Closed-Form Solution for Thermo-Elasto-plastic Strains in Semiconductor Chip Bonding Structures*

Akio YASUKAWA**

A closed-form solution is presented for calculating the strains produced by temperature changes in semiconductor chip bonding structures; it is based on beam theory and elasto-perfect plastic bonding layer assumptions. This solution agrees well with three-dimensional FEM and experimental results. The parameters that govern the strains produced in bonding structures are derived from this solution. These parameters are charted in order to obtain the maximum shearing strains of bonding layers and the maximum normal stresses of chips.

Key Words: Electronic Equipment, Thermal Stress, Plasticity, Structural Analysis, Solder, Bonding

1. Introduction

Evaluation of the strains produced in semiconductor chip bonding structures during thermal cycling is one of the most fundamental objectives in semiconductor device design. This is because these strains often cause fatigue failure of the bonding layers and brittle fracture of the chips. Simple analytical closed-form solutions are needed to provide insight into strain behavior so that device structural design can be improved.

One of the most significant features of chip bonding structures is that they contain plates made of different materials. One of the earliest studies of these structures was done by Timoshenko⁽¹⁾, who analyzed structures in which two beams made of different materials were rigidly connected. He did not, however, address the bonding layer strains. Taylor and Yuan⁽²⁾, Chen and Nelson⁽³⁾, and Suhir⁽⁴⁾ did analyze bonding layer stresses. They did not, however, take

into account the effects of the inelastic behavior of the bonding materials. This inelastic behavior is common in most solder materials used for chip bonding layers.

In this paper we present a simple closed-form solution in which the inelastic behavior of bonding layers is taken into account.

2. Derivation of the Solution

The problem to be solved is the three-dimensional deformation of structures in which the upper member (the chip) and the lower member (the substrate) are bonded by a bonding layer, as shown in Fig. 1. We use a coordinate system in which the x - y plane coincides with the middle plane of the bonding

Fig. 1 Semiconductor chip bonding structure

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** Mechanical Engineering Research Laboratory, Hitachi, Ltd., 502 Kandatsu, Tsuchiura 300, Japan

layer, and the x and y axes are parallel to two right-angled sides of the chip.

To solve this problem, the three-dimensional deformation is approximated as a combination of the two-dimensional deformations in the y - z and the z - x planes, and the two bonded members are modelled as beams. The z - x plane deformation is analyzed in terms of a model shown in Fig. 2. An external force F_{UN} and a bending moment M_{UN} act on both ends of the upper member. F_{LN} and M_{LN} act similarly on the lower member. The deformations produced by temperature changes can be obtained by assigning F_{UN} , M_{UN} , F_{LN} , and M_{LN} some values which correspond to the temperature changes⁽⁶⁾. In this section, the solution to the problem is derived based on the ideas in Ref. (6).

2.1 Basic equations

In this section we derive the basic equations which generally hold in any relationship between shear stress τ and shear strain γ of a bonding layer, as expressed by

$$\tau = \tau(\gamma). \quad (1)$$

Bonding layer shear strain γ is expressed by

$$\gamma = \frac{u_L - u_U}{h}, \quad (2)$$

where u_U and u_L are the x -directional displacements of the upper and lower surfaces of the bonding layer, respectively, and h is the thickness of the bonding layer.

The upper surface of the bonding layer is the lower surface of the upper member. From this and beam theory, the following equation can be derived:

$$\frac{du_U}{dx} = \varepsilon_{UL} = \frac{F_U}{B_U} - \frac{e_U}{\rho}, \quad (3)$$

where ε_{UL} is the upper-member lower surface strain.

For the lower-member upper surface strain ε_{LU} , the following equation can likewise be derived:

$$\frac{du_L}{dx} = \varepsilon_{LU} = \frac{F_L}{B_L} + \frac{e_L}{\rho} \quad (4)$$

where F_U and F_L are the normal forces acting on the cross sections of the upper and lower members, respectively, B_U and B_L are the extensional rigidities of the upper and lower members, respectively, e_U and e_L are the distances from the bonding layer middle plane to the neutral plane of the upper and the lower members, respectively, and ρ is the radius of curvature of the members.

From the equilibrium of forces and moments in

Fig. 2 Multi beam model

Fig. 3, we obtain

$$F_{UN} + F_{LN} - F_U - F_L = 0, \quad (5a)$$

and

$$M_{UN} + M_{LN} + e_U F_{UN} - e_L F_{LN} - e_U F_U + e_L F_L - M_U - M_L = 0, \quad (5b)$$

where M_U and M_L are the bending moments acting at the cross sections of the upper and lower members, respectively, and are obtained from beam theory as

$$M_U = D_U/\rho \text{ and } M_L = D_L/\rho, \quad (6)$$

where D_U and D_L are the bending rigidities of the upper and lower members, respectively.

Because the normal forces of the upper and lower members are changed by the shear stresses acting on the bonding layer, we obtain

$$\frac{dF_U}{dx} = -\frac{dF_L}{dx} = (1-\beta) \cdot \tau, \quad (7)$$

where β is the void area ratio of the bonding layer.

It is noted that force F , moment M , and rigidities B and D introduced so far are defined as values per unit width of the bonding layer in order to simplify the equations.

From the above equations, the following is derived:

$$\frac{d^2\gamma}{dx^2} = \frac{1-\beta}{h} \left[\frac{1}{B_U} + \frac{1}{B_L} + \frac{(e_U + e_L)^2}{D_U + D_L} \right] \tau. \quad (8)$$

Next, we consider boundary conditions.

From the symmetry of the structure, we obtain

$$\gamma = 0 \text{ at the center } (x=0). \quad (9)$$

On the other hand, $F_U = F_{UN}$ and $F_L = F_{LN}$ at the edge ($x = l_x/2$, where l_x is the x -directional length of the bonding layer). Combining this and Eqs. (2)-(7), we obtain

$$\frac{d\gamma}{dx} = \frac{1}{h} \left[\frac{F_{LN}}{B_L} - \frac{F_{UN}}{B_U} + \frac{M_{UN} + M_{LN}}{D_U + D_L} (e_U + e_L) \right] \text{ at } x = l_x/2. \quad (10)$$

As a result, γ can be obtained by solving Eq. (8) under the conditions of Eqs. (9) and (10), if Eq. (1) is determined. We will derive the equation for γ by giving a practical shape to Eq. (1) in the next section.

Once the shear strain equation $\gamma(x)$ is derived, the deformations and strains of the bonded members are readily obtained by

$$U_U = \varepsilon_{U0}(z) \cdot x + S_U(z) \cdot h \cdot [\gamma_0 \cdot x / (l/2) - \gamma(x)] / (S_{U0} + S_{L0}), \quad (11)$$

$$\varepsilon_U = \partial U_U / \partial x, \quad (12)$$

Fig. 3 Cross-sectional force equilibrium

$$\phi = \partial U_U / \partial z, \quad (13)$$

$$1/\rho = \partial \phi / \partial x, \quad (14)$$

where U_U is the x -directional displacement, ε_U is the x -directional strain of the upper member, ϕ is the deflection angle, $1/\rho$ is the curvature, S_{U0} and S_{L0} are the values of $S_U(z)$ and $S_L(z)$, respectively, at $z=0$, and $\varepsilon_{U0}(z)$, $S_U(z)$, $S_L(z)$, and γ_0 are defined as

$$\varepsilon_{U0}(z) \equiv F_{UN}/B_U - (e_U - z) \cdot (M_{LN} + M_{UN}) / (D_U + D_L), \quad (15)$$

$$S_U(z) \equiv 1/B_U + (e_U - z) \cdot (e_U + e_L) / (D_U + D_L), \quad (16)$$

$$S_L(z) \equiv 1/B_L + (e_L + z) \cdot (e_U + e_L) / (D_U + D_L), \quad (17)$$

and

$$\gamma_0 \equiv [l_x / (2h)] [(F_{LN}/B_L - F_{UN}/B_U) + (e_U + e_L) \cdot (M_{UN} + M_{LN}) / (D_U + D_L)]. \quad (18)$$

Equation (11) is also derived from Eqs. (2) - (7). Similar equations can be obtained for the lower member.

2.2 Strain equations

In this section we derive practical equations for calculating shear strain by assuming that the bonding layer is elasto-perfect plastic:

$$\tau = \begin{cases} G \cdot \gamma & (\text{when } G \cdot \gamma \leq \tau_Y) \\ \tau_Y & (\text{when } G \cdot \gamma > \tau_Y), \end{cases} \quad (19)$$

$$(20)$$

where G is the shear modulus and τ_Y is shear yield stress.

By combining and solving the previously mentioned basic equations with the above equation, we obtain the results of the following subsections 2.2.1 and 2.2.2.

2.2.1 Bonding layer strain In the range of small temperature changes, the entire bonding layer is in an elastic state, and the shear strain is expressed by

$$\gamma = \gamma_{e \max} \cdot \sinh(\lambda \xi) / \sinh(\lambda), \quad (21)$$

where ξ is a nondimensional coordinate defined by

$$\xi \equiv x / (l_x / 2). \quad (22)$$

From Eq. (21) it is determined that strain is maximum at the bonding layer edge ($\xi=1$). This maximum strain, $\gamma_{e \max}$, is expressed by

$$\gamma_{e \max} = \gamma_0 \cdot \tanh(\lambda) / \lambda, \quad (23)$$

where γ_0 is already defined in Eq. (18), and is interpreted as the upper limit value of the maximum shear strain when bonding layer shear rigidity decreases. The λ is a nondimensional parameter that expresses the ratio of the compliance of the bonded members to that of the bonding layer, and is defined by

$$\lambda \equiv \sqrt{[(S_{U0} + S_{L0}) / S_B]}, \quad (24)$$

where S_{U0} and S_{L0} are the compliance of the upper and lower members, respectively, as defined by the values of $S_U(z)$ and $S_L(z)$ of Eqs. (16) and (17) at $z=0$. S_B is the compliance of the bonding layer and is defined by

$$S_B \equiv h / [(1 - \beta) G (l_x / 2)^2]. \quad (25)$$

When temperature increases further, a plastic

region is produced at the bonding layer edge, where strain is maximum, and expands to the center, as shown in Fig. 4.

The condition for producing this plastic region is expressed as

$$G \cdot \gamma_{e \max} > \tau_Y. \quad (26)$$

In this case the shear strain γ is

$$\gamma = \gamma_0 \cdot \zeta \cdot \sinh(\lambda \xi) / \sinh(\lambda \xi_Y) \quad (27 \cdot a)$$

$$\text{at } 0 \leq \xi \leq \xi_Y,$$

and

$$\gamma = \gamma_0 [\lambda^2 \zeta (\xi^2 - \xi_Y^2) / 2 + (1 - \lambda^2 \zeta) (\xi - \xi_Y) + \zeta] \quad (27 \cdot b)$$

$$\text{at } \xi_Y < \xi \leq 1,$$

where ζ is nondimensional shear strain and is defined by

$$\zeta \equiv \tau_Y / (G \cdot \gamma_0) \quad (28)$$

The ξ_Y is a nondimensional coordinate of the boundary between the elastic and plastic regions and has the following relation with ζ and λ :

$$1 / [\lambda / \tanh(\lambda \xi_Y) + \lambda^2 (1 - \xi_Y)] = \zeta. \quad (29)$$

The calculated results for this equation are shown in Fig. 4, from which ξ_Y is easily obtained if the values of λ and ζ are given.

2.2.2 Bonded member strains The bonded member strains are obtained by substituting the previously mentioned bonding layer strains into Eqs. (11) and (12):

$$\varepsilon_U = \varepsilon_{U0}(z) + \varepsilon_{UF}(z) \cdot f(\xi), \quad (30)$$

where ε_U is the x -directional strain of the upper member, $\varepsilon_{U0}(z)$ is already defined by Eq. (15), and $\varepsilon_{UF}(z)$ is defined by

$$\varepsilon_{UF}(z) \equiv (2h\gamma_0/l_x) \cdot [S_U(z) / (S_{U0} + S_{L0})]. \quad (31)$$

The $f(\xi)$ is defined by

$$f(\xi) = 1 - \cosh(\lambda \xi) / \cosh(\lambda), \quad (32 \cdot a)$$

when the entire bonding layer is in an elastic state

Fig. 4 Effects of nondimensional yield stress, ζ , and compliance ratio, λ , on the nondimensional coordinate, ξ_Y , of the boundary between an elastic region and an inelastic region

($G \cdot \gamma_{e \max} \leq \tau_Y$), and by

$$f(\xi) = 1 - \lambda \zeta \cosh(\lambda \xi) / \sinh(\lambda \xi_Y) \quad \text{at } 0 \leq \xi \leq \xi_Y, \quad (32 \cdot b)$$

$$f(\xi) = \lambda^2 \zeta (1 - \xi) \quad \text{at } \xi_Y < \xi \leq 1, \quad (32 \cdot c)$$

when a plastic region is produced ($G \cdot \gamma_{e \max} > \tau_Y$).

Similar equations can be obtained for the lower member.

The equations so far obtained are also applicable to the case where there are gradients in temperature and changes in material properties in the vertical direction in either or both of the upper and lower members, by using equivalent values⁽⁵⁾ for F_{UN} , F_{LN} , M_{UN} , and M_{LN} .

Hereafter, for simplicity, we consider the case where the material properties are constant for the upper and lower members, respectively, and temperature change $\Delta\theta$ is uniform over the whole structure. In this case, $F_{UN} = B_U \alpha_U \Delta\theta$, $F_{LN} = B_L \alpha_L \Delta\theta$, and $M_{UN} = M_{LN} = 0$, and therefore, from Eqs. (15) and (18), we obtain

$$\varepsilon_{U0}(z) = \alpha_U \Delta\theta \quad \text{and} \quad (33)$$

$$\gamma_0 = l_x (\alpha_L - \alpha_U) \Delta\theta / (2h), \quad (34)$$

where σ_U and α_L are the linear thermal expansion coefficients of the upper and lower members, respectively.

The upper member stresses, σ_U , is obtained by subtracting the free thermal expansions from the strain of Eq. (30) and multiplying this by the elastic constant. It is found from Eqs. (30) and (32) that the maximum stress of the upper member is produced at the center of its lower surface and is expressed as

$$\sigma_{U \max} = \sigma_{U0} [1 - 1 / \cosh(\lambda)] \quad \text{when } G \cdot \gamma_{e \max} \leq \tau_Y, \quad \text{and} \quad (35 \cdot a)$$

$$\sigma_{U \max} = \sigma_{U0} [1 - \lambda \zeta / \sinh(\lambda \xi_Y)] \quad \text{when } G \cdot \gamma_{e \max} > \tau_Y, \quad (35 \cdot b)$$

where $\sigma_{U \max}$ is the upper member x -directional maximum stress, and σ_{U0} is defined by

$$\sigma_{U0} \equiv E_U (\alpha_L - \alpha_U) \Delta\theta \cdot S_{U0} / (S_{U0} + S_{L0}), \quad (36)$$

where E_U is the upper member Young's modulus. This σ_{U0} is interpreted as the upper limit of $\sigma_{U \max}$ when the bonding layer rigidity increases (λ and ζ increase).

2.3 Extension to three-dimensional deformations

The equations so far obtained are based on the assumption that y -directional stress, σ_y , equals 0. Although this assumption holds if the y -directional dimensions of the structure are very small compared with the other-directional dimensions, it does not hold in many cases because chips usually have nearly regular square shapes. In this section we consider an approximate solution to this nearly regular square chip case. The validity of this approximation is examined by comparing the solution with the FEM results in a later section.

A basic assumption introduced here is that both

x - and y -directional displacement distributions can be obtained from Eq. (11), provided that y -directional quantities are used instead of x -directional quantities in the case of y -directional displacement, and also provided that $E/(1-\nu)$ is used instead of E in both the x - and y -directional cases to take into account the member rigidity increase effects caused by the interaction of the x - and y -directional displacements.

Strains, deflection angles, and curvatures are easily obtained by combining the x - and y -directional quantities, which are obtained from the previously mentioned equations based on the above assumption.

An exception is the vertical (z -directional) strain ε_z in the bonding layer. Because horizontal thermal expansion of the layer is constrained by the upper and lower members, ε_z is expressed by

$$\varepsilon_z = \alpha \cdot \Delta\theta + [\nu/(1-\nu)](2\alpha \cdot \Delta\theta - \varepsilon_x - \varepsilon_y), \quad (38)$$

where α is the linear thermal expansion coefficient of the bonding layer, and ε_x and ε_y are normal strains in the x and y directions, respectively. The ε_x is expressed as

$$\varepsilon_x = (\varepsilon_{ULx} + \varepsilon_{LUx})/2 + (\varepsilon_{ULx} - \varepsilon_{LUx})z/h, \quad (39)$$

where ε_{ULx} and ε_{LUx} are the x -directional strains of the lower surface of the upper member and the upper surface of the lower member, respectively. The ε_y is obtained by substituting y -directional quantities instead of x -directional quantities into the above equation.

The upper member stress is expressed as

$$\sigma_{Ux} = [E_U/(1-\nu_U^2)] \cdot [(\varepsilon_{Ux} - \alpha_U \Delta\theta) + \nu_U (\varepsilon_{Uy} - \alpha_U \Delta\theta)], \quad (40)$$

where σ_{Ux} is the x -directional stress of the upper member, and E_U , ν_U , ε_{Ux} , and ε_{Uy} are the upper member's Young's modulus, Poisson's ratio, x -directional normal strain, and y -directional normal strain, respectively. The y -directional stress, σ_{Uy} , is obtained by substituting y -directional quantities instead of x -directional quantities into the above equation.

From Eqs. (40) and (30), it is found that the upper member maximum stress, $\sigma_{U \max}$, is produced at the center of the lower surface. When the upper and lower members have exactly regular square shapes, ε_{Ux} and ε_{Uy} have equal values at the center, and therefore $\sigma_{U \max}$ is expressed by Eqs. (35) and (36), provided that $E/(1-\nu)$ is used instead of E .

3. Verification of the Solution

3.1 Comparison with FEM results

In this section we compare the proposed solution with three-dimensional FEM results in order to evaluate the validity of the assumption introduced in the previous section for calculating the three-dimensional strain distribution.

The elastic stress and strain produced in the

structure shown in Fig. 1 with the material properties shown in Table 1 are calculated under a uniform temperature of 100°C. In the FEM calculation, only a quarter of the structure can be modelled because of its symmetry, and is divided into FEM elements, as shown in Fig. 5. These elements are three-dimensional solids with six planes and have displacement functions containing second-order terms. The FEM results (values at the center of each element) are shown along with the results of the proposed equations in Figs. 6, 7, and 8.

For the upper member stresses, as shown in Fig. 6, there is good agreement between the proposed solution and the FEM results near the chip center, although there are some minor differences near the edge. The chip center stress is especially important because it sometimes causes chip cracking. For the bonding layer strain distribution, as shown in Fig. 7, there is also good agreement between the proposed solution and the FEM results. Moreover, for the behavior of the bonding layer maximum strain change due to the bonding layer shear modulus change, as shown in Fig. 8, there is fair agreement between the proposed solution and the FEM results.

Concerning the FEM results of the bonding layer shear strain, the average values over the bonding layer thickness were plotted to match them with the proposed solution in this section. The bonding layer strain distribution of the thickness direction will be examined later in section 5.

Table 1 Constants used in calculations

Member	E (GPa)	ν	G (GPa)	α (K ⁻¹)	L×W×H (mm)
Upper member	170	0.3	65	3×10^{-6}	5×5×0.2
Bonding layer	2.9	0.45	1	20×10^{-6}	5×5×0.05
Lower member	270	0.3	104	6.5×10^{-6}	10×10×2

E: Young's modulus, ν : Poisson's ratio
G: Shear modulus, α : Linear thermal expansion coefficient
L: Length, W: Width, H: Thickness

Fig. 5 Three-dimensional FEM mesh

3.2 Comparison with experiment

In this section we compare the proposed solution with experimental results.

A silicon plate was bonded to a copper plate using 95Pb-5Sn solder, a strain gauge was attached to the silicon plate, temperature cycling was performed, and the relationship between the temperature and the strain was measured. The silicon plate has a regular square shape, sides of 4 mm length, thickness of 0.175 mm, a {100} plane, and a <110> side direction. The copper plate is also square, with sides of 20 mm length and thickness of 4 mm. The bonding size is the same as the chip size, the bonding layer thickness is 0.02 mm, and the void area ratio is 30%. For the temperature change pattern, the rates of temperature increase

Fig. 6 Comparison of the proposed solution and the three-dimensional FEM results for the distribution of the upper member stress, σ_{Ux}

Fig. 7 Comparison of the proposed solution and the three-dimensional FEM results for the distribution of the bonding layer shear strain, γ_{zx}

Fig. 8 Comparison of the proposed solution and the three-dimensional FEM results for the effects of shear modulus, G , on the maximum bonding layer shear strain, γ_{zxmax}

and decrease were both 0.05°C/s , and there was no hold time at either the maximum or the minimum temperature.

Measured strain values are shown in Fig. 9. The strain values calculated using the proposed equations under the conditions corresponding to the experimental ones are also shown. These values were calculated at the center of the silicon plate by taking into account the effects of the solder yield stress changes due to temperature changes at each stage of the temperature change history. The values from Ref. (7) were used for the yield stress values, and the strain rate dependence of the yield stress was taken into account. The strain rate values are estimated using

$$\dot{\gamma}_0 = l_x(a_L - a_U) \dot{\theta} / (2h), \quad (41)$$

where γ_0 is the estimated value of the strain rate and $\dot{\theta}$ is the temperature change rate.

The effects of residual stress must be taken into account when calculating the strains produced by temperature cycling. This is done by shifting the original temperature θ_0 when the temperature change direction varies (for example, when it varies from an increase to a decrease at the maximum temperature). θ_0 is the origin of the temperature change, $\Delta\theta$, in the proposed equations, and is expressed by $\Delta\theta = (\theta - \theta_0)$, where θ is the present temperature. This means that θ_0 is the temperature where residual stress is 0. The shift values of θ_0 can be easily obtained from the condition that the structure deforms elastically during unloading processes that range from the time at which the temperature change direction varies to the time at which residual stress becomes zero.

As shown in Fig. 9, the proposed solution agrees well with the experimental results.

3.3 Comparison with incremental method results

In this section we compare the proposed solution with results obtained using the method in Ref. (7) which is based on the incremental method.

Fig. 9 Comparison of the proposed solution and the experimental results for the upper member strain, ϵ_U

The shear strain range distribution produced in a bonding layer is calculated for a structure which consists of a silicon plate, 95Pb-5Sn solder bonding layer, and an alumina plate. The silicon plate has a regular square shape, 10 mm side length, and 0.4 mm thickness. The bonding layer is 0.05 mm thick and the alumina plate is 4 mm thick. The temperature cycling has maximum temperature stages of 150°C and minimum temperature stages of -55°C , with 30-minute hold times at both stages.

The results for the proposed equations were obtained by using the method in Ref. (8) to take into account the effects of solder layer creep. Creep is essential when there is hold time in the temperature cycling. In this method, the strain range during the temperature cycling is easily obtained through only a one-time strain calculation for a unidirectional temperature change by using the substantial yield stress, $\Delta\tau_y$, instead of the actual yield stress, τ_y .

The calculated results are shown in Fig. 10 along with the incremental method results. As shown in this figure, the proposed solution agrees well with the incremental method results.

4. Strain-governing Parameters

In this section we investigate the parameters that govern the bonding layer maximum shear strain and the bonded member maximum stress. The maximum strain is important in evaluating the fatigue failure of bonding layers. The maximum stress is important in evaluating the brittle failure of bonded members.

4.1 Maximum bonding layer strain

Equations (23), (27), and (29) show that the parameters which govern the maximum shear strain, $\gamma_{\max}$, can be reduced to just three nondimensional fundamental parameters: ζ , λ , and γ_0 . The effects of these parameters were calculated using Eqs.(23), (27), and (29). As shown in Fig. 11, $\gamma_{\max}/\gamma_0$ increases and approaches 1 ($\gamma_{\max}$ approaches γ_0) as ζ becomes smaller. Therefore, γ_0 is considered to be the upper

Fig. 10 Comparison of the proposed solution and the incremental method for the distribution of the bonding layer strain range, $\Delta\gamma$

limit of shear strain when $\zeta \rightarrow 0$.

Figure 11 also shows that the values of ζ at which $\gamma_{\max}$ approaches the upper limit become smaller as λ becomes larger. The λ expresses the compliance of the bonded members and is made nondimensional by taking the ratio of the compliance to the bonding layer compliance. It is, therefore, considered that the above-mentioned $\gamma_{\max}-\zeta-\lambda$ relation shows that the bonding layer strains can be reduced by making the bonded members compliant to adsorb most of the thermal expansion mismatch, even if a low-yield stress bonding material is used.

4.2 Maximum bonded member stress

For the maximum stress, $\sigma_{U\max}$, of the bonded members, it is found from Eqs. (35) and (29) that the fundamental parameters are ζ , λ , and σ_{U0} . The effects of these parameters are shown in Fig. 12. As shown, $\sigma_{U\max}$ increases and approaches σ_{U0} as λ becomes larger. Therefore, σ_{U0} is considered to be the upper limit value of $\sigma_{U\max}$. It is also shown that $\sigma_{U\max}$ can be decreased by decreasing ζ , even if λ is large. This relationship shows that the bonded member stress can be decreased by using a low-yield stress bonding material.

5. Limitations of Proposed Solution

The following two points are important when discussing the limitations of the proposed solution.

Fig. 11 Effects of nondimensional yield stress ζ and the compliance ratio λ on the maximum normalized bonding layer shear strain, $\gamma_{\max}/\gamma_0$

Fig. 12 Effects of nondimensional yield stress ζ and the compliance ratio λ on maximum normalized upper member stress $\sigma_{U\max}/\sigma_{U0}$

(i) The proposed solution is based on the assumption that the thermal expansion mismatch of two bonded members is mainly absorbed by bonding layer shearing deformation and bonded member extensional and bending deformations. It is, therefore, not applicable to cases where other types of deformations are predominant.

(ii) The bonding layer shear strains obtained by the proposed equations are the mean values over the thickness of the bonding layers, as expressed in Eq. (2). Distributions along the thickness direction are not obtained.

Concerning (i) above, there are problems for cases where the bonding layer rigidity is extremely large. For example, it is observed that chip cracking occurs at bonding edges if chips are bonded using hard solder with high yield stress or bonded directly to substrates without using bonding materials, using such methods as anodic bonding. The stresses which cause this chip edge cracking are beyond the applicable range of the proposed solution, because these stresses are dominated by the local deformations at the chip edges.

The objective of the proposed solution is to analyze structures in which compliant bonding materials, such as solder, are used. It is observed that chip cracking occurs near centers if compliant bonding materials are used. The stresses which cause this chip center cracking can be calculated with high accuracy by using the proposed equations.

Concerning the previously mentioned (ii), there are two problems. One is the problem of local strain concentration at the edges of the bonding layer and the other is the problem of nonhomogeneity in the bonding material.

We first examine the former: the edge strain concentration problem. At the bonding layer edges, strains are distributed along the thickness direction and concentrate locally; moreover, strain singularities are produced in some cases^{(9),(10)}. These local strains cannot be derived directly using the present equations. It is useful in design to obtain these local strains by multiplying the strain concentration factors by the proposed strain values, which are thought to be nominal values. Obtaining these concentration factors and examining their effects on strength are left for future study. For example, it would be useful to investigate the behavior of strain singularity by extending the methods in Ref. (9) and (10).

We now examine the characteristics of the edge strains. The results of FEM analysis for one sample structure are shown in Fig. 13, which shows bonding layer strain distributions near the bonding layer/upper member interface (1.75 μm from the interface)

Fig. 13 FEM results and the proposed solution for local strain distributions of a bonding layer edge

and near the bonding layer middle plane ($17.5\text{ }\mu\text{m}$ from the interface). These distributions are the result of elastic analysis of the structure in Table 1 (bonding layer thickness of $50\text{ }\mu\text{m}$).

As shown in Fig. 13, the difference between the near-interface strains and the near-middle plane strains becomes larger when the distance from the edge becomes shorter. The near-interface strains seem to have singularity near the edge. The region where these local strains are predominant is limited to $20\text{ }\mu\text{m}$ from the edge. Outside of the near-edge region, there is no significant difference between the interface strains and the middle plane strains, and they also agree well with the proposed solution.

It is thought that the effects of these local strains on strength depend on the bonding material used. For example, for Pb-Sn solder, which is commonly used as the bonding material, it is observed that it has nonhomogeneous microstructures with many micrograins, and thermal fatigue cracks are produced at the boundaries of these grains⁽¹¹⁾. It is thought that these cracks are produced by the boundary sliding of the grains which have diameters greater than $20\text{ }\mu\text{m}$. In this case, the nonhomogeneity of the bonding materials must be taken into account to obtain the true strains. From the viewpoint of design, however, a simple method is also needed where average strains are used to evaluate strength. These strains are analyzed by assuming homogeneity of the bonding mate-

rial and are averaged over a certain volume of the material.

6. Conclusions

The following results were obtained by examining the strain produced in semiconductor chip bonding structures during temperature changes.

(1) An analytical solution was obtained in which the elastoplastic behavior of bonding layers is taken into account.

(2) It was confirmed that the proposed solution agrees well with both three-dimensional FEM results and incremental method results.

(3) The fundamental parameters that govern the strains and stresses of bonding structures were determined using the proposed solution.

(4) These parameters were charted so that the maximum bonding layer shear strains and the maximum bonded member stresses can easily be obtained.

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