The Law of Similarity on the Fracture Behavior of Continuous Fiber-Reinforced Composite Laminates with Different Ply Thicknesses*

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The macroscopic fracture behavior of composite laminates is now being clarified to be controlled by the extension of microscopic cracks. On the basis of released elastic strain energy accompanied by microcrack extension, we present simple equations which enable us to predict the fracture behavior of the laminate with arbitrary ply thickness by using the experimental results of the laminate with standard ply thickness. We come to the conclusion that the damaged region ahead of notch tips in laminates extends to a size proportional to the ply thickness under the same apparent critical stress intensity factor. The difference in the maximum apparent critical stress intensity factor sustained by the laminate with different ply thicknesses is also quantitatively examined. The predicted fracture behavior is in good agreement with the experimental results of continuous glass fiber-reinforced epoxy cross-ply laminates with three different ply thicknesses.

Key Words: Composite Material, Fracture Behavior, Continuous Fiber-Reinforced Composite Laminate, Ply Thickness, Microcrack Extension, Released Elastic Strain Energy

1. Introduction

Continuous fiber-reinforced composite laminates have great advantages not only of specific strength and stiffness but also of high degree of freedom for material designing. In the practical application of this kind of materials, then, we have to design not only the elastic properties but also the fracture behavior for the efficient use of their potential.

The fracture behavior of composite laminates is generally very much complicated and is known to be greatly influenced by lamination geometry such as fiber orientations and lay-up sequences. It is becoming increasingly important that we have to obtain systematic knowledge about the relationship between the fracture behavior and the structure of continuous fiber-reinforced composite laminates.

Until now, many authors have discussed the fracture process of continuous fiber-reinforced composite laminates. In particular, a great number of experi-

ments have been conducted on various kinds of laminates in order to verify the dependence of fracture behavior on lamination geometry. For example, Garg⁽¹⁾ conducted detailed observations of the fracture behavior of notched graphite/epoxy cross-ply laminates with various lay-up sequences and examined the fracture toughness. Ochiai and Peters⁽²⁾ surveyed the factors which affect the fracture behavior and stated that an increase in ply thickness led to higher toughness, while Herakovich⁽³⁾ presented the finding that thicker ply resulted in lower strength.

On the other hand, several papers were recently coming out in which they directly discussed the extension and multiplication of microscopic cracks inside the materials, such as intralaminar and interlaminar cracks, during the fracture process of the laminates. Dvorak and Laws⁽⁴⁾ and Laws and Dvorak⁽⁵⁾ extensively studied the accumulation of intralaminar cracks in cross-ply laminates and reported that ply cracking took place under the condition of a constant strain energy release rate. The same kind of conclusion was obtained by O'Brien⁽⁶⁾ and Crossman and Wang⁽⁷⁾ with respect to the delamination onset along straight edges in graphite/epoxy laminates under tensile load. Dam-

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age extension ahead of notch tips in composite laminates was also found by the present authors⁽⁸⁾ to be an extension and bifurcation process of microcracks with a constant strain energy release rate. Kortschot and Beaumont⁽⁹⁾ calculated by using a finite element method the extension of microcracks in graphite/epoxy cross-ply laminates and proposed a damage-based strength model, which explains the effect of notch size and lay-up sequence on damage growth and notched strength.

These reports suggest that microcrack extension, which is controlled by the constant strain energy release rate, results in the macroscopic fracture behavior of composite laminates. Hence, if we could understand how lamination geometry influences the release of elastic strain energy, it would naturally explain the difference in the fracture behavior among the laminates with different lamination geometries. In this paper, we try to present the law of similarity on the fracture behavior of continuous fiber-reinforced composite laminates with different ply thicknesses on the basis of released elastic strain energy accompanied with microcrack extension, which is quite an important problem for material designing. Fiber orientations of a laminate will be determined by the requirements for its elastic properties. However, in the case of microcrack extension under a constant strain energy release rate, two laminates of the same fiber orientation with different ply thicknesses will never fracture in the same way, because the macroscopic critical stress which provides the critical strain energy release rate is different for each laminate. We derive some essential equations in which we can clearly understand how ply thickness affects the damage extension behavior ahead of notch tips. These equations also enable us, without any elaborate numerical computations, to predict the fracture resistance curves of the laminates with various ply thicknesses.

2. The Influence of Ply Thickness on the Condition of Microcrack Extension

In this paper, we deal with the extension of microcracks ahead of notch tips in continuous fiber-reinforced cross-ply laminates. The appearance of the damaged region ahead of a notch tip is typically illustrated in Fig. 1, where a tensile load is applied in the direction perpendicular to the notch which is parallel to one of two orthogonal fiber directions. The damage consists of two kinds of microcracks. One is the intralaminar microcracks extending parallel to the fibers in each ply, and the other is the interlaminar cracks spreading along the interface between 0° -plies and 90° -plies. In this paper, 0° -ply and 90° -ply indicate

the plies in which fibers run in the direction parallel and perpendicular to the load, respectively. In this chapter, we focus our attention on these two kinds of microcracks and consider the influence of ply thickness on their extension behavior.

Let us first examine the extension of intralaminar cracks in two laminates with different ply thicknesses, i.e., a laminate with standard ply thickness and another laminate with n -times larger ply thickness. If these two laminates are made of the same fibers and matrix and have the same fiber volume fraction, their macroscopic elastic properties are approximately identical. Figure 2 shows the cross sections A-A' in Fig. 1. The width of intralaminar cracks is almost equal to the ply thickness, so that n -times wider cracks extend in the laminate with n -times larger ply thickness. In Fig. 2, cracks would suddenly appear when the intralaminar cracks extended across section A-A' in Fig. 1. Now we consider the deformation of these cross sections as two-dimensional problems and examine the elastic strain energy released by the appearance of these intralaminar cracks.

When the area of a crack increases by δA in the domain of volume V , the energy release rate G is given by Kamiya and Sekine⁽⁸⁾ in the following equation.

Fig. 1 Schematic view of the damage ahead of a notch in cross-ply laminate

Fig. 2 Intralaminar cracks in the laminates with the standard ply thickness and n -times larger ply thickness

$$G = -\frac{V}{2} \tau_{ij} \tau_{kl} \frac{\delta c_{kl}^{ij}}{\delta A} \quad (1)$$

where c_{kl}^{ij} is the macroscopic compliance averaged in the domain V and δc_{kl}^{ij} means its increment associated with the crack extension. τ_{ij} is also the macroscopic stress averaged in V . Now suppose two similar domains, with the similitude ratio n , in each laminate as shown in Fig. 2. Then, Eq. (1) gives us the energy released per unit area of the intralaminar cracks,

$${}^1G = \frac{{}^1V}{2} {}^1\tau_{ij} {}^1\tau_{kl} \frac{\delta {}^1c_{kl}^{ij}}{\delta {}^1A} \quad (2)$$

$${}^nG = \frac{{}^nV}{2} {}^n\tau_{ij} {}^n\tau_{kl} \frac{\delta {}^nc_{kl}^{ij}}{\delta {}^nA} \quad (3)$$

where left-hand superscripts 1 and n denote the quantities related to the laminates with the standard ply thickness and n -times larger ply thickness, respectively. Since these two domains have perfectly similar configurations including the cracks, the macroscopic compliances are identical to each other before and after the appearance of cracks. By taking account of the similitude ratio n , Eq. (3) can be transformed into the following equation.

$${}^nG = \frac{{}^nV}{2} {}^n\tau_{ij} {}^n\tau_{kl} \frac{\delta {}^nc_{kl}^{ij}}{\delta {}^nA} \quad (4)$$

According to Griffith's criterion, the energy released by the crack extension would turn into the surface energy of newly created crack surfaces. Remember these two laminates are made of the same constituent materials, so that the surface energy, i.e., the critical strain energy release rates which drive the cracks, should be identical to each other. Then we can equate Eq. (4) with Eq. (2) and obtain the following relation of the critical stress τ_{ij}^c concerning the intralaminar crack extension in the two laminates.

$${}^n\tau_{ij}^c {}^n\tau_{kl}^c \frac{\delta {}^nc_{kl}^{ij}}{\delta {}^nA} = \frac{1}{n} {}^1\tau_{ij}^c {}^1\tau_{kl}^c \frac{\delta {}^1c_{kl}^{ij}}{\delta {}^1A} \quad (5)$$

In the case where we consider the corresponding points in the specimens of the same configuration, i.e., the stress tensors are identical except for a scalar factor, Eq. (5) presents us with the result that the laminate with n -times larger ply thickness requires only the stress of $\sqrt{1/n}$ in order to drive interlaminar cracks.

Next, let us examine the extension of interlaminar cracks. Notice that all of the 0°- and 90°-plies at the notch tip have intralaminar cracks in each fiber direction. These intralaminar cracks cause interlaminar cracks ahead of the notch tip, so that we can see interlaminar cracks running after the intralaminar cracks inside the damaged region as shown in Fig. 1. Figure 3 shows cross section B-B' in Fig. 1. The extension of these interlaminar cracks can be regarded as a two-dimensional debonding problem, as you can see in Fig. 3, where 0°-plies are debonded from 90°

-plies. When 0°-plies are debonded in Fig. 3 by δl under the tensile stress σ acting on 0°-plies in the direction of fibers, the elastic strain energy released per unit thickness of the laminate, δW , can be written as

$$\delta W = \frac{1}{4} \sigma^2 \delta l \left(\frac{1}{E_L} - \frac{1}{E_L + E_T} \right) \quad (6)$$

where E_L and E_T are the Young's moduli of plies in the directions parallel and perpendicular to fibers respectively. Since interlaminar cracks also extend under a constant critical strain energy release rate G_d , the critical stress σ_d which results in the extension of the interlaminar cracks can be obtained by using the relation $G_d = \delta W t_p / \delta l$,

$$\sigma_d = \sqrt{\frac{4G_d}{t_p \left(\frac{1}{E_L} - \frac{1}{E_L + E_T} \right)}} \quad (7)$$

where t_p is the ply thickness, which is common in both 0°- and 90°-plies. Equation (7) tells us that the critical stress which drives the interlaminar cracks is also inversely proportional to the square root of the ply thickness. Concerning the two laminates with the standard and n -times larger ply thickness, we obtain the same kind of relationship as in the case of intralaminar cracks,

$${}^n\sigma_d = {}^1\sigma_d / \sqrt{n}. \quad (8)$$

Now we come to the conclusion that microcracks extend more easily in the laminate with larger ply thickness. In the next chapter we attempt to research how this fact affects the fracture behavior in the stress field ahead of notch tips.

3. The Law of Similarity on the Damage Extension Behavior Ahead of Notch Tips

In this chapter, we attempt to clarify the effect of ply thickness and the critical stress obtained in the previous chapter. We introduce the stress field ahead of a crack tip in an anisotropic material⁽¹⁰⁾ as an approximation of the notch tip stress field in the laminates, which is expressed in the form

Fig. 3 Two-dimensional debonding problem of the interlaminar cracks ahead of the notch tip

$$\tau_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta). \quad (9)$$

In Eq.(9), r and θ represent the polar coordinates with the origin at the crack tip, while K is the stress intensity factor and $f_{ij}(\theta)$ is a known function of θ .

Consider first the fracture resistance curve of the laminate with the standard ply thickness, which is the relationship between the apparent critical stress intensity factor 1K_c at the notch tip and the extension of microcracks $\Delta^1 a(\theta)$ measured from the notch tip in the direction of r with the angle θ . Since the increment of the compliance due to the extension of microcracks is small enough compared to the compliance of the laminate, the disturbance of the stress field caused by microcracks may be ignored in order to estimate $\Delta^1 a$. In the vicinity of extending microcracks, Eq.(9) gives us the relation

$${}^1\tau_{ij}^c = \frac{{}^1K_c}{\sqrt{2\pi \Delta^1 a(\theta)}} f_{ij}(\theta) \quad (10)$$

where ${}^1\tau_{ij}^c$ is the critical stress mentioned in the previous chapter. Equation (10) can be transformed into

$${}^1K_c = \frac{{}^1\tau_{ij}^c}{f_{ij}(\theta)} \sqrt{2\pi \Delta^1 a(\theta)} \quad (11)$$

which directly shows the fracture resistance curve. Note that you can use any combination of i, j to calculate Eq.(11), since the stress intensity factor is common to all components of the stress tensor as presented in Eq.(10).

We next consider the case of the laminate with n -times larger ply thickness. In this case, the critical stress is written as

$${}^n\tau_{ij}^c = \frac{{}^nK_c}{\sqrt{2\pi \Delta^n a(\theta)}} f_{ij}(\theta). \quad (12)$$

Substituting Eqs.(10) and (12) for the critical stress in Eq.(5), we obtain the following relation,

$$\frac{{}^nK_c^2}{\Delta^n a(\theta)} = \frac{1}{n} \frac{{}^1K_c^2}{\Delta^1 a(\theta)} \quad (13)$$

Equations (11) and (13) will lead to the fracture resistance curve of the laminate with arbitrary ply thickness of n :

$${}^nK_c = \frac{{}^1\tau_{ij}^c}{f_{ij}(\theta)} \sqrt{2\pi \frac{\Delta^n a(\theta)}{n}}. \quad (14)$$

The first glance at Eq.(14) tells us that the microcracks extend proportional to the square of the apparent critical stress intensity factor. It also shows that these microcracks will form the damaged region extending in a self-similar manner, i.e. the shape of the damaged region remains constant while it grows in scale, since the variation in $\Delta^n a$ with respect to θ is determined only by f_{ij} which is independent of the applied load. We also find another conclusion in Eq.(14) that the size of the damaged region appears just proportional to the ply thickness under the same apparent critical stress intensity factor.

In the previous chapter, we considered the interlaminar cracks as a debonding problem in order to clarify the conditions of their extension. Now that we have to quantitatively estimate the interlaminar crack extension, we attempt to set the debonding model into the displacement field ahead of the notch tip, while attention is focused on the displacement at the front of the interlaminar cracks. Again, we utilize the displacement field ahead of crack tips as the first-order approximation. Then, taking the polar coordinates r and θ at the crack tip, the displacement v in the direction of fibers in the 0° -plies, i.e., perpendicular to the crack, is given in the following equation⁽¹⁰⁾.

$$v = K g(\theta) \sqrt{r} \quad (15)$$

where K is the stress intensity factor and $g(\theta)$ is a known function of θ . Here attention is focused on the maximum extension $2l$, as a representative measure of the interlaminar crack extension behavior, which is observed immediately ahead of the notch tip as shown in Fig. 1. Then θ in Eq.(15) should be set at 90° , which means the plane perpendicular to the crack, and we can examine the 0° -plies debonded at the notch tip. When 0° -plies are debonded as shown in Fig. 3, taking account of the symmetry with respect to the notch plane, their elongation δ at the tips of the interlaminar cracks is given by

$$\delta(l) = \frac{\sigma_d l}{E_L} \quad (16)$$

where l is the half-length of the interlaminar cracks at the notch tip and σ_d and E_L are, respectively, the critical stress and Young's modulus of a ply in the direction of fibers as mentioned in the previous chapter. According to our concept of the first-order approximation of the displacement field around the notch tip, the elongation δ should be equal to the displacement v given by Eq.(15) at $r=l$ with θ set at 90° . Then, we get the following relation between the apparent critical stress intensity factor K_{dc} and the half-length of the interlaminar cracks at the notch tip l .

$$K_{dc} = \frac{\sigma_d \sqrt{l}}{E_L g(90^\circ)} \quad (17)$$

In view of the relationship expressed in Eq.(8), we obtain a kind of delamination resistance curve of a laminate with an arbitrary ply thickness,

$${}^nK_{dc} = \frac{{}^1\sigma_d \sqrt{l/n}}{E_L g(90^\circ)} \quad (18)$$

where n is the similitude ratio of ply thickness stated before. Equation (18) tells us that the interlaminar cracks also extend proportional to both the square of the apparent critical stress intensity factor and the ply thickness, which is the same as in the case of intralaminar crack extension.

Now let us finally consider the failure of the 0° -

plies at the notch tip, which will correspond to the maximum load sustained by cross-ply laminates with notches. The probability that a brittle material of length $2l$ will be broken by the tensile stress σ is known to be expressed in the following equation⁽¹¹⁾.

$$F(\sigma, 2l) = 1 - e^{-2lN(\sigma)} \quad (19)$$

Here $N(\sigma)$ represents the number of flaws per unit length of the fiber, which will lead to failure under tensile stress less than σ . Introduction of the Weibull distribution as $N(\sigma)$ gives

$$N(\sigma) = (\sigma/\sigma_0)^m \quad (20)$$

where m and σ_0 are Weibull parameters. Remember that different ply thicknesses result in different values of critical stress in 0° -plies. If all 0° -plies had the same value of $F(\sigma, 2l)$ as the state of failure regardless of the thickness, we could find different critical lengths of interlaminar cracks at which 0° -plies would break. Again, we consider two laminates with the standard and n -times larger ply thickness. Then Eq.(19) leads to the following relation according to the concept of the same probability of failure

$${}^1l_b N({}^1\sigma_d) = {}^n l_b N({}^n\sigma_d),$$

which is immediately transformed with Eqs.(8) and (20) into

$${}^n l_b = {}^1 l_b n^{m/2} \quad (21)$$

where subscript b refers to the value at which failure takes place. Equation (21) suggests that the interlaminar cracks of the laminate with thicker plies will spread more widely until failure. By using Eqs.(18) and (21), we obtain the failure envelope which gives the maximum apparent stress intensity factor sustained by the laminates with various ply thicknesses as a function of the maximum interlaminar crack extension, i. e.,

$${}^n K_{dc}({}^n l_b) = {}^1 K_{dc}({}^1 l_b) \left(\frac{{}^n l_b}{{}^1 l_b} \right)^{\frac{m-2}{2m}} \quad (22)$$

We have derived several equations which describe the fracture behavior of laminates, i. e., Eqs.(14), (18), and (22) for instance. Notice that in these equations the fracture behavior of the laminate with n -times larger ply thickness is figured by referring to the case of the standard ply thickness. In this sense these equations give us the law of similarity on the fracture behavior of laminates with different ply thicknesses. Once we see the fracture behavior of the laminate with the standard ply thickness, fracture behavior of a laminate with an arbitrary ply thickness can be seen with these equations without knowing the values of parameters, such as ${}^1\tau_{ij}^c$, $f_{ij}(\theta)$, E_L , ${}^1\sigma_c$, $g(90^\circ)$, and so on. In the following chapter we will examine the adequacy of these equations with some experimental results.

Table 1 Pre-preg lay-up sequence and ply thickness

Laminate	Prepreg lay-up sequence	Ply thickness mm
A	$[(0^\circ/90^\circ)_{27}/0^\circ]$	0.11
B	$[0^\circ_2/(90^\circ_3/0^\circ_3)_8/90^\circ_3/0^\circ_2]$	0.32
C	$[0^\circ_5/(90^\circ_9/0^\circ_9)_2/90^\circ_9/0^\circ_5]$	0.92

Table 2 Notch length and thickness of the specimen

Specimen	a mm	t mm
A-1	30.0	6.1
A-2	29.9	6.3
B-1	29.9	6.1
B-2	30.1	6.3
C-1	29.9	6.3
C-2	29.9	6.5

Fig. 4 Compact tension specimen

4. Materials and Experiment

Materials used in this report are cross-ply laminates made by Nippon Steel Chemical Co., Ltd., which contain $13\text{-}\mu\text{m}$ -diameter E-glass fibers as reinforcement and epoxy resin as matrix. The fiber volume fraction is approximately 45 %. Each laminate consists of 55 sheets of pre-preg with thickness 0.11 mm. Three different lay-up sequences shown in Table 1 provide three laminates A, B and C with different ply thicknesses. In Table 1 is also listed average ply thickness measured in each laminate.

Compact tension specimens were made of these laminates as shown in Fig. 4. Notches in the specimens were machined parallel to the fiber direction of the surface plies by using a carbon blade. All of the radii of the notch tips are 0.25 mm. The notch length and thickness t of each specimen are listed in Table 2.

The experiment was performed with a screw-driven testing machine at a constant crosshead speed of 0.2 mm/min for all specimens. During the experiment, we recorded the applied load and photographed

the damage ahead of the notch tip with transparent lighting. We continued to increase the load until the specimens could sustain no more load, i.e., until passing the maximum load, except for the case of specimen C-1 which was unloaded before its maximum load. After the experiment, the extension lengths of intra- and interlaminar cracks were measured with the photographs and plotted against the apparent critical stress intensity factor at the notch tip.

5. Results and Discussion

Figure 5 shows the relation between the apparent critical stress intensity factor K_c and the extension of the intralaminar microcracks, Δa_0 , in the direction parallel to the notch. All of the specimens had approximately the same elastic compliance in the early stage of loading, which means that the three laminates A, B and C have macroscopically the same elastic properties. However, as you can see in Fig. 5 their fracture behavior is never unique. Three solid lines in Fig. 5 indicate the relative differences in the fracture resistance curves predicted by Eq.(14), which are in good agreement with the experimental data. In the case of laminate C, with the progress of microcrack extension, the experimental data deviate in the direction of higher load from the predicted curve. Taking account of the fact that the specimens used in this experiment have the ligament of only 20 mm, this tendency is supposed to come from the configuration of the specimen. In Fig. 5 are also shown the numerical simulation results obtained by using the method of Kamiya and

Fig. 5 Apparent critical stress intensity factor against the extension of intralaminar cracks

Sekine⁽⁸⁾ on the basis of the constant strain energy release rate of intralaminar crack extension, in which the disturbance of the stress field caused by the specimen configuration and intralaminar crack extension is correctly accounted. To say nothing of the agreement with the experimental results, it is noteworthy that the simulation results are going along with the curves given by Eq.(14) up to about 4 mm of intralaminar crack extension. The symbol $\star$ in Fig. 5 represents the simulation result of laminate C for the case of a 2TCT specimen, which is twice as large as the specimen shown in Fig. 4. This result is clearly in better agreement with the predicted curve over a wider range of microcrack extension. Therefore we can see that the disturbance of the stress field caused by the intralaminar cracks is small, and that Eq.(14) gives a fairly good approximation within the small range of the intralaminar crack extension up to the length of about 20 % of the specimen ligament.

On the other hand, the extension of interlaminar cracks at the notch tip measured in the direction perpendicular to the notch is plotted against the apparent critical stress intensity factor in Fig. 6. Theoretical predictions by Eq.(18) are also presented in solid lines. In the case of the intralaminar crack extension parallel to the notch, compressive stress acting on the ligament of the compact tension specimen would deviate the actual fracture resistance curves from the theoretical prediction. However in the case of interlaminar cracks, Eq.(18) provides the perfect explanation for each laminate since the deformation just ahead of the notch tip may scarcely be affected by the finite configuration of the specimen. The broken line in Fig. 6 represents the failure envelope given by Eq.(22) with the Weibull parameter $m=4.0$ ⁽¹²⁾. The intersections of the broken line and the fracture resistance curves indicate the maximum apparent critical stress intensity factor sustained by

Fig. 6 Apparent critical stress intensity factor against the extension of interlaminar cracks

the laminates. As you can see, Eq.(22) gives good predictions of the drastically different failure points for the three laminates. The actual maximum stress intensity factor of laminate A obtained in the experiment looks slightly higher than the prediction. There are two possible reasons come into our mind for this finding. One is the effect of notch tip radius which results in smaller deformation than in the case of a crack tip. The other comes from the Weibull parameter $m=4.0$ which is the value obtained for the case of a single fiber. The fabrication process may damage the fibers, which will result in a lower value of m inside the composite materials. The value of 3.7 for m actually improves the accuracy of the prediction.

As can be seen in Figs. 5 and 6, the equations derived here are proven to provide a reliable law of similarity on the fracture behavior of continuous fiber-reinforced cross-ply laminates with different ply thicknesses. It is clear that the microcracks in the laminates with different ply thicknesses should have the same critical strain energy release rate, which leads to the law of similarity on the fracture behavior in the stress field ahead of notch tips.

6. Conclusions

The difference in the fracture behavior of continuous fiber-reinforced cross-ply laminates with different ply thicknesses has been examined from the viewpoint of the elastic strain energy released by the extension of intra- and interlaminar microcracks. The microcracks in laminates are known to extend under the condition of a constant elastic strain energy release rate. If the constituent materials of the laminates, such as the matrix and fibers, are identical, this critical strain energy release rate is supposed to be independent of the ply thickness of the laminate. In this case, the critical stress which drives microcracks should be inversely proportional to the square root of ply thickness. We introduced the crack tip stress field for an approximation of the stress field ahead of the notch tip, and came to the following conclusions. The damaged region ahead of the notch tip, which consists of microcracks, extends to the size proportional to the square of the apparent critical stress intensity factor, and it also extends in proportion to the ply thickness under the same apparent critical stress intensity factor. We also examined the failure of plies at the notch tip and explained the difference in the maximum apparent stress intensity factors sustained by the laminates with different ply thicknesses. The maximum apparent stress intensity factor sustained by a laminate corresponds to the failure of plies at the notch tip in which fiber runs in the direction of the applied load. Since the critical stress acting on the

plies in the damaged region is inversely proportional to the square root of the ply thickness, a laminate with thicker ply can sustain a larger apparent critical stress intensity factor until failure.

The fracture behavior predicted by the simple equations obtained here perfectly coincides with the experimental results on the glass fiber-reinforced epoxy cross-ply laminates with three different ply thicknesses. These equations present the law of similarity, which enables us to predict the fracture behavior of laminates with arbitrary ply thickness by using the experimental results of only one laminate with the standard ply thickness. We believe that the knowledge of the similarity discussed here is quite important information for designing the fracture behavior of composite laminates.

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References

- (1) Garg, A., Fracture Behavior of Cross-Ply Graphite/Epoxy Laminates, *Eng. Frac. Mech.*, Vol. 22, No. 6 (1985), p. 1035.
- (2) Ochiai, S. and Peters, P. W. M., Effects of Notch Length, Specimen Thickness, Ply Thickness and Type of Defect on the Fracture Behavior of Angle-Ply Graphite-Epoxy Composites, *J. Mater. Sci.*, Vol. 17, No. 8 (1982), p. 2324.
- (3) Herakovich, C. T., Influence of Layer Thickness on the Strength of Angle-Ply Laminates, *J. Compos. Mater.*, Vol. 16, No. 3 (1982), p. 216.
- (4) Dvorak, G. J. and Laws, N., Mechanics of First Ply Failure in Composite Laminates, *AMD*, Vol. 74 (1985), p. 59.
- (5) Laws, N. and Dvorak, G. J., Progressive Transverse Cracking in Composite Laminates, *J. Compos. Mater.*, Vol. 18, No. 3 (1988), p. 900.
- (6) O'Brien, T. K., Characterization of Delamination Onset and Growth in a Composite Laminate, *ASTM STP 775* (1982), p. 140.
- (7) Crossman, F. W. and Wang, A. S. D., The Dependence of Transverse Cracking and Delamination on Ply Thickness in Graphite/Epoxy Laminates, *ASTM STP 775* (1982), p. 118.
- (8) Kamiya, S. and Sekine, H., The Simulation of Damage Extension in Continuous Fiber-Reinfor-

- ced Cross-Ply Laminates, JSME Int. J., Ser.I, Vol. 34, No. 2 (1991), p. 214.
- (9) Kortschot, M. T. and Beaumont, P. W. R., Damage Mechanics of Composite Materials, Compos. Sci. Tech., Vol. 39, No. 3 (1990), p. 289, Vol. 40, No. 2 (1991), p. 147, (four-part series papers).
- (10) Liebowitz, H. ed., Fracture, Vol. 2 (1968), p. 125, Academic Press.
- (11) Olshansky, R. and Mauer, R. D., Tensile Strength and Fatigue of Optical Fibers, J. Appl. Phy., Vol. 47, No. 10 (1976), p. 4497.
- (12) Niki, E., Proc. 2nd Simpo. Compos. Mater., (in Japanese), (1969), p. 1.
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