Anisotropy in Multiaxial Creep of Nickel-Based Single-Crystal Superalloy CMSX-2* (Experiments and Identification of Active Slip Systems)

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Combined tension-torsion creep tests of a nickel-based single-crystal superalloy CMSX-2 are performed at 900°C under stationary as well as nonstationary stress conditions using thin-walled tubular specimens with the crystal direction [001] oriented in the axial direction. Marked effects of anisotropy are observed both on the magnitude of creep rate and on the noncoaxiality between creep rate and stress. To identify operative slip systems in the present creep tests, the experimental data of steady-state creep are analyzed on the basis of a crystal creep theory of the power-law type. It is shown that the experiments are simulated well if we assume simultaneous operation of the $\{111\}\langle 112 \rangle$ octahedral and $\{100\}\langle 110 \rangle$ cubic slip systems.

Key Words: Creep, Anisotropy, Combined Stress, Nickel Base, Single-Crystal Superalloy, CMSX-2, Slip System, Thin-Walled Tube

1. Introduction

Nickel-based single-crystal superalloys have been used as materials for gas turbine blades. They may exhibit strong anisotropy in creep and plasticity because the crystal directions are ordered in them. In order to describe the anisotropy, it is necessary to identify active slip systems. Hence, the active slip systems in nickel-based single-crystal superalloys have been studied, mainly in uniaxial stress states so far⁽¹⁾⁻⁽³⁾.

For primary creep at intermediate temperatures such as 750 to 775°C, the $\{111\}\langle 112 \rangle$ slip systems were found to be active by measuring lattice rotations induced in creep⁽⁴⁾⁻⁽⁶⁾. Operation of these slip systems was ascertained also by the analysis, based on a crystal theory, of primary creep of Mar-M200 thin-walled tubes subjected to combined tension and internal pressure at 760°C⁽⁷⁾. For steady-state creep of

Mar-M200 at 760°C and 850°C, on the other hand, the $\{111\}\langle 110 \rangle$ slip systems were deduced to be operative from observations of dislocation networks and a consideration of activation energies in creep⁽⁸⁾. However, lattice rotations in creep of Mar-M247 at 774°C⁽⁵⁾ and SRR 99 at 750°C⁽⁶⁾ favor the operation of the $\{111\}\langle 112 \rangle$ slip systems even in steady-state creep⁽²⁾.

We thus have a discrepancy with respect to the active slip systems in steady-state creep of nickel-based single-crystal superalloys. This results in confusion in constitutive modeling of creep of such alloys. Sheh and Stouffer⁽²⁾, based on the results of lattice rotations mentioned above, assumed that the $\{111\}\langle 112 \rangle$ octahedral slip systems are exclusively active in creep of nickel-based single-crystal superalloys. On the other hand, Ghosh et al.⁽⁹⁾ adopted the $\{111\}\langle 110 \rangle$ octahedral slip systems, together with the $\{100\}\langle 110 \rangle$ cubic slip systems, for simulating tertiary creep of SRR 99 in the temperature range 750-950°C, because the $\{111\}\langle 110 \rangle$ slip systems have a lower dislocation energy than the $\{111\}\langle 112 \rangle$ slip systems. It is, however, noted that since they examined uniaxial creep only in the [001] and [111] directions, both

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the $\{111\}\langle 110\rangle$ and $\{111\}\langle 112\rangle$ slip systems allow us to simulate well their experiments.

In the present work, in order to study anisotropic effects and active slip systems in multiaxial creep of a nickel-based single-crystal superalloy CMSX-2, combined tension-torsion creep tests of this material are performed at 900°C, and the experimental data of steady-state creep are analyzed using a crystal creep theory of the power-law type. Thin-walled tubular specimens with the crystal direction $[001]$ oriented in the axial direction are employed. The tests include a multiaxial test of nonstationary stress, which will be referred to as the nonstationary test. This test is useful because it yields creep data in several stress states without scatter of the material property.

Incidentally, Ohno et al.⁽¹⁰⁾ carried out a similar study for unidirectionally solidified IN738LC thin-walled tubular specimens, which were subjected to combined tension and torsion at 850°C. Applying the crystal creep theory to the experimental data of the minimum creep rate, they found that the behavior is simulated well if the $\{111\}\langle 112\rangle$ octahedral and $\{100\}\langle 110\rangle$ cubic slip systems are assumed to operate simultaneously.

2. Material and Specimen

Figures 1(a) and (b) show two types of thin-walled tubular specimens used in the present work. The material is a nickel-based single-crystal superal-

(a) A-type

(b) B-type

Fig. 1 Thin-walled tubular specimens

loy CMSX-2, which has the chemical composition shown in Table 1. Each specimen was machined from a mother tube of outside diameter 22 mm, inside diameter 7 mm and length 150 mm. All the mother tubes were cast for the crystal direction $[001]$ so as to be oriented in the axial direction (see Table 3 shown later). The A-type specimens (Fig. 1(a)) were used in most of the present tests, while some torsional tests were performed using the B-type specimens (Fig. 1(b)) which have a short gauge section to allow large deformation without torsional buckling.

The mother tubes and the specimens were subjected to the following heat treatments: the solution heat treatment of 1280°C×30 min+1290°C×15 min+1295°C×15 min+1305°C×3 h/AC followed by the stabilization treatment of 980°C×5 h/AC before machining, and the aging treatment of 870°C×20 h/AC after machining.

The single-crystal superalloy CMSX-2 at 900°C has mechanical properties of the 0.2% proof stress of 583 MPa and the tensile strength of 931 MPa in the $[001]$ direction⁽¹¹⁾.

3. Loading Parameters in Experiments

The present material has anisotropy which induces nonuniform distributions of creep rate and stress in the gauge section of the specimens. Nevertheless, the following equivalent stress σ_{eq} and stress angle θ , which are defined for thin-walled tubes of Mises-type isotropic materials subjected to tensile stress σ and torsional stress τ , were taken as the experimental parameters in the present tests:

$$\sigma_{eq} = [\sigma^2 + (\sqrt{3}\tau)^2]^{1/2}, \quad (1)$$

$$\theta = \tan^{-1}(\sigma/\sqrt{3}\tau). \quad (2)$$

Then, deviations of experiments from predictions derived for Mises-type isotropic materials can be regarded as due to anisotropic effects. For steady-state creep of Mises-type isotropic materials, we have predictions written as

$$\dot{\epsilon}_{eq}^c = f(\sigma_{eq}), \quad (3)$$

$$\Theta = \theta, \quad (4)$$

where $(\dot{})$ indicates the differentiation with respect to time t , and $\dot{\epsilon}_{eq}^c$ and Θ denote the equivalent creep

Table 1 Chemical composition (wt%)

Cr	Co	W	Mo	Ta	Al	Ti	Mn
7.9	4.6	7.9	0.6	6.0	5.62	1.0	<10ppm
Si	P	Nb	Fe	Ag	Sn	Pb	
0.006	<50ppm	<0.10	0.03	<1ppm	<5ppm	<0.5ppm	
Se	Bi	B	S	C	N	O	
<0.5ppm	<0.2ppm	<10ppm	3ppm	14ppm	1ppm	1ppm	

rate and creep rate angle defined for Mises-type isotropic materials, respectively. Let $\dot{\epsilon}^c$ and $\dot{\gamma}^c$ be overall tensile and torsional creep rates detected at the ridges of the specimens shown in Fig. 1. Then $\dot{\epsilon}_{eq}^c$ and θ are expressed as

$$\dot{\epsilon}_{eq}^c = [(\dot{\epsilon}^c)^2 + (\dot{\gamma}^c/\sqrt{3})^2]^{1/2}, \quad (5)$$

$$\theta = \tan^{-1}[\dot{\epsilon}^c/(\dot{\gamma}^c/\sqrt{3})]. \quad (6)$$

Equation (3) represents that the second invariant of creep rate is a function of the second invariant of deviatoric stress, and Eq. (4) expresses the coaxiality between creep rate and deviatoric stress.

4. Experimental Conditions

Combined tension-torsion creep testing machines⁽¹²⁾ were used. Each specimen, heated in an electric furnace, was soaked at the test temperature for about 20 hours prior to testing. The temperature in the gauge section was measured by means of two PR thermocouples and controlled within $900 \pm 2^\circ\text{C}$.

Creep tests were performed under stationary and nonstationary stresses by specifying the values of σ_{eq} and θ . For the stationary tests, σ_{eq} and θ were kept constant in the stress states shown in Fig. 2. For the nonstationary test, θ was changed in the order of 45° , 25° , 90° , ..., 45° with the hold times given in Table 2 under a constant effective stress of $\sigma_{eq}=300\text{ MPa}$, as shown in Fig. 3.

Measuring the elongation and twist angle between the ridges of each specimen, we obtained the (overall) tensile strain ϵ and (overall) torsional strain γ in the gauge section, i.e., $\epsilon = (l - l_0)/l_0$ and $\gamma = r_0\alpha/l_0$, where l , r and α indicate the length, average radius and twist angle of the gauge section, respectively, and $()_0$ denotes the initial state. For the B-type specimens, it is noted that the portions outside the gauge section between the ridges were found to deform almost elastically in the creep tests, and this elastic deformation was subtracted to obtain ϵ and γ , and that l_0 was taken to be $l_0=3.5\text{ mm}$ by taking account of the rounded corners in the gauge section. The B-type specimens were used only in some stationary torsional tests of $\theta=5^\circ$ (see Fig. 2).

Fig. 2 Stationary creep tests (● A-type specimen, ▲ B-type specimen)

The creep tests were performed under constant true stresses, rather than constant loads, by reducing the weights of axial force W and torsional torque M with the increase of the axial elongation as follows: $W = (l_0/l)W_0$ and $M = (l_0/l)^{3/2}M_0$, where W_0 and M_0 denote the initial values of W and M , respectively⁽¹⁰⁾.

5. Experimental Results

Some creep curves obtained in the stationary tests are shown in Figs. 4(a) to (d), where the effective stress σ_{eq} is identical ($\sigma_{eq}=300\text{ MPa}$) but the stress angle θ has different values. From the creep curves shown in Figs. 4(a) to (d), we observe the following. (1) The creep curves show obvious dependence on θ , suggesting marked anisotropy in the present creep tests. (2) They have little primary creep strain, and creep rates are nearly constant up to $t=150$ or 200 h . Creep rates at $t=100\text{ h}$ were thus taken as the steady-state creep rates.

Figure 5 shows variations of ϵ and γ obtained in the nonstationary test. It is seen from this figure that $\dot{\epsilon}$ and $\dot{\gamma}$, which are nearly constant in each stage, depend conspicuously on the stress angle θ . Anisotropic effects are thus marked in the nonstationary test as well. It is, however, noted that creep rates in the first and last stages, where the stress states are the same (i.e., $\theta=45^\circ$), differ by a factor of about two. This may be due to a stress-history effect.

The steady-state creep rates obtained in the stationary and nonstationary creep tests were substituted into Eqs. (5) and (6) to calculate the

Table 2 Stress hold times in nonstationary creep test

Sequence No.	θ (deg.)	Hold time (h)
1	45.0	72
2	25.0	24
3	90.0	24
4	67.5	24
5	5.0	30
6	45.0	66

Fig. 3 Nonstationary creep test (numbers 1, 2, ... 6, indicating the sequence of loading)

effective creep rate $\dot{\epsilon}_{eq}^c$ and the creep rate angle θ . The resulting values of $\dot{\epsilon}_{eq}^c$ and θ are plotted in Figs. 6(a) and (b) as functions of the stress angle θ ,

Fig. 4 Creep curves by stationary tests under $\sigma_{eq}=300$ MPa

respectively. The isotropic predictions given by Eqs. (3) and (4) are also shown by solid lines in these figures. The numbers attached to $\diamond$ in Figs. 6(a) and

Fig. 5 Result of nonstationary creep test ($\sigma_{eq}=300$ MPa)

(a) Magnitude of creep rate

(b) Angle of creep rate

Fig. 6 Experimental data of steady-state creep under $\sigma_{eq}=300$ MPa

(b) represent the sequence of loading in the non-stationary test.

We see that the experimental results, which show some scatter, do not satisfy the isotropic predictions as follows. In Fig. 6(a) $\dot{\epsilon}_{eq}^c$ is not constant but has a minimum around $\theta=45^\circ$ and in Fig. 6(b) creep rate and stress are not coaxial with each other (i.e., $\theta \neq \theta$). These experimental features are definitely due to anisotropy of the present material.

The steady-state creep rates in the tensile tests of $\theta=90^\circ$ and torsional tests of $\theta=5^\circ$ are plotted as a function of applied stress in Fig. 7. The experimental data in the figure are fitted well by the relations

$$\dot{\epsilon} = 2.42 \times 10^{-29} \sigma^{9.8}, \quad (7)$$

$$\dot{\gamma}/\sqrt{3} = 1.47 \times 10^{-28} (\sqrt{3} \tau)^{9.5}, \quad (8)$$

where units of stress, time and strain are MPa, h and mm/mm, respectively. These relations were used to determine creep constants of the slip systems, as will be described in section 7.

Crystal directions were measured for the specimens used in the stationary tests at $\sigma=300$ MPa and the nonstationary test (Table 3). The measurement was done at the axial ends of the specimens after the tests. As shown in Table 3, differences between the axial and [001] directions are less than 6.5° , so that the specimens can be regarded as [001]-oriented.

Table 3 Crystal directions in specimens

Test No.	Degrees from [001]	[101]	[111]	σ_{eq} (MPa)	θ (deg.)
2	5.0	40.6	50.0	300	90
4	3.2	42.4	51.6	300	60
5	6.2	39.0	49.4	300	30
7	2.9	42.2	52.5	300	5
10	6.4	38.7	49.5	300	Non-stationary

Fig. 7 Relationship between steady-state creep rate and applied stress in tensile tests ($\theta=90^\circ$) and torsional tests ($\theta=5^\circ$)

6. Crystal Theory of Creep

The crystal theory of creep used to analyze the present experiments is described briefly here. The details of such a theory, which is based on crystal slip, are found, for example, in the monograph by Asaro⁽¹³⁾.

Three groups of slip systems, i.e., $\{111\}\langle 110 \rangle$ octahedral, $\{111\}\langle 112 \rangle$ octahedral, and $\{100\}\langle 110 \rangle$ cubic slip systems, may operate in the present material. They will be indicated as oct1, oct2 and cub from now on, respectively. The creep rate due to each group of slip systems is written as

$$\dot{\epsilon}_S^c = \sum_{k=1}^{N_S} \dot{\gamma}_S^{(k)} \alpha_S^{(k)} \quad (S=\text{oct 1, oct 2, cub}), \quad (9)$$

where $\dot{\gamma}_S^{(k)}$ and $\alpha_S^{(k)}$ denote the slip rate and orientation tensor of the k th slip system in group S , respectively, and N_S indicates the number of slip systems in S :

$$N_{\text{oct1}} = N_{\text{oct2}} = 12, N_{\text{cub}} = 6. \quad (10)$$

The orientation tensor $\alpha_S^{(k)}$ is written in terms of the unit vectors of the slip direction and slip plane, $b_S^{(k)}$ and $\nu_S^{(k)}$, as follows:

$$\alpha_S^{(k)} = (1/2)(b_S^{(k)} \otimes \nu_S^{(k)} + \nu_S^{(k)} \otimes b_S^{(k)}), \quad (11)$$

where $\otimes$ stands for the tensor product.

For steady-state creep, we may assume a simple power law for $\dot{\gamma}_S^{(k)}$:

$$\dot{\gamma}_S^{(k)} = K_S \text{sign}[\tau_S^{(k)}] |\tau_S^{(k)}|^{n_S}, \quad (12)$$

where K_S and n_S ($S=\text{oct1, oct2, cub}$) are material constants, and $\tau_S^{(k)}$ denotes the shear stress acting on the k th slip system in S . With stress tensor σ , $\tau_S^{(k)}$ is expressed as

$$\tau_S^{(k)} = \text{tr}(\sigma \alpha_S^{(k)}), \quad (13)$$

where tr indicates the trace. Although sophisticated equations with internal variables have been studied for $\dot{\gamma}_S^{(k)}$ in nickel-based single-crystal superalloys^{(2),(3),(14)-(16)}, Eq. (12) was used in the present work. This equation is valid for steady-state creep.

On the other hand, elastic strain of nickel-based single-crystal superalloys which have cubic symmetry is written in the following form with respect to the material principal axes:

$$\epsilon_{ij}^e = \begin{cases} \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \text{tr} \sigma, & i=j, \\ \frac{1}{2G} \sigma_{ij}, & i \neq j, \end{cases} \quad (14)$$

where E , G and ν are material constants. They are independent of each other because of the cubic symmetry.

7. Models for Analysis

Figure 8 illustrates the lattice directions on the cross section of the present specimens. The symmetry seen in the figure allows us to consider only a quarter of the cross section for analyzing combined tension-

torsion creep in this study. Therefore, we considered the quarter shown in Fig. 9 and divided it into m elements. In the figure, the polar coordinates (r, φ, z) , together with the rectangular coordinates (x, y, z) , are used. The m elements have the same size, and each element has a representative point located at the center. The z -axis (i.e., the specimen axis) is assumed to be $[001]$ -oriented.

In combined tension-torsion creep of the present specimens, stress and strain rate are distributed, in general, nonuniformly in the gauge part due to anisotropy of the material. To analyze the experiments, therefore, we used two simplified models based on uniform stress and uniform strain rate, which are described in the succeeding subsections. Incidentally, performing 3D finite-element analysis of the present problem, Takeuchi and Ohno⁽¹⁷⁾ found that uniform distributions of σ_{zz} and $\sigma_{z\varphi}$ tend to prevail in the specimen having long gauge length (i.e., A-type), while the specimen with short gauge length (i.e., B-type) tends to have uniform distributions of $\dot{\epsilon}_{zz}$ and $\dot{\epsilon}_{z\varphi}$. Moreover, Méic and Cailletaud⁽¹⁶⁾ performed 3D finite-element viscoplastic analysis of a single-crystal thin-walled tube with a fairly long gauge section, and they found that $\sigma_{z\varphi}$ tends to be uniform under cyclic torsion.

7.1 Model I (uniform stress model)

We assume that tensile and torsional stresses σ_{zz} and $\sigma_{z\varphi}$ are distributed uniformly in the gauge section. Then, we have

$$\sigma = \sigma_{zz:i} \quad (i=1, 2, \dots, m), \quad (15)$$

Fig. 8 Lattice directions on the cross section in gauge part

Fig. 9 Elements and coordinates for analysis

$$\tau = \sigma_{z\varphi:i} \quad (i=1, 2, \dots, m), \quad (16)$$

where $(\)_{:i}$ denotes the representative point of the i th element. We assume further that the overall tensile and torsional strain rates detected at the ridges, $\dot{\epsilon}$ and $\dot{\gamma}$, are equal to the averages of local strain rates:

$$\dot{\epsilon} = (1/m) \sum_{i=1}^m \dot{\epsilon}_{zz:i}, \quad (17)$$

$$\dot{\gamma}/2 = (1/m) \sum_{i=1}^m \dot{\epsilon}_{z\varphi:i}. \quad (18)$$

7.2 Model II (uniform strain rate model)

If tensile and torsional strain rates $\dot{\epsilon}_{zz}$ and $\dot{\epsilon}_{z\varphi}$ are distributed uniformly in the gauge section, we have

$$\dot{\epsilon} = \dot{\epsilon}_{zz:i} \quad (i=1, 2, \dots, m), \quad (19)$$

$$\dot{\gamma}/2 = \dot{\epsilon}_{z\varphi:i} \quad (i=1, 2, \dots, m). \quad (20)$$

Moreover, the equilibrium of force in the axial and circumferential directions are expressed as

$$\sigma = (1/m) \sum_{i=1}^m \sigma_{zz:i}, \quad (21)$$

$$\tau = (1/m) \sum_{i=1}^m \sigma_{z\varphi:i}. \quad (22)$$

7.3 Computation by Models I and II

Models I and II, in combination with the crystal creep theory described in section 6, enable us to compute the overall tensile and torsional strain rates $\dot{\epsilon}$ and $\dot{\gamma}$.

Splitting $\dot{\epsilon}_{zz:i}$ and $\dot{\epsilon}_{z\varphi:i}$ into elastic and creep components and using Eq. (14), we have

$$\dot{\epsilon}_{zz:i} = \dot{\sigma}_{zz:i}/E + \dot{\epsilon}_{zz:i}^c \quad (23)$$

$$\dot{\epsilon}_{z\varphi:i} = \dot{\sigma}_{z\varphi:i}/2G + \dot{\epsilon}_{z\varphi:i}^c. \quad (24)$$

Here it is noted that the elastic shear relation in Eq. (14) keeps its form under any rotation of the coordinate axes with respect to the z -axis, i.e.,

$$\dot{\epsilon}_{z\varphi}^e = \sigma_{z\varphi}/2G. \quad (25)$$

This means that $\sigma_{z\varphi}$ is distributed uniformly in the initial elastic state. We substitute Eqs. (23) and (24) into Eqs. (17) and (18), respectively, and we note that Eqs. (17) and (18) are also valid in Model II because Eqs. (19) and (20) satisfy Eqs. (17) and (18). Thus we can derive the following relations for both Models I and II:

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{1}{m} \sum_{i=1}^m \dot{\epsilon}_{zz:i}^c, \quad (26)$$

$$\dot{\gamma} = \frac{\dot{\tau}}{G} + \frac{2}{m} \sum_{i=1}^m \dot{\epsilon}_{z\varphi:i}^c. \quad (27)$$

When $\dot{\sigma} = 0$ and $\dot{\tau} = 0$, these equations are reduced to

$$\dot{\epsilon} = (1/m) \sum_{i=1}^m \dot{\epsilon}_{zz:i}^c, \quad (28)$$

$$\dot{\gamma} = (2/m) \sum_{i=1}^m \dot{\epsilon}_{z\varphi:i}^c. \quad (29)$$

Computation by Model I is simple because $\sigma_{zz:i}$ and $\sigma_{z\varphi:i}$ are prescribed by Eqs. (15) and (16). Setting $\sigma_{zz:i} = \sigma$ and $\sigma_{z\varphi:i} = \tau$ ($i=1, 2, \dots, m$) in accordance with Eqs. (15) and (16), we compute $\dot{\epsilon}_{zz:i}^c$ and $\dot{\epsilon}_{z\varphi:i}^c$ ($i=1, 2, \dots, m$) using the crystal theory. Then Eqs. (28)

and (29) give $\dot{\epsilon}$ and $\dot{\gamma}$. For Model II, on the other hand, $\sigma_{zz:i}$ and $\sigma_{z\varphi:i}$ change from the initial elastic state, $\sigma_{zz:i}(0)=\sigma$ and $\sigma_{z\varphi:i}(0)=\tau$, to the steady state. Values of $\sigma_{zz:i}$ and $\sigma_{z\varphi:i}$ in the steady state are computed incrementally using the following relations derived from Eqs. (19), (23) and (28) and Eqs. (20), (24) and (29), respectively:

$$\dot{\sigma}_{zz:i} = E \left\{ (1/m) \sum_{k=1}^m \dot{\epsilon}_{zz:k}^c - \dot{\epsilon}_{zz:i}^c \right\}, \quad (30)$$

$$\dot{\sigma}_{z\varphi:i} = 2G \left\{ (1/m) \sum_{k=1}^m \dot{\epsilon}_{z\varphi:k}^c - \dot{\epsilon}_{z\varphi:i}^c \right\}, \quad (31)$$

where $\dot{\epsilon}_{zz:i}^c$ and $\dot{\epsilon}_{z\varphi:i}^c$ are evaluated using the crystal theory. If $\sigma_{zz:i}$ and $\sigma_{z\varphi:i}$ in the steady state are obtained, Eqs. (28) and (29) give $\dot{\epsilon}$ and $\dot{\gamma}$ in this state.

8. Results of Analysis

We began the analysis by assuming Model I and employing the three groups of slip systems individually, i.e.,

$$\dot{\epsilon}^c = \dot{\epsilon}_S^c \quad (S = \text{oct 1, oct 2, cub}). \quad (32)$$

The results obtained for $\sigma_{eq}=300$ MPa are shown in

(a) Magnitude of creep rate

(b) Angle of creep rate

Fig. 10 Results by Model I based on each of oct1, oct2 and cub ($\sigma_{eq}=300$ MPa)

Figs. 10(a) and (b). The number of elements, m , was taken as $m=6$. The two cases of $m=6$ and $m=12$ agreed within the difference of 0.1% with respect to $\dot{\epsilon}_{eq}$. Model II gave nearly the same results as those of Model I, as discussed later.

Table 4 shows the material constants employed in the analysis. The creep constants K_s and n_s ($S = \text{oct1, oct2, cub}$) in the table were determined as follows. It is seen from Fig. 10(a) that, in the present tests, both octahedral groups, oct1 and oct2, are more effective in tensile creep than in torsional creep while the cubic group, cub, has the opposite feature. Hence, the creep constants of oct1 and oct2 were determined for Model I based on $\dot{\epsilon}_{oct1}^c$ and $\dot{\epsilon}_{oct2}^c$, respectively, so as to represent the tensile creep tests of $\theta=90^\circ$ shown in Fig. 7, while those of cub were determined using the torsional creep test data of $\theta=5^\circ$ in Fig. 7.

Let us compare the results of analysis based on Eq. (32) with those of the experiments. We see in Fig. 10(a) that oct2 and cub are in good agreement with the experimental results in the ranges $\theta > 40^\circ$ and $\theta < 40^\circ$, respectively. On the other hand, oct1 is less successful even in the range $\theta > 40^\circ$ because of the following discrepancy. Although oct1 predicts a maximum of $\dot{\epsilon}_{eq}^c$ around $\theta=65^\circ$, it is hardly observed in the experiments. We now discuss Fig. 10(b) dealing with the noncoaxiality between creep rate and stress. The group of oct2 yields good results again when $\theta > 40^\circ$ but when $\theta < 40^\circ$ both octahedral groups, oct1 and oct2, are unsuccessful in simulating the experiments. On the other hand, the prediction by cub, $\theta=0$, is fairly close to the results of experiments in the range $\theta < 30^\circ$. It is noted that the cubic slip systems, cub, always give $\dot{\epsilon}_{zz}^c=0$ (i.e., $\theta=0$). This is because any cubic slip system has $(b_{cub}^{(k)})_z=0$ resulting in $(a_{cub}^{(k)})_{zz}=0$, so that Eq. (9) gives $(\dot{\epsilon}_{cub}^{(k)})_{zz}=0$.

The discussion above suggests that the experiments would be simulated well in the whole range of θ if slip systems oct2 and cub operate simultaneously. We therefore made further computation by assuming

$$\dot{\epsilon}^c = \dot{\epsilon}_{oct1}^c + \dot{\epsilon}_{cub}^c, \quad (33)$$

$$\dot{\epsilon}^c = \dot{\epsilon}_{oct2}^c + \dot{\epsilon}_{cub}^c. \quad (34)$$

Results for Eqs. (33) and (34) are shown in Figs. 11(a) and (b). It is seen from these figures that the simple superposition of $\dot{\epsilon}_{oct2}^c$ and $\dot{\epsilon}_{cub}^c$ by Eq. (34) gives

Table 4 Material constants at 900°C

$K_{oct1}=4.82 \times 10^{-26}$	$n_{oct1}=9.8$
$K_{oct2}=2.04 \times 10^{-26}$	$n_{oct2}=9.8$
$K_{cub}=9.61 \times 10^{-26}$	$n_{cub}=9.5$
$E=83400$	$G=96100$
$\nu=0.34$	

MPa (stress), h (time), mm/mm (strain)

Fig. 11 Results by Models I and II based on simultaneous operation of octahedral and cubic slip systems ($\sigma_{eq} = 300$ MPa)

a good simulation of the experiments.

Figures 11(a) and (b) include results by Model II in addition to those by Model I. The same material constants were employed for Model II. When torsion prevails over tension (i.e., $\theta < 45^\circ$), Models I and II show some difference in their results with regard to $\dot{\epsilon}_{eq}$. This difference is, however, small in comparison with the experimental scatter, as seen in Fig. 11(a). In other words, either model can be used to discuss the present experiments as far as the overall tensile and torsional creep rates, $\dot{\epsilon}^c$ and $\dot{\gamma}^c$, are concerned.

9. Conclusions

In the present work, first, combined tension-torsion creep tests of a nickel-based single-crystal superalloy CMSX-2 were performed at 900°C using thin-walled tubular specimens with the crystal direction [001] oriented in the axial direction. The tests include a nonstationary multiaxial test as well as stationary multiaxial tests. Marked anisotropic effects were thus observed in steady-state creep both on the magnitude of creep rate and on the noncoaxiality between creep

rate and stress.

Then, to identify active slip systems in the present creep tests, the experimental data of steady-state creep were analyzed using a crystal creep theory of the power law type. It was thus found that the anisotropic effects in steady-state creep are expressed well if the $\{111\}\langle 112 \rangle$ octahedral and $\{100\}\langle 110 \rangle$ cubic slip systems operate simultaneously.

We used two simple models, Models I and II, to analyze the experiments. In these models, the specimens were assumed to have uniform distributions of stress and strain rate. In a companion paper⁽¹⁷⁾, it is shown that the overall tensile and torsional strain rates $\dot{\epsilon}$ and $\dot{\gamma}$ obtained using the models are in good agreement with those of 3D-finite element analysis, and that deviations of the axial direction from [001] in the present specimens do not have significant influence on $\dot{\epsilon}$ and $\dot{\gamma}$.

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