Mechanics of Drilling*

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In this overview the process of drilling is regarded from a continuum mechanical point of view. Starting with drilling into a material that is free of stresses, with drilling only as a process of cutting, the case of drilling into a pre-stressed material is investigated more precisely. The drilled material has either a metallic or a granular behavior. In particular, materials with a strain-softening behavior at the point of failure is considered. A complete analytical solution is presented for conditions of rotational symmetry, while for biaxial symmetry semi-analytical and numerical solutions are developed. The theoretical analysis is in good agreement with the experimental results.

Key Words : Drilling, Test Drilling, Stress-Relief-Drilling, Borehole Bumps, Translatory Rock Bursting, Strain-Softening, Granular Material, Plastic Domain, Yielding

1. Introduction

Drilling is one of the most important technological methods of material processing. In the following, the process of chipping^{(1),(2)}, hence the proper behavior of the material in the near vicinity of the contact interface between the borer and the material, is not examined in detail, since the interactive loads and the global behavior of the material can be determined without that. Rather, this interface will be idealized, from the continuum mechanical point of view, as a surface of discontinuity of the point velocities. The borer is assumed to consist of a conical head and a cylindrical shaft (Examples cf. Fig. 1 a, b), while the helicyclical grooves on it (used for the transportation of the fines) are disregarded.

The main interest of the analysis lies with the drilling into a pre-stressed material, for in the case of drilling into a material that is free of stresses, drilling is reduced to a pure cutting process. There is no cutting on the cylindrical surface of the shaft because of the missing normal stress. Cutting only takes place at the head of the borer. With a given diameter d of

the borer's head, the vertex angle β of the borer and the shear yield limit K of the drilled material, the drilling moment M is approximately given by

$$M = \frac{d^3 K \pi}{12 \sin \beta/2} \quad (1)$$

for a metallic material, provided the circumferential frictional shear stress is constant over the surface of the borer's head. The shear yield limit for a granular or a brittle, rock-like material is a function of the normal stress, which is generated by the axial drilling force N . Assuming the well-known MOHR-COULOMB yield criterion to hold (cf. Ref.(2)) the drilling moment M is now approximately given by

Fig. 1 Drilling with an annular bit a) and a twist drill b)

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$$M = \frac{d}{12 \sin \beta/2} (4N \tan \Phi + d^2 K \pi) \quad (2)$$

Φ denoting the internal angle of friction and K the cohesion. Subsequently K and Φ will always be considered as given constants, possibly assuming different values in different domains of the material. Especially the temperature rise and its influence on the shear yield limit in the cutting area may be neglected because the corresponding decrease of K seems to be compensated by the high local rate of shear strain⁽²⁾, Fig. 4. 2.

2. Drilling into a Pre-Stressed Material

2.1 General

For a pre-stressed material, especially a material under pressure, only the state of stress, strain and strain rate around the cylindrical shaft of the borer is to be investigated. As a matter of fact, if the length l of the part of the shaft inside the material and the average initial radial pressure, σ_i (Fig. 1) are large enough, the head only opens the borehole up. Behind the head the broken material, forming around the shaft, flows into the hole until its resultant width corresponds to the width of the shaft, $2r_i$. This motion of the material towards the shaft continues until the pre-stress vanishes. Until then the transported material may exceed by far the volume of the resultant borehole, i.e., $2\pi r_i l$. Therefore the effect of the head is disregarded. Whether this is admissible or not, may indirectly be judged by comparing the loads M and N at the head with the corresponding quantities M' and N' along the shaft. They are given by the equations,

$$N' = 2\pi r_i l (K + \sigma_i \tan \Phi) \sin \beta' \quad (3)$$

$$M' = 2\pi r_i^2 l (K + \sigma_i \tan \Phi) \cos \beta' \quad (4)$$

where the feed angle β' can be determined from the axial (feed) velocity of the borer, w and its angular velocity, Ω according to

$$\tan \beta' = \frac{w}{r_i \Omega} \quad (5)$$

$N_0 = N + N'$, $M_0 = M + M'$ are the total loads transferred by the drilling machine or the operator, respectively. The effect of the head of the borer may be disregarded if,

$$N \ll N', M \ll M'. \quad (6)$$

First a model with rotational symmetry is chosen, that means, there is no dependence of stress, strain or strain rate on the coordinate φ in hoop direction. This simple model has the advantage of the existence of a closed analytical solution.

2.2 Drilling into pre-stressed metallic material

At the beginning, the process of drilling into a pre-stressed metallic material or a material with a behavior similar to metals is considered. Due to the pre-stress, a stress concentration in the vicinity of the

borehole is generated^{(3),(4)}. If the initial outer pre-pressure σ_a is big enough, then the (in our example: TRESCA) equivalent stress exceeds the limit of elasticity, and a brittle-plastic domain is set up around the borehole. In the case of a rotationally symmetric distribution of the pre-pressure σ_a , the plastic region assumes an annular shape. Then the yield condition reads

$$\sqrt{\left(\frac{\sigma_r - \sigma_\varphi}{2}\right)^2 + \tau^2} - K = 0, \quad (7)$$

where σ_r , σ_φ denote the radial or hoop pressures, respectively, while $\tau = \tau_{r\varphi}$ is the circumferential shear stress acting against the rotation of the borer. The shear stress that is transmitted at the edge of the borehole, $r = r_i$, assumes its highest possible value, namely the yield limit K , itself. Therefore the edge of the borehole is a slip line. The corresponding radial pressure σ_i , being initially equal to the pre-stress σ_a , may be expected to decrease during the boring process ("stress-relief drilling"). Anyway, the stresses in the plastic domain are the following⁽⁵⁾,

$$\sigma_r = \sigma_i - K(\ln \tan |\alpha| + \cos 2|\alpha|) \quad (8)$$

$$\sigma_\varphi = \sigma_i - K(\ln \tan |\alpha| - \cos 2|\alpha|) \quad (9)$$

$$\tau = K \left(\frac{r_i}{r} \right)^2. \quad (10)$$

The parameter α describes the angle between the radial direction of the polar coordinate system r , φ and the direction of the lower principal stress. At the edge of the borehole $r = r_i$, the angle α becomes $\alpha = \pi/4$. The radial coordinate r is connected to the angle α by the function

$$r = \frac{r_i}{\sqrt{\sin 2|\alpha|}}. \quad (11)$$

The plastic domain is separated from the outer region, remaining still elastic, by the interfacial radius r_p . The state of stress in the elastic domain is described by the well-known equations for a thick-walled tube or an infinite plate with a circular hole⁽⁶⁾, but with a superimposed shear stress τ . The interfacial radius r_p is determined by the continuity of the stresses at the transition from the plastic domain into the elastic one at the interfacial radius. r_p is not remaining constant but will continuously increase, starting with the radius of the borehole itself. As mentioned before, the internal pressure σ_i at the edge of the borehole is going to decrease gradually. The growth of the plastic region will stop when the internal pressure has completely vanished. The borehole is now stabilized, provided the extension of the surrounding material is large enough for the corresponding enlargement of the plastic region. Otherwise a complete plastification would take place even before $\sigma_i = 0$ was reached, leading to a total failure of the surrounding material.

A decrease of the material density due to plastic

deformation and a corresponding enhanced amount of the quantity of the fines of drilling, as in the subsequently treated case of a rock-like material, does not occur for a metallic material, for the rate of volumetric strain $\dot{\epsilon}_v$ is zero for most metals.

2.3 Drilling into a pre-stressed brittle or rock-like material

Drilling into a pre-stressed brittle or rock-like material is of considerable practical significance. One of the most important cases of application is the test and stress-relief drilling in hard-coal mining^{(7),(8)} for the recognition and prevention of the danger of a translatory rock bursting. Here, horizontal drillings into a coal seam are carried out. With the help of the amount of the fines of drilling, the state of stress inside the coal seam can be estimated in the case of test drilling usually carried out with small radii of the borehole, r_i , whereas in the case of stress-relief drilling (big holes), the stresses in the seam — and thus the danger of a translatory rock bursting — can be reduced^{(9)–(11)}. Surface instabilities, like borehole breakouts, splitting cracks, exfoliations or warpings are described in^{(12)–(15)}, but are not of interest in the case of stress-relief or test drilling, because here large displacements in an extensive area around the hole take place, leading even to its total closure.

The analytical procedure is similar to metallic materials. The dependence of the yield locus on the hydrostatic pressure is observed, for instance by using the MOHR-COULOMB yield criterion. If plain strain conditions are assumed in the cross section of the borehole, this yield criterion, with the cohesion K and the constant internal angle of friction Φ , is given by the equation⁽¹⁶⁾

$$\sqrt{\left(\frac{\sigma_r - \sigma_\varphi}{2}\right)^2 + \tau^2} - \frac{\sigma_r + \sigma_\varphi}{2} \sin \Phi - K \cos \Phi = 0. \quad (12)$$

The corresponding equations for the stresses in the plastic domain can be taken from^{(2),(17),(18)}. The shear stress that is acting at the edge of the borehole, τ_i , depends as before on the radial pressure σ_i , according to

$$\tau_i = \sigma_i \tan \Phi + K. \quad (13)$$

At the beginning of the drilling process, the material is still solid, therefore showing a maximum density. Then the flow rule associated to the MOHR-COULOMB yield criterion may be assumed to hold⁽²⁾. If it is maintained also for the further process, the edge of the borehole remains a slip line, so that the value of the angle α_i at the edge of the borehole between the r -direction of the coordinate system and the direction of the lower principal pressure obeys

$$|\alpha_i| = \pi/4 - \Phi/2. \quad (14)$$

In Fig. 2 a field of slip lines in the annular plastic area

for a granular medium is shown, taking account of the additionally acting shear stress.

Due to the choice of the associate flow rule, too large values for the volumetric increase are calculated, compared to those obtained from experiments. Therefore the analytical results represent an upper bound. However, by using a non-associated flow rule with an angle of dilatancy, which replaces the angle of internal friction, the analysis can be improved (cf. Ref. (2)). It turns out, that for the stresses σ_r , σ_φ to be counted positive in the pressure domain,

$$\sigma_r < \sigma_\varphi \quad (15)$$

holds in such a way, that actually a motion of the material towards the borehole occurs, as it could be expected before. The surplus volume is transported outwards through the borehole and can now be measured as the amount of fines of drilling. This is composed firstly of the mass of material having initially filled the borehole itself, secondly the mass caused by the brittle-plastic flow, and thirdly the mass caused by the superimposed elastic radial inward deformation of the material at the edge of the borehole. The latter part is very small compared with the other parts, and therefore it is neglected. At test and stress-relief drilling in hard-coal mining, the total amount of fines of drilling can exceed the mass of the borehole extensively. An empirical critical value for a threatening coal mine bump (translatory rock bursting) is obtained at the factor of 4.

In order to identify the geometric parameters of the simplified rotationally symmetric analysis with the *in situ* situation in coal mines, an infinite cylindrical coal "stratum" is considered, the radius of which, r_a , corresponds to half the thickness of the real coal seam. This cylinder is embedded in a likewise infinite rigid matrix simulating the stiff rock layers above and below the seam. Loading the cylinder at its outer edge r_a with the initial outer pre-pressure σ_a , corresponding to the rock pressure in the massive coal, and then drilling with the shaft radius r_i (without any displacements at the outer edge r_a), the outer radial stress σ_a will decrease to the lower residual outer stress σ_R due

Fig. 2 Field of shear lines in a ring shaped brittle-plastic region with an additionally acting shear stress at the hole's edge for a granular material obeying the MOHR-COULOMB criterion

to brittle-plastic yielding. In Fig. 3 the residual stress σ_R is plotted for several borehole ratios r_a/r_i versus the initial outer pre-stress. The deviation from the straight line $\sigma_R = \sigma_a$ (if $r_a/r_i = \infty$) indicates the stress relief. This deviation is the larger, the smaller the ratio r_a/r_i would be chosen. Therefore, from the practical point of view, the ratios, i.e., $r_a/r_i \leq 10$ may be attributed to the process of stress-relief drilling, whereas $r_a/r_i > 10$ belongs to test drilling (see below).

The corresponding mass of fines of drilling Q , which is related to the mass of the material initially filling the borehole, is shown in Fig. 4. It can easily be recognized that Q is a sensitive indicator for the pre-stress σ_a in the case of a small diameter of the shaft of the borer as it is used at test drilling ($r_a/r_i > 10$, say).

In Figs. 5 and 6, the interfacial radius r_p and the related mass of fines of drilling Q are plotted as a function of the internal angle of friction, always assuming a constant cohesion K . For an internal angle of friction Φ tending towards zero, the solution for a granular material tends to the solution for a metallic

Fig. 3 Residual outer pressure σ_R versus initial outer pre-pressure σ_a for several ratios of borehole radii r_a/r_i ; K : cohesion

Fig. 4 Mass of fines of drilling Q related to the mass of material initially inside the borehole versus initial outer pre-pressure σ_a for several ratios of borehole radii r_a/r_i ; K : cohesion

(incompressible) material. Then the cohesion K corresponds to the shear yield limit. The two parts before and behind the maximum of the curve of the related mass of fines of drilling can be interpreted as follows: Starting with a zero internal angle of friction, the interfacial radius r_p becomes very large. However, as there is no dilatancy in the plastic region, no increased mass of fines of drilling can be expected. On the other hand now, because of the increasing brittle-plastic resistance, the interfacial radius decreases with a growing internal angle of friction. Dilatancy, caused by plastic yielding, grows with an increasing internal angle of friction. The second effect is the dominating one for small angles of friction, and thus the mass of fines of drilling grows. From a certain value of the angle onward, the first effect predominates, and the mass decreases.

Due to the fact that the elastic displacements are very small compared to the plastic ones, the values of the interfacial radius r_p , the residual outer pressure σ_R and the quantity of fines of drilling Q are independent of YOUNG's modulus E . For an increasing POISSON's ratio $\nu < 1/2$, r_p , σ_R and Q are decreasing. But the influence of POISSON's ratio becomes weaker with an increasing ratio r_a/r_i , since there is no

Fig. 5 Final interfacial radius r_p versus internal angle of friction Φ , if $\sigma_a/K=10$, $r_a/r_i=10$; r_i : borehole radius

Fig. 6 Related mass of fines of drilling Q versus internal angle of friction Φ , if $\sigma_a/K=10$, $r_a/r_i=10$

influence at all for an infinite outer radius r_a .

3. Drilling into a Pre-Stressed Strain-Softening Material

Experiments carried out in a triaxial compression cell show a jump-like strain-softening behavior for hard-coal and other material⁽¹⁹⁾, used as model material for hard-coal ("Model-ARALDIT", see below). The internal angle of friction, Φ_G at the onset of brittle plastic deformation remains nearly constant, so that $\Phi_G = \Phi_R$ holds with $\Phi = \Phi_R$ in the residual state where the cohesion falls down nearly discontinuously from K_G to K_R immediately after the first failure (Fig. 7). For hard-coal K_G is about four times higher than K_R , and for Model-ARALDIT a similar relation was observed. The following discussion refers to drilling into such sort of strain-softening material.

Inside the plastic domain, the material is completely broken and belongs to the state of residual strength with the residual cohesion K_R being, besides K_G and Φ ($=\Phi_G=\Phi_R$), a characteristic material value. The state of stress in the elastic domain just at

Fig. 7 Axial pressure p versus axial strain $\epsilon = 1 - h/h_0$ at a triaxial compression test of a cylindrical specimen (initial height h_0) for a constant circumferential pressure q , idealized; material: hard-coal, Model-ARALDIT⁽¹⁹⁾

Fig. 8 Radial distribution of radial stress σ_r , hoop stress σ_ϕ and shear stress τ for a material with two different cohesions, first $K_R = K$ if $r < r_p$ and second $K_G = 4K$ if $r = r_p$; further parameters: $\sigma_i/K = 0$, $\sigma_a/K = 10$, $\Phi_G = \Phi_R = 8^\circ$ (σ_i : instantaneous borehole pressure, σ_a : initial outer pre-pressure, r : radius)

the elastic-plastic interface r_p , was in the previous analysis assumed to be the same one as on the inner, brittle-plastic side of the interface. However, at the instant of a sudden failure of the material in the brittle-plastic domain, called a borehole bump, and being correlated to a jump of the cohesion K from K_G to K_R , also the hoop pressure σ_ϕ undergoes a sudden jump towards a lower value; cf. Fig. 8 as an example.

The stress of Fig. 8 belongs for any given value of the dimensionless pre-pressure σ_a/K , and if $K_G/K_R = 4$, to one point on the lower trajectory 1) of Fig. 9. For the same pre-pressure σ_a/K , a second solution to the problem can also be found, starting from the assumption that now also $K_G = K (=K_R)$ holds so that a continuous stress distribution arises again. This assumption corresponds to the situation that due to any perturbation of the whole system — detonation waves from remote sources, vibrations of the drilling machine, etc. — the broken domain suddenly propagates outward. This means a further increase of the quantity Q of the fines of drilling corresponding to the upper trajectory 2) in Fig. 9.

The two trajectories of Fig. 9 represent limit curves. The true values of Q may be expected between them, suddenly jumping towards the upper curve if there is any (small or large, observable or non-observable) perturbation acting on the system so that a borehole bump takes place.

In the same way also the residual outer stress σ_R may jump downward between the two limit trajectories of Fig. 10.

The phenomenon of a borehole bump is frequently observed under mining conditions. It could also be confirmed in experiments with radially pre-stressed, cylindrically shaped specimens which were drilled in axial direction. The marked experimental values in Figs. 9 and 10 were obtained after first loading the specimen radially, and then drilling it under pressure. These experiments were carried out using the epoxy resin ARALDIT, which has — under certain condi-

Fig. 9 Theoretically established trajectories of the related mass of fines of drilling Q versus the initial outer pre-pressure σ_a if, 1) $K_R = K$, $K_G = 4K$, 2) $K_R = K_G = K$; experimental values: $\diamond$

tions of preparation — a similar mechanical behavior toward hard-coal, but is easier to handle (“Model-ARALDIT”; cf. Ref.(11) and (19)). The quantity of the fines of drilling as well as the residual stress were measured after piercing the whole specimen. One could see that the experimental values scatter actually between both theoretical limit curves. Depending on the number of borehole bumps occurring, or on their audible strength, the measured values are closer to the upper or to the lower trajectory. The strength of the borehole bumps was judged only in a subjective manner.

Besides experiments with Model-ARALDIT or besides stress-relief drillings in coal mines, borehole bumps could also be observed in laboratory experiments with hard-coal^{(20),(21)}. *In situ*, the miners distinguish intuitively between 20 meters bumps or 100 meters bumps, depending on the maximal distance from the borehole at which they are still audible⁽²²⁾.

4. Drilling under Biaxial Pre-Stress

In the preceding chapters the case of drilling under conditions of rotational symmetry was discussed. If there is no such symmetry, although polar coordinates are still used, the relevant quantities become functions of the circumferential coordinate φ as well. The plastic domain gets a non-circular shape to be determined. However, regarding the borehole and the interior of the brittle-plastic domain, it is assumed that the rotational symmetry is kept further on, provided the hole is still a circle, thus generating rotationally symmetric boundary conditions which determine the solution in the plastic domain. Due to this assumption, the previously obtained stress and displacement fields remain valid. On the other hand, the radius $r_p = r_p(\varphi)$ of the interface between the elastic and the brittle-plastic domain becomes a function of the circumferential coordinate φ as well. The stresses and the deformations in the elastic domain

Fig. 10 Theoretically established trajectories of the residual outer pressure σ_R versus the initial outer pre-pressure σ_a if, 1) $K_R = K$, $K_G = 4K$, 2) $K_R = K_G = K$; experimental values: $\diamond$

are determined by using the complex functions method^{(23),(24)}. Similar to the procedure described in Ref.(25)–(29), but regarding also the shear stress τ_i at the edge of the borehole caused by the rotating borer, a solution is found by means of a conformal mapping of the unit circle $\gamma = e^{i\theta}$ onto the shape of the elastic-plastic interface. Here θ is the polar angle in an auxiliary complex plane, i the complex unit, and γ a complex number. The case of a yield criterion with an infinite cohesion and an internal angle of friction equal to $\pi/2$ has been treated in^{(30),(31)}, but it is not relevant for drilling. The conformal mapping function ω , which determines the points of the elastic/brittle-plastic interface in complex cartesian coordinates of the cross sectional plane of the borehole, is expanded in a LAURENT series according to

$$\omega(\gamma) = \sum_{k=-1}^{\infty} C_k \gamma^k. \quad (16)$$

This analytic function $\omega(\gamma)$ or its still unknown coefficients C_k govern the elastic stress and strain fields. They may be determined by the boundary conditions consisting, for instance, of a constant vertical pressure σ_V and a constant horizontal pressure σ_H acting at an infinite vertical or horizontal distance from the borehole, in combination with the continuity of the shear stress and the normal stress at the elastic/brittle-plastic interface. More general boundary conditions are also possible.

The conditions for the tangential stress along the interface have to be taken into account. For a non-strain-softening material, the tangential stress must also be continuous. Otherwise, the jump within the tangential stress field caused by the two different cohesions K_R or K_G on the brittle-plastic or on the elastic side of the interface has to be observed. Then the coefficients C_k follow by minimizing numerically the quadratic error functional arising from the difference between the tangential stresses from both sides of the interface. Normally a finite number of the complex coefficients $C_{-1}, \dots, C_n$ is sufficient for the solution of the problem. As a qualitative test of the computation only one brittle-plastic domain, enclosing the whole borehole, must arise.

In Fig. 11 elastic-plastic interfaces for one specific biaxial stress state at infinity and for a borehole free of normal stress σ_i are shown. If on both sides of the interface the same cohesion $K_R = K_G = K$ exists so that no jump of the tangential stress can occur, the shape 2) is found. The larger axis of the elliptically formed interface belongs to the direction of the smaller pressure at infinity. If on the brittle-plastic side of the interface the residual cohesion K_R is smaller than the cohesion K_G on the elastic side of the interface, shape 1) arises. Both shapes represent

limits for all possible interfaces at otherwise constant conditions σ_H , σ_V , τ_i and σ_i . Shape 1) belongs to the smallest interface immediately before a borehole bump, while shape 2) characterizes the largest interface after a bump. All other interfaces are enclosed between 1) and 2). It is remarkable that, before the

Fig. 11 Elastic plastic interfaces under biaxial pressure $\sigma_V/K=10$, $\sigma_V/\sigma_H=5/4$ at infinity if, 1) $K_R=K$, $K_C=4K$, 2) $K_R=K_C=K$

Fig. 12 Shape of the brittle-plastic domain around a borehole after^{(26),(27)}; material: Model-ARALDIT

Fig. 13 Related mass of fines of drilling Q versus vertical pressure σ_V at infinity for a horizontal pressure $\sigma_H/\sigma_V=0.75$ at infinity; experimentally measured curves from drilling with a continuously increasing pressure; experimental values $\diamond$ for a drilling under various fixed pre-stresses each; theoretical limit values, $\star$: before, $\circ$: after a borehole bump

bump, the larger diameter of the maximal interface points into the direction of the larger pre-pressure at infinity. Also differently shaped interfaces have been found, although without observing the shear stress τ_i at the borehole. Fig. 12 shows a brittle-plastic domain found on a specimen from Model-ARALDIT^{(25)-(27),(32)}.

In Fig. 13 the experimentally obtained mass of fines Q of drilling into a square shaped specimen of Model-ARALDIT⁽³²⁾ is compared with theoretical values. The drawn curves represent drilling experiments under a continuously increasing vertical and horizontal loading of the specimen, starting with the load zero. The stepwise growth of Q is caused by drilling bumps due to the increasing pressure. The marked experimental values refer to drillings, each carried out with various specimens under constant pre-stress. These values are below the drawn curves (although very close to the lower one), because under constant pre-stress the external perturbations are small and borehole bumps are retarded (lower limit), while an increasing load constitutes a perturbation by its own, thus supporting borehole bumps (upper limit). Theoretically obtained values for both cases are in a good agreement with the experiments, especially for lower loadings. For higher loads the specimens are too small in comparison to the diameter of the borer. Therefore the assumption, that the pre-stress acts at an infinite distance becomes invalid.

5. Conclusion

In an overview the important technological process of drilling is discussed from the continuum mechanical point of view for metallic, brittle or rock-like materials with or without strain-softening. The size and the shape of the plastic domain around a borehole, the stress-relief and the quantity of fines of drilling can be determined, and the phenomenon of borehole bumps can be explained. The theoretical findings are compared with experimental results; in most cases a good agreement is obtained. As a result the acquired knowledge should contribute to a better qualitative and quantitative understanding of the basic features of the process, allowing also an effective use under non-standard conditions.

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