or} \quad A + BuM = u, \quad (3)$$

where $A, B = \text{constant}$. From Eq. (3), it is evident that deformation u is proportional to the product uM . Application of Eq. (3) to measured and calculated values (Fig. 14) shows that both sets of values satisfy the relation. Next, we determine the coefficients A and B .

4.1.1 Coefficient A Figure 15 shows the relationship between coefficient A and wall thickness t . Coefficient A represents the deformation at the center of the wall surface with bending moment zero. This value, however, is too large to be taken as an initial deformation. Coefficient A can be considered as representing the wall surface deformation directly after buckling. The figure shows that the absolute value of coefficient A decreases as the wall thickness increases.

4.1.2 Coefficient B We assume that coefficient B can be represented as

$$B = K_m \cdot f(\theta) \cdot g(t) \quad (4)$$

where K_m is the coefficient of loading mode. Function $f(\theta)$ is related to temperature and includes the material properties; function $g(t)$ is related to thickness. If Eq. (4) holds, the following relations can be obtained:

$$r_1 = \frac{B(\text{mode 2})}{B(\text{mode 1})} = \frac{K_{m2}}{K_{m1}} = \text{constant}$$

and

$$r_2 = \frac{B(\text{room temp.})}{B(300^\circ\text{C})} = \text{constant.} \quad (5)$$

(b) Loading mode 2 at room temperature

Fig. 14 Relationship between deformation u and product uM

Table 3 shows values of r_1 and r_2 . As the wall thickness increases, r_1 and r_2 increase slightly, but these changes are within the test value dispersion. Thus, coefficient B can be represented by Eq. (4). Coefficient B , like coefficient A , decreases as the wall thickness increases. Figure 16 shows $\log B - \log t$ relationships of the test results for loading mode 1 at

Table 3 Coefficients r_1 and r_2

Thickness t mm	Coefficient r_1	Coefficient r_2
2.0	1.21	0.414
2.5	1.28	0.429

Fig. 15 Relationship between coefficient A and wall thickness t

Fig. 16 Relationship between coefficient B and wall thickness t

room temperature. It gives

$$g(t) = t^{-1.53} \quad \text{where } t = \text{wall thickness.} \quad (6)$$

On the other hand, the form of $f(\theta)$ is not determined by data at only two different temperatures. To determine the form of $f(\theta)$, we need data at another temperature.

4.2 Ultimate strength

To determine the ultimate strength of a flat plate with simply supported edges under a compressive force, we use Kármán et al.'s study.⁽²⁾ In Fig. 17, a plate with length l and width b is compressed. All forces are supported by two slender fragments of width w and loading is uniform. If stress is σ , total force P is given by

$$P = 2w\ell\sigma, \quad (7)$$

where w represents effective width, which is given by

$$w = \frac{\pi}{\sqrt{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma}} t. \quad (8)$$

If the stress distribution in the flange of a square tube is represented by the dotted line in Fig. 18, we can derive the following equation,

$$2w\sigma = \sigma_1 b = (M/I) \cdot (b/2) \cdot b. \quad (9)$$

If moment of inertia I of the square tube is represented by inside width b_i and wall thickness t , and if $t/b_i \ll 1$, $b_i = b$, $\sigma = \sigma_y$ (yield stress), substituting Eq. (8) into Eq. (9) yields

Fig. 17 Effective width and stress distribution of a flat plate

Fig. 18 Assumption of stress distribution in flanges and webs of a square tube

$$M_{1k} = \frac{2}{3} \frac{\pi}{\sqrt{3(1-\nu^2)}} \sqrt{E\sigma_y} b t^2 \quad (10)$$

$$M_{2k} = r_m M_{1k} = \frac{2}{3} \frac{\pi}{\sqrt{3(1-\nu^2)}} r_m \sqrt{E\sigma_y} b t^2, \quad (11)$$

where M_{1k} and M_{2k} are the ultimate bending moments for loading modes 1 and 2. Next, we discuss the effect of wall thickness, loading mode and temperature. Furthermore, we discuss the validity of the formula derived from Kármán's theory.

4.2.1 Effect of wall thickness Figure 19 shows the $\log M_c - \log t$ relation obtained from the test results of loading mode 2. The plotted points are nearly in a line, and we have

$$M_{rc} = 3.96 t^{1.57} \quad (\text{room temp.}) \quad (12)$$

$$M_{hc} = 2.56 t^{1.50} \quad (300^\circ\text{C}). \quad (13)$$

According to Eqs. (10) and (11), the ultimate bending moment is proportional to the square of the wall thickness. However, experimental results are represented by Eqs. (12) and (13), which show that the ultimate bending moment is proportional to the wall thickness to the power of 1.5–1.6.

Fig. 19 Relationship between ultimate bending moment M_c and wall thickness t

Fig. 20 Effect of loading mode on ultimate bending moment

4.2.2 Effect of loading modes Figure 20 shows the effect of loading mode on the ultimate bending moment. Evidently, the ratio between ultimate bending moments M_{1c} in loading mode 1 and M_{2c} in loading mode 2 does not depend on the wall thickness, temperature, etc. Thus, from the gradient of the best-fit line passing through the origin, we have

$$r_m = \frac{\text{Ultimate bending moment in loading mode 2}}{\text{Ultimate bending moment in loading mode 1}} = \frac{M_{2c}}{M_{1c}} = 0.91. \quad (14)$$

4.2.3 Effect of temperature θ By substitution of the values of the material constants in Table 3 into Eq. (10), a ratio r_θ can be obtained:

$$r_\theta = \frac{M_{k1}(300^\circ\text{C})}{M_{k1}(20^\circ\text{C})} = 0.59. \quad (15)$$

Figure 21 shows the effect of temperature on the ultimate bending moment, proving that the ratio between ultimate bending moments M_{hc} at 300 °C and M_{rc} at room temperature does not depend on wall thickness, loading direction, etc. Thus, from the gradient of the best-fit line passing through the origin, we have

$$r_\theta = \frac{\text{Ultimate bending moment at } 300^\circ\text{C}}{\text{Ultimate bending moment at room temp.}} = \frac{M_{hc}}{M_{rc}} = 0.62. \quad (16)$$

This value agrees well with that in Eq. (15).

4.2.4 Validity of the formula derived from Kármán's theory Figure 22 shows the relationship between the estimated values M_k calculated from Eqs. (10) and (11) and test values M_c . The calculated values provide a conservative estimation. The best estimate is represented by a line passing through the origin at gradient 1.42, and the ultimate bending moment M_c is given by

$$M_c = 1.42 M_k. \quad (17)$$

Fig. 21 Effect of temperature on ultimate bending moment

Fig. 22 Relationship between estimated values M_k and test values M_c

Although the value of the coefficient of Eq. (17) is not 1 but 1.42, this may be the result of loading rates. The material constants currently in use are for a strain rate of about 0.05/min. In tests, a strain rate of about 0.25/min is likely generated at the outermost end. However, there have been no tests sufficiently

quantitative to draw any conclusions.

5. Conclusions

The relationship between buckling wave growth and bending moment was represented by an empirical formula obtained by analogy with an equation derived from the results of bending theory for plates having an initial curvature. A formula was also obtained for the ultimate bending moment by applying Kármán's formula for critical loads in plates to square tubes. These experimental formulas were proven to represent the behavior of test and analysis values very well.

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