Analysis of Shear Buckling of Cylindrical Shells*

Kunio KOKUBO**, Hideaki NAGASHIMA***,
Masaaki TAKAYANAGI*** and Akira MOCHIZUKI***

Cylindrical shells subjected to lateral loads buckle in shear or bending modes. The bending buckling mode is a diamond-shaped pattern or an elephant-foot bulge at the edge. The effects of geometrical imperfections and combined loading on buckling loads and postbuckling behavior are investigated by developing a special-purpose finite element analysis (FEA) program using the 8-node isoparametric shell element. Four types of imperfections are considered and the relationships between the imperfection amplitude and the buckling load are obtained. Three types of loading are taken into consideration. For short cylindrical shells, shear buckling is dominant but elephant-foot bulges appear with an increase in bending moments and axial loads. Effects of axial loads on shear buckling and elephant-foot bulges are calculated. Postbuckling behavior which is closely related to imperfection sensitivity is also investigated.

Key Words: Computational Mechanics, Buckling, Inelasticity, Finite-Element Method, Cylindrical Shell, Imperfection, Combined Loading, Postbuckling

1. Introduction

A great number of experiments and analyses have been performed to investigate buckling problems of cylindrical shells subject to axial force and external pressure and much of that data can be used in vessel design. For cylindrical shells subject to lateral loads, many studies⁽¹⁾⁻⁽⁹⁾ have been performed in the past few years concerning the seismic design of vessels. As is well known, the buckling phenomena of shell structures are affected by various parameters, for example, geometrical imperfections, boundary conditions and combined loading modes. As many studies are necessary to estimate effects of the parameters, and as it is not easy to estimate each parameter separately in experiments, numerical simulations are used to provide accurate approximations of buckling problems.

The shear buckling mode is dominant for short cylindrical shells subject to lateral loads⁽¹⁾⁻⁽⁵⁾. With increasing bending moment due to lateral loads, a bending buckling mode occurs even in a cylindrical shell that is expected to buckle in a shear mode⁽⁸⁾. The bending buckling modes have diamond-shaped patterns at the boundary for thin shells, but an elephant-foot bulge appears instead for thick cylindrical shells^{(4),(9)}. The bending buckling mode also becomes dominant for a cylindrical shell under the combined loading of bending loads, axial loads and internal pressure⁽⁴⁾.

In this study a special-purpose finite element analysis (FEA) program using the 8-node isoparametric shell elements is developed and effects of geometrical imperfections and combined loading on the buckling of cylindrical shells are investigated. Four types of imperfections (local dents, Fourier harmonic components, axisymmetric imperfections and their superpositions), and three types of loading (lateral loads, axial loads and pure bending moments), are taken into account in the analyses. Postbuckling behavior is also investigated in some respects.

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** Department of Mechanical Engineering, Kogakuin University, 2665-1 Nakano-machi, Hachioji-shi, Tokyo 192, Japan

*** Mechanical Engineering Research Laboratory, Hitachi, Ltd., 502 Kandatsu-machi, Tsuchiura-shi, Ibaraki-ken 300, Japan

2. Method of Finite-Element Analysis

As it is important to take into account the exact geometrical shape of the shell to estimate the effect of geometric imperfections, the 8-node isoparametric curved shell elements are used in the formulation. As our main objective is parametric study of the buckling strength of cylindrical vessels that buckle in shear and bending modes, the total Lagrangian formulation is adopted.

Strain ϵ is expressed by a sum of the linear term ϵ_1 and nonlinear term ϵ_2 of the displacement u of the shell. Stress σ is expressed by the product of Young's modulus tensor E and strain ϵ in the elastic stress range.

$$\epsilon = \epsilon_1(u) + \epsilon_2(u) \quad (1)$$

$$\sigma = E\epsilon \quad (2)$$

The first variation $\Delta\epsilon$ and second variation $\Delta^2\epsilon$ of strain in the case of the displacement $u + \Delta u$ are expressed respectively as

$$\Delta\epsilon = \epsilon_1(\Delta u) + \epsilon_{11}(u, \Delta u) \quad (3)$$

$$\Delta^2\epsilon = \epsilon_{11}(\Delta u, \Delta u) \quad (4)$$

where

$$\epsilon_{11}(u, \Delta u) \equiv \epsilon_2(u + \Delta u) - \epsilon_2(u). \quad (5)$$

Let us assume that the equilibrium configuration u corresponding to a load p is known. The incremental equilibrium equation to determine Δu that corresponds to a load increment Δp can be written as follows.

$$\int \{ \sigma^T \delta \Delta^2 \epsilon + \delta \Delta \epsilon^T E \Delta \epsilon \} dv = \int \Delta p^T \delta \Delta u dv \quad (6)$$

where dv is the integral over the total shell volume and the suffix T represent the transpose of tensors.

The middle surface of the 8-node shell element can be expressed by the two curvilinear coordinates x^1 and x^2 . The displacement u that corresponds to a point t at the position normal to the middle surface is assumed to be expressed by the displacement u_0 of the middle surface and $t^{(10)}$,

$$u = u_0 + t u_1 \quad (7)$$

where the displacement u_1 is the rotation of the normal n , as shown in Fig. 2, and can be expressed by

the rotation angle θ , where θ_{x^1} and θ_{x^2} are components of angle θ in x^1 and x^2 directions, respectively.

$$\begin{aligned} u_1 &= \{u_1, v_1, w_1\} \\ &= \{-\sin \theta \cdot \cos \beta, -\sin \theta \sin \beta, (1 - \cos \theta)\}, \end{aligned} \quad (8)$$

where β shown in Fig. 2 is given as follows

$$\cos \beta = \theta_{x^1} / \theta, \quad \sin \beta = \theta_{x^2} / \theta.$$

The displacements u_0 and u_1 are expressed by the interpolation function N_i and the nodal displacements $\bar{u}_0$ and $\bar{u}_1$.

$$u_0 = [N_i] \bar{u}_0, \quad u_1 = [N_i] \bar{u}_1 \quad (9)$$

As the displacement u_1 can be expressed in terms of θ_{x^1} and θ_{x^2} , the nodal displacements $\bar{u}_0$ and $\bar{u}_1$ can be finally expressed by the displacement $\bar{u} = (u, v_0, w_0, \theta_{x^1}, \theta_{x^2})$ using Eq. (8).

$$\delta u = \{ \delta u_0 + t \delta u_1 \} = C \delta \bar{u}, \quad (10)$$

where $C(\theta_{x^1}, \theta_{x^2})$ is given as the function of θ_{x^1} , θ_{x^2} using Eq. (8) and (9)⁽¹⁰⁾. Each term in the equilibrium equation (6) can be written as

$$\begin{aligned} \delta \Delta \epsilon &= \epsilon_1(\delta u) + \epsilon_{11}(\Delta u, \delta \Delta u) \\ &= B(\Delta u) \delta \Delta u \\ &= BC \delta \Delta \bar{u} \end{aligned} \quad (11)$$

$$\begin{aligned} \sigma^T \delta \Delta^2 \epsilon &= \sigma^T \epsilon_{11}(\Delta u, \delta \Delta u) \\ &= \Delta u^T \sigma^T \left(\frac{\partial B}{\partial \bar{u}} C + B \frac{\partial C}{\partial \bar{u}} \right) \delta \Delta u. \end{aligned} \quad (12)$$

The incremental equilibrium equation (6) can be written as

$$\begin{aligned} \int \delta(\Delta \bar{u})^T \left\{ C^T B^T E B C \Delta \bar{u} \right. \\ \left. + \sigma^T \left(\frac{\partial B}{\partial \bar{u}} C + B \frac{\partial C}{\partial \bar{u}} \right) \Delta \bar{u} \right\} dv \\ = \delta(\Delta \bar{u})^T K_t \Delta \bar{u} = \int \delta(\Delta u)^T \Delta p dv. \end{aligned} \quad (13)$$

K_t is known as the tangent stiffness matrix and can be expressed as follows.

$$K_t \equiv \int \left\{ C^T B^T E B C + \sigma^T \left(\frac{\partial B}{\partial \bar{u}} C + B \frac{\partial C}{\partial \bar{u}} \right) \right\} dv \quad (14)$$

The incremental equation can be solved efficiently using Riks' method in pre- and postbuckling states⁽¹¹⁾. For a structure with no imperfections, the calculations beyond the bifurcation point are not always easy. In

Fig. 1 8-node isoparametric shell element

Fig. 2 Rotation of normal vector

this case the structures with negligibly small imperfections are considered for calculations, instead of a perfect structure.

For elasto-plastic analyses, the so-called D^{ep} matrix⁽¹²⁾ is used instead of Young's modulus tensor E . The reduction in stiffness due to plasticity is taken into account in D^{ep} matrix. The D^{ep} matrix is used for elements in which plastic deformation occurs. The elastic matrix E is used for elements in which elastic deformation and unloading of plastic deformation occur.

3. Finite-Element Model

The FEA model for a cylindrical shell subject to lateral and axial loads and bending moments is shown in Fig. 3. The shell is clamped at the bottom and loaded on the upper part of the ring. The effective length $l=125$ mm, the radius $R=125$ mm, and the thickness $t=0.31$ mm ($R/t=400$) or 0.63 mm ($R/t=200$) are all shown in Fig. 3. The geometry of the shell corresponds to that of the shell studied in Ref. (7) Height H of the loading point can be changed and the moment PH at the boundary become larger with increasing H . Axial forces are added on the rigid ring. The uniform moment are added by distributing axial forces on the ring in $\cos \theta$ mode. Effects of combined loading can be examined by superposing these loads.

An elastic linear eigenvalue analysis is conducted

Fig. 3 FEA model of cylindrical shell

Fig. 4 Components of buckling modes (linear eigenvalue analysis)

for the buckling modes of the cylindrical shell, in which the axisymmetric finite-element and Fourier expansion methods are used. Various values of the component w_n of Fourier expansion of the normal buckling mode deflection are shown in Fig. 4. It can be seen that the shear buckling mode has a wide range of Fourier harmonic numbers while the diamond-patterned bending mode has lower Fourier harmonic numbers. The dominant Fourier harmonic numbers n in the shear buckling modes of shells with thicknesses 0.31 mm and 0.63 mm are 12 and 16 , respectively. Taking these results into account, we determine the number of discretized elements in circumferential direction is about 30 – 40 for a thick shell and 40 – 50 for a thin shell. For bending buckling, the fine discretizations are used on the compression side near the clamped boundary.

For elasto-plastic analyses, the stress and strain relation is assumed to be bilinear. Young's modulus coefficient E , the yield stress σ_y and the hardening coefficient $\bar{H}$ are assumed to be as follows.

$$E = 200 \text{ GPa}$$

$$\sigma_y = 320 \text{ MPa}$$

$$\bar{H} = 0.6 \text{ GPa}$$

The buckling modes obtained in nonlinear analyses are shown in Fig. 5. The distributions of wrinkles in the shear buckling mode and the elephant-foot bulge are quite different. The elephant foot bulge is found along the boundary. For simple support boundary conditions, the bulge approaches the lower boundary. The buckling modes in a circumferential direction are shown in Fig. 6. Although wrinkles of the shear mode spread over a wide range of angles, the center of which is nearly $\theta=90^\circ$. A convex deformation of the elephant-foot bulge also spreads over a wide range of angles, the center of which is $\theta=0^\circ$.

Fig. 5 Buckling modes

Fig. 6 Buckling modes

The relationships between the load P and the displacement $\bar{w}$ for thin and thick shells are shown in Fig. 7 and are compared with experimental values from Ref. (7). Load P_{cr} represents the buckling load obtained through calculation with no imperfections. The experimental buckling load of a shell ($R/t=400$) is nearly equal to the value for a shell with an imperfection, as obtained from FEA. For the thick shell ($R/t=200$), the experimental value of the buckling load is somewhat greater than that obtained from FEA. The differences in prebuckling nonlinearity between experiments and analyses are observed in Fig. 7. The differences may be due to the fact that the relationship between stress and strain is not well approximated by a bilinear relationship.

4. Geometrical Imperfection

It is well known that the shear buckling mode occurs in a short cylindrical shell subject to lateral loads. Four types of imperfections are considered in numerical analyses. The imperfections of types (I), (II) and (III) are shown in Fig. 8.

Type (I) corresponds to a local dent on the shell wall; the initial normal deflection w_0 of the shell can be expressed as

$$w_0 = \delta_I \sin \frac{\pi x}{l} \{1 + \cos n(\theta - \theta^*)\}.$$

The important parameters of type (I) are n and θ^* , where θ^* is the angle of the center of a local dent, $\delta_I > 0$ indicates a local convex imperfection and $\delta_I < 0$ indicates a local concave imperfection. The total amplitude δ of type (II) is given as $\delta = 2\delta_I$.

Type (II) is an imperfection over the entire shell surface and the Fourier harmonic component of the deflection, and can be written as follows:

$$w_0 = \delta_{II} \cos n\theta \sin \frac{\pi x}{l}.$$

The total amplitude δ is given as $\delta = 2\delta_{II}$.

Type (III) is the axisymmetric imperfection of the shell.

$$w_0 = \delta_{III} \sin \frac{m\pi x}{l}$$

The total amplitude of the imperfection δ can be expressed as $\delta = \delta_{III}$ ($m=1$) and $\delta = 2\delta_{III}$ ($m \geq 2$).

The fourth type is a superposition of these imperfections. The total amplitude of the imperfection can

Fig. 7 Load-displacement relationships (experiment and analysis)

Fig. 8 Initial imperfections

be expressed as the sum of their amplitudes. The effects of these imperfections on the elastic shear buckling of shells with $R/t=400$ are investigated.

For type (I) imperfections θ^* , n and δ_I are the parameters of importance. The buckling load varies with θ^* in case of $n=14$ and $\delta_I = \pm 0.75t$, as shown in Fig. 9. The reduction in buckling load is largest at $\theta^* = 90^\circ$. The effect becomes negligible when θ^* approaches 30° , which can be understood through inspection of the buckling mode shown in Fig. 5. The difference in buckling loads is small in both cases of $\delta > 0$ and $\delta < 0$, which indicates that the critical point is

close to the symmetric buckling point⁽¹³⁾. The relationships between the buckling load P_{cr} and the imperfection amplitude δ as a function of n are shown in Fig. 10. The imperfections of $n=14$ have a greater effect than those of $n=7$ for type (I).

Effects of type (II) imperfections are also investigated and compared with imperfections of buckling modes in Ref.(10). For type (II) the most effective mode shows the best correspondence with n which is dominant in the buckling mode. For $R/t=400$, n is nearly 14-16. The numerical calculations also verify the predictions, as shown in Fig. 11. The buckling load of $\delta/t=1.5$ ($n=14$, type (II)) is nearly 80% of the buckling load P_{cr}^0 of perfect shells. In the comparison of the results of $n=14$ in Fig. 9 and Fig. 11, it is shown that effects of type (II) are greater than those of type

Fig. 9 Effect of imperfection

Fig. 10 Effect of imperfection

Fig. 11 Effect of imperfection

(I). The deformation modes of the shell with type (II) imperfections of $n=22$ are shown in Fig. 12. At the first stage of deformation the modes are affected by the imperfections and approach the buckling mode with increasing deformation.

For axisymmetric imperfections, $m=1$ and $\delta_{III} > 0$ indicate a barrel-type imperfection and the buckling load increases with the amplitude of imperfections, as shown in Fig. 13. For $\delta_{III} < 0$ the buckling load decreases with the amplitude of imperfections. Effects of type (II) imperfections are shown for comparison in this figure.

For the superposed imperfections of type (III) and type (II) the effects are calculated and shown in Fig. 14. The effects are almost the sum of the effects of the individual imperfections.

For thick shells ($R/t=200$), plastic shear buckling occurs. The effects of imperfections ($n=12$, type (II)) are calculated and compared with those of elastic shear buckling in Fig. 15. Although effects of imperfections in both cases are not as important in comparison with the buckling under axial compressions, effects of imperfections must be taken into account in vessel design. It is possible to estimate the

Fig. 12 Deformation modes ($R/t=400$)

Fig. 13 Effect of imperfection

shear buckling load of a vessel by taking account of some of the typical geometrical imperfections treated here.

5. Effect of Combined Loading

Lateral load P , axial force F and bending moment M are considered in the calculation. These three types of loading are shown in Fig. 3.

First, elastic linear eigenvalue buckling analyses are conducted in which the lateral load P ($H/R=1.0$) and the pure bending moment M are applied to a shell of $R/t=400$. The interaction curve between the pure bending moment and the lateral load is shown in Fig.

Fig. 14 Effect of imperfection

Fig. 15 Effect of imperfection

Fig. 16 Interaction between lateral load and bending moment

16, in which the buckling load P_{cr} and M_{cr} are non-dimensionalized by the loads $\bar{P}_{cr}$ and $\bar{M}_{cr}$, which represent the buckling loads in the absence of other loads. Bending moment PH at the clamped boundary caused by the lateral load P is not included in the bending moment M_{cr} . The corresponding buckling modes are shown in Fig. 17. The bending buckling mode is a diamond shaped mode at the boundary. Considering the fact that the shear buckling mode and bending buckling mode are quite different, it can be expected that the interaction between the bending moment and the lateral force is small. Figure 16 also confirms that the interaction is small. Similar interaction curves by linear eigenvalue analysis are also obtained in Ref.(14).

For thick shells ($R/t=200$), the elephant-foot bulge appears instead of the diamond-shaped bending mode, and the interaction curve cannot be obtained by linear eigenvalue analyses. For thick shells the shear and bending plastic buckling modes were shown in Fig. 5. An interaction curve between the lateral load P_{cr} and the bending moment M_{cr} is obtained in Fig. 18. The moment ($M_{cr} + HP_{cr}$) at the boundary is also shown in this figure; it shows that the interaction between P_{cr} and ($M_{cr} + HP_{cr}$) is very small.

The lateral buckling load P_{cr} of the shell ($R/t=200$) is calculated by changing the height of the loading point. With the increase in the moment HP_{cr} at the boundary, an elephant-foot bulge appears instead of the shear mode as shown in Fig. 19. For shear buckling, the effects of H on buckling loads are small and

Fig. 17 Buckling modes ($R/t=400$)

effects of H become larger for bending buckling.

For the seismic design of vessels the axial force must be taken into account, as well as the lateral load. The effects of axial forces are calculated for shear and bending buckling. Shear buckling corresponds to $H/R=1.0$ and bending buckling corresponds to $H/R=2.5$, as shown in Fig. 20. Effects of axial forces are not very significant for the shear buckling mode, but the buckling mode changes to the bending mode with increasing axial force. For the bending buckling load, the effects of axial force become very important. Figure 21 gives a summary of Figs. 19 and 20 for shells of $R/t=200$. Effects of boundary conditions and axial forces and the height of the loading point are summarized in the figure.

Fig. 18 Interaction between lateral load and bending moment

Fig. 19 Effect of loading point

Fig. 20 Effect of axial force

ized in the figure.

6. Postbuckling Behavior

As the load-carrying capacity in the postbuckling state is closely related to imperfection sensitivity of shells, as pointed out by Koiter⁽¹³⁾, analyses of postbuckling behavior are important for the buckling problem. For the shell of $R/t=200$, the postbuckling behavior in a shear mode is calculated as shown in Fig. 22. The reduction in shell stiffness in the postbuckling state is not very drastic in comparison with that of buckling under axial compression. This may be due to the fact that a tension stress field appears on the shell wall in the same manner as in buckling of a plate under shear loads. The load carrying capacity after buckling can be expected to a certain extent, as shown in Fig. 22, and the imperfection sensitivity is not very significant for shear modes as already seen in Fig. 15.

The buckling load of an imperfect shell ($\delta/t=1.0$, type (II)) is slightly smaller than that of a perfect structure, but the difference of the stiffness of the shell becomes smaller with increasing deformations in the postbuckling states. It can be concluded that the initial imperfection does not affect postbuckling behavior. The deformation modes obtained through ana-

Fig. 21 Effect of loading point and axial force

Fig. 22 Postbuckling behavior

lyses in postbuckling states show the same patterns as the buckling mode.

As nonlinearity in prebuckling states depends upon the stress-strain relationship adopted in the calculations, the prebuckling nonlinearity is small due to the bilinear stress-strain relationship. Although more exact approximations of stress and strain relationships should be adopted to predict prebuckling nonlinearity, postbuckling behavior as a whole can be obtained by the bilinear stress and strain relationship.

7. Conclusions

The effects of geometrical imperfections and combined loading on the buckling of cylindrical shells and postbuckling behavior are investigated by means of nonlinear finite-element analyses. The following findings are obtained.

(1) The effects of geometrical imperfections on shear buckling of cylindrical shells are not very significant in comparison with those of buckling under axial compressions.

(2) The elephant-foot bulge becomes dominant with an increase in bending moment or axial compression.

(3) Axial compression has a greater effect on bending buckling than on shear buckling.

(4) The interaction between bending buckling and shear buckling is small.

(5) The load-carrying capacity after shear buckling does not decrease drastically.

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