Method for Assessing Truss System Fatigue Failure Probability Due to Random Vibration

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In this study, the system fatigue failure probability of two-dimensional, multiple-story statically indeterminate truss structures due to stationary random vibration will be investigated. For the example of aeronautical industries, in order to maintain a high standard of structural safety, certain structural components or systems in use should be timely replaced by new ones before their failure probabilities reach a certain critical level. After undergoing a session of random vibration, a truss system can easily have trillions of possible damage modes, the identification and collapse judgement of this gigantic number of modes present an insurmountable analysis workload. In this light, a combinational pivotal decomposition method (CPDM) will be employed in a general manner. The fatigue failure probability of each truss member will be predicted beforehand with the aid of the finite element method, stress crossing analysis and associated material S-N-P fatigue curves. Then, the CPDM method will be incorporated with the component fatigue analysis to assess the truss system failure probability in an innovative, systematic and efficient manner. This is important for judicious decision-making concerning replacement timing of associated truss components or systems.

** Key Words: **Truss, Random Vibration, Fatigue, System Reliability

1. Introduction

Probabilistic fatigue mechanics and structural system reliability have become active research areas in recent years, as indicated by the increase in publications and associated conferences. Some papers[1]–[4] synthesize the power of the finite element method and probabilistic concepts, others are devoted especially to truss structures[5]–[9]. On the other hand, a few papers discuss the probabilistic fatigue mechanics and associated random vibration[10]–[17]. In addition, some authors concern themselves with system reliability of structures[18]–[23], and assess the associated research direction and needs[24,25].

The assessment of system survivability or corresponding failure probability is an essential issue in design analysis for many important structures, such as the truss boom of an attack helicopter (Figs. 1 and 2). System failure of the boom due to combat may cost 1000-fold more than a repairable nonsystem failure.

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Fig. 1 Four-story truss structure and a sample failure mode

Fig. 2 Truss-simulated boom portion of a helicopter
dimensional, multiple-story statically indeterminate truss structures due to random-vibration-induced fatigue will be investigated. In the aeronautical industries, in order to maintain a high standard of structural safety, and thus a low accident rate, structural components or systems in use should be replaced by new ones before their critical system failure probabilities reach a certain dangerous level.

In the following, first a general ingenious combinational pivotal decomposition method (CPDM) will be briefly reviewed. The method is useful for this kind of complicated structural survivability problems which has a large number of possible damage modes, since even simply the identification and pass/fail judgement of these possible damage modes constitute a formidable analysis workload. Therefore, the CPDM method is employed in a general manner to circumvent the original dilemma. Furthermore, the fatigue failure probability of each truss member due to random vibration will be predicted with the aid of the finite element method, stress crossing analysis and associated family of S-N-P material fatigue in a probabilistic and discrete sense. Finally, the CPDM method is incorporated with the random-vibration-induced fatigue analysis to assess the system failure of the objective truss structure. With judicious fusion of these concepts and techniques, it is feasible to evaluate the structural system survivability in a systematic and efficient manner.

2. Combinational Pivotal Decomposition Method for System Assessment

The system survivability of structures is an essential issue in design analysis not only for airborne, seaborne or land-based military war machines but also for many important civilian structures. Undergoing a session of random vibration, two-dimensional, multiple-story statically indeterminate truss structures may have millions or even trillions of possible damage modes for which the identification and pass/fail judgement present an insurmountable workload for existing analysis techniques. In order to solve this dilemma, an ingenious combinational pivotal decomposition method (CPDM) has been developed to establish a systematic and efficient method of evaluating the system survivability of this type of structure. The CPDM method makes use of selected key bars in a truss structure as well as the pivotal decomposition concept in an enhanced manner to circumvent the original dilemma.

For the purpose of explaining the general CPDM method, the typical four-story truss structure shown in Fig. 1 is used here. The statistically independent failure probability for each truss member is temporarily assumed to be $P_f$ throughout the structure. For the said truss structure, there will be more than two million ($2^{21}=2097152$) possible damage modes. For each truss bar, the probability of failure $P_f$ and probability of survival $P_s$ together will equal unity. The system failure is defined as the appearance of any movable hinged node due to destruction of truss member(s) in the system, since it implies disastrous truss system collapse and/or loss of its function as a truss structure with free-moving node(s). The three pivotal bars (3, 8, 13) can have eight pass/fail combinations of ppp, pff, pfp, ffp, fff, ffp and fpp. These are used as pre-subscripts for conditional system survivability. For example, $(pppP_s)_{out}$ used in Eq. (3) represents the system survivability while bar 3 is safe (P), yet bar 8 and bar 13 have failed (ff). Furthermore, for an example of combat survivability of a truss system exposed to different configurations of gunfire, the cross bars 5, 6, 10, 11, 15, 16, 20 and 21 in Fig. 1 may have probabilities of failure different from the remainder of the four-story truss system. Then, by CPDM method, the necessary equations to calculate the system survivability are given below, where $P_{out}$ and $P_{in}$ are for cross bars, while $P_{in}$ and $P_{out}$ are for the remainder of the bars in the truss system.

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2}) + 2P_{p2}P_{p1}P_{p2})^2 (1)
\]

\[
(\text{pppP}_s)_{out} = (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out}
\]

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2} + 2P_{p2}P_{p1}P_{p2})
\]

\[
(\text{pppP}_s)_{out} = (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out}
\]

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2} + 2P_{p2}P_{p1}P_{p2})^2 (2)
\]

\[
(\text{pppP}_s)_{out} = (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out} + (\text{pppP}_s)_{out}
\]

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2} + 2P_{p2}P_{p1}P_{p2})^2 (3)
\]

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2} + 2P_{p2}P_{p1}P_{p2})^2 (4)
\]

\[
(\text{pppP}_s)_{out} = P_{pp}(P_{pp}P_{p2} + 3P_{pp}P_{p1}P_{p2} + 2P_{p2}P_{p1}P_{p2})^2 (5)
\]

Thus, the system $P_s$ can be obtained by summing these equations for eight combinational pivotal cases appropriately. Although the above example is for combat survivability, with appropriate modification, the CPDM method is applicable to a truss system with distinct component failure probabilities under random vibration, as discussed below.

3. Random-Vibration-Induced System Fatigue Failure Probability

Before the system failure of a particular structure can be assessed, it is necessary to obtain the failure probability of each structural component of the
system. Again, the typical four-story truss shown in Fig. 1 will be used as an example to explain the processing procedures. The said truss is assumed to be subjected to a horizontal stationary Gaussian random force with zero-mean at node 10 with a power spectral density (PSD) function as shown in Fig. 3. \( f_1 \) and \( f_2 \) are the lower and upper frequency limits for this force PSD input, respectively. While the force PSD primarily excites a particular natural frequency and mode of the truss, the response displacement PSD and subsequent stress PSD will be a narrow-band type. For such a stationary Gaussian process, let \( \nu^* \) be the average frequency of the positive-slope crossing rate of the stress level \( \sigma \), which is expressed as

\[
\nu^* = \frac{1}{2\pi} \sum_{\sigma}^{\sigma_t} \frac{\sigma^*}{\sigma^*} e^{-\left[ \nu^*/(2\sigma^*)^2 \right]^2},
\]  

(7)

where \( \sigma \) and \( \sigma^* \) are the standard deviations of the stress and stress rate, respectively, for a zero-mean Gaussian distribution. These two standard deviations can be easily obtained by the following equations with the aid of matrix analysis, or with the finite element method in more complicated cases:

\[
\sigma^2 = \int_{f_1}^{f_2} |H(f)|^2 \tilde{S}_{pp}(f) df
\]  

(8)

\[
\sigma^2 = \int_{f_1}^{f_2} |H(f)|^2 \tilde{S}_{PP}(f) df,
\]  

(9)

where \( H(f) \) and \( H_i(f) \) are the stress and stress rate frequency response functions, respectively, for each truss component, and \( \tilde{S}_{PP} \) is the PSD of the given stationary Gaussian forcing function. On the other hand, the stress peak probability density function \( f_6(q) \) is the well-known Rayleigh distribution as shown in Fig. 4.

\[
f_6(q) = \frac{q}{\sigma^2} e^{-q^2/(2\sigma^2)}
\]  

(10)

With \( t \) being the duration of random force excitation, the number of stress peaks \( N_q(q, \sim q) \) in a stress range \( q_1 \) to \( q_2 \) can be easily obtained as

\[
N(q, \sim q) = \frac{v_q}{v} \int_{q_1}^{q_2} f_6(q) dq,
\]  

(11)

where \( v_q^* \) is the average positive-slope zero-stress crossing rate obtained from Eq. (7) for \( s = 0 \). As an important safety maintenance measure, when the risk of fatigue failure of an airborne structural system reaches a certain level, it would be opportune to replace it with new parts or system. Moreover, assuming that no alternating stress lower than the endurance limit \( s_e \) will affect any fatigue failure probability, it would be appropriate to discretize the stress range of \( s_e \) to \( s_u \) (ultimate strength) into a finite number of segments, e.g., \( M \) segments, to accommodate the limited data availability of the associated family of S-N-P fatigue curves. Theoretically, \( M \) can be as large as the number of stress reversals, but as commonly known, the high-end and low-end tails of a probability distribution curve are usually less accurate due to the practical difficulties in testing an extremely large number of specimens. Thus, to avoid this problem, it is appropriate to discretize the horizontal axis of the stress-peak Rayleigh distribution curve into a finite number of stress segments. Subsequently, the number of stress peaks corresponding to each stress segment can easily be obtained with the aid of Eq. (11) and Fig. 4. Note that each truss component of the four-story truss has its unique peak-stress Rayleigh distribution curve similar to Fig. 4. The representative peak stress of each stress segment and the corresponding number of stress peaks can be used in conjunction with the family of S-N-P curves\(^{180}\), as shown in the example in Fig. 5, to obtain its corresponding survivability \( L_m \). A structural component should pass all the fatigue ordeals of each stress segment in order to survive. Assuming these ordeals are statistically independent, then the failure probability \( P_f \) of the structural component can be estimated as

\[
P_f = 1 - \prod_{m=1}^{M} L_m,
\]  

(12)

where \( \Pi \) is the notation for sequential multiplications as indicated by the range of its subscripts and/or superscripts. Equation (12) is valid only when fatigue is a chance failure or a constant-failure-rate phenomenon. Note that the effects of cumulative damage due to the number of stress reversals and corresponding stress peaks have been accounted for in the \( L_m \) and associated Eq. (12). For longer duration under a given stationary random excitation force, i.e., a greater number of stress reversals, each \( L_m \) will generally

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**Fig. 3** Example of the power spectral density of a stationary Gaussian input force

**Fig. 4** A typical Rayleigh distribution of stress peaks for a Gaussian narrow-band process
Fig. 5  Family of S-N-P Curves for AA 7075-T6 aluminum alloy (P = Probability of failure)(293)

decrease, as reflected by the associated S-N-P curve. Although experience shows that many theoretical fatigue life estimations are often be offset by a high percentage from actual data, for these types of difficult fatigue problems, such estimations are still useful for many design purposes. In that light, it can be assumed that the elegant Eq. (12) will give a comparatively reasonable estimation of component failure probability under given conditions. Naturally, many other existing damage rules(293) can be suitably employed, but the component failure reported in many other papers is not the focus of this study. On the other hand, the traditional controversial sequence effect due to various groupings of stress reversals is not intended by Eq. (12). Likewise, the failure probability $P_s$ of each remaining structural component can be obtained by the same concept and procedure involving Eqs. (7)～(12) presented in this section. Subsequently, these component failure probabilities can be substituted into the following system survivability equations derived by CPDM method

\[
\begin{align*}
(p_{12}s_2P_s)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{sm}}{P_{sm}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{sn} \right) \left[ 1 + \sum_{m=12,15,16} P_{sm} \right] \\
&= \left( \prod_{n=12} P_{sn} \right) \left[ 1 + \sum_{m=17}^{21} P_{sm} \right]
\end{align*}
\]

(13)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(14)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(15)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(16)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(17)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(18)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(19)

\[
\begin{align*}
(p_{12}s_2)_{soi} &= (P_{s1}s_{12}P_{s2}) \\ 
&= \left( \prod_{n=12} \frac{P_{s1}}{P_{s2}} \right) \left[ 1 + \sum_{m=12}^{15} \frac{P_{s1m}}{P_{s2m}} \right] \\ 
&= \left( \prod_{n=12,15,16} P_{s1n} \right) \left[ 1 + \sum_{m=12,15,16} P_{s1m} \right] \\
&= \left( \prod_{n=12} P_{s1n} \right) \left[ 1 + \sum_{m=17}^{21} P_{s1m} \right]
\end{align*}
\]

(20)

The sum of the above eight equations gives the system survivability of the four-story truss system. The corresponding system failure probability can subsequently be obtained. For a numerical example, let each horizontal and vertical truss member be 0.25 m and all cross bars be 0.3535 m in length with the same cross section of 4.0 cm². The material used is AA 7075-T6 aluminum alloy which is one of the common airframe materials with the family of S-N-P fatigue curves(293) shown in Fig. 5. This material has a specific weight of 2.81 and a Young's modulus of 71 GN/m². A constant modal damping ratio of 0.01 may be assumed for this structure. As examples, the first and second natural mode shapes with natural frequencies 177.6 Hz and 772.9 Hz, respectively, are shown in Fig. 6. The power spectral density of input stationary Gaussian random force acting horizontally at node 10 for 56,300 seconds is shown in Fig. 3. With the aid of the well-known ANSYS finite element program, the stress power spectral density of each truss component
can be obtained and integrated to attain the square of the zero-mean standard deviation $\sigma_t$, which can be used in Eq. (10) to obtain the Rayleigh distribution of peak stresses for this bar. An example of stress power spectral density is shown in Fig. 7 as a frequency function. The stress PSD and the PSD of stress time rate can be integrated and substituted into Eq. (7) to obtain the average positive zero-stress crossing rate $\bar{\nu}$ of 177.6 crossings/sec. Thus an average of 10 million cycles will be experienced by each truss component for the above given duration of random excitation. A Rayleigh distribution similar to that in Fig. 4 is then discretized into a few segments of peak stress between endurance stress and ultimate stress, to accommodate Eqs. (11), (12) and data availability of the associated family of S-N-P fatigue curves. By Eqs. (7) ~ (12), $P_s$ of each truss component can be obtained. Subsequently, with the aid of Eq. (13) through Eq. (20), the system failure probability of the four-story truss system can be obtained as 0.0125. Although this example shows a single-input random force for the sake of simplicity, in fact, as long as its spectral contents primarily excite a narrow-band response, multiple random forces at multiple application points on the structure are permissible.

4. Conclusions

Assessing system failure probability is one of the crucial design analysis tasks for many important structures. In this study, the method of assessing the system failure of truss structures due to stationary random-vibration-induced fatigue has been investigated. The focus is on predicting system-level failure probability, instead of that at the component level. Therefore, there is no attempt to innovate new fatigue analysis scheme on the component level. Aeronautical industries traditionally have a high standard structural safety requirement, thus timely replacement of important structural components or systems becomes necessary before their critical system failure probabilities reach a certain dangerous level. For a typical truss structure, the component fatigue failure probability due to random vibration can be predicted first using stress crossing analysis, S-N-P curves and other associated techniques. Nevertheless, a truss system subjected to random excitation may present a formidable amount of possible damage modes. Thus it become an extremely difficult analysis task for, e.g., a raw fault tree analysis, truth table analysis, or even the Monte Carlo sampling analysis. In that light, the CPDM method is developed in a more general manner to overcome the difficulties. Consequently, several techniques are incorporated to establish a systematic method for assessing the truss system fatigue failure probability in an innovative and efficient manner. Naturally, due to the inherent difficulty in testing a large number of samples, the availability of accurate S-N-P fatigue curves for various materials is another concern when applying the above analysis scheme. In some cases, limited S-N curve data can be used in conjunction with some reasonably assumed reliability distribution curve at each peak-stress level. Although the structural random response of the given example is for narrow-band Gaussian response with zero-mean, in fact, as long as the component survivability can be obtained by means of any fatigue analysis technique or damage theory, the CPDM method can be employed to obtain the system failure probability. For more complicated truss structures such as electric power transmission truss towers, the pivotal bars can be arranged in a hierarchical manner to attain the best efficiency for CPDM treatment. Incidentally, the analysis objects of CPDM method are not limited to truss systems; for example, it has been successfully used for an efficient assessment of the electric power system reliability of multiple-engine aircraft.

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