Kinking Analysis by Hyper-Elastic Finite Element Method*

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'Kinking' is known as an unstable behavior of a twisted slender rod, which occurs after torsional buckling. It is simulated by a hyper-elastic finite element code developed by the authors, with hexahedral solid elements. The mechanical characteristics and criterion of kinking are successfully discussed. An incompressible constraint to model a rubber-like material is introduced in the simulation by the Lagrange multiplier method and the penalty method. It is found that the former method is computationally more efficient in the case of large deformation such as kinking.

Key Words: Finite Element Method, Torsion, Hyper-Elasticity, Mooney-Rivlin Material, Lagrange Multiplier Method, Penalty Method

1. Introduction

The objective of this paper is two-fold. The first aim is to simulate 'kinking', which is known as an interesting spatial unstable behavior of a twisted slender rod, by the hyper-elastic finite element code developed by the authors, and the second aim is to compare and assess the robustness of some kinds of hexahedral solid elements through the kinking simulations.

A strict definition of kinking is not well established, but it is often observed in practice. For instance, when we twist an elastic string to some degree, a kink will occur somewhere on the string^{(1),(2)}. This phenomenon is based on the unstable transformation of torsional strain energy to bending strain energy, and the formation of a loop is understood to absorb one twist of the torsion.

A phenomenon resembling kinking can be

simulated theoretically by solving the elastica equation⁽³⁾ formulated by Euler and Kirchhoff, and recently Miyazaki and Kondo⁽⁴⁾ have explored the feasibility. In the elastica equation, however, the following assumptions are introduced:

The shear deformation of the rod and the axial deformation of the neutral axis are negligible.

The cross section of the rod is undeformed during deformation.

The bending and torsional moments are proportional to the curvature and torsion, respectively.

On the other hand, it is unnecessary to introduce these assumptions if the finite element method using solid elements is employed. Furthermore, the analysis can be performed under arbitrary boundary conditions. In addition, more realistic simulation can be realized by adopting the hyper-elastic constitutive equation, which is known as an appropriate model for rubber-like materials.

It is known, however, that rubber-like materials are almost incompressible even under finite deformation. The introduction of incompressibility constraint to the finite element formulation is thus inevitable, and several methods have been proposed already.

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Oden⁽⁵⁾, Yamada and Kikuchi⁽⁶⁾, and others assumed full incompressibility and used mixed elements based on the Lagrange multiplier method. Also used are the displacement based elements with penalty constraints, such as those by Häggblad and Sundberg⁽⁷⁾, Peeken et al.⁽⁸⁾ and Simo and Taylor⁽⁹⁾. Among them, Peeken et al.⁽⁸⁾ also showed numerical examples of slightly compressible material by utilizing the fact that there exists some relationship between the penalty parameter and Poisson's ratio under infinitesimal deformation. The formulation by Simo and Taylor⁽⁹⁾ is somewhat different from the usual manner. Usually the penalty terms are added to the total potential energy, but their method is based on the stress strain relation containing the penalty terms. More general compressible elements which perform well even under incompressible conditions, i.e., u/p formulation based elements, have been proposed by Sussman and Bathe⁽¹⁰⁾, and van den Bogert et al.⁽¹¹⁾ In the main part of this paper, 8(displacement)/1(pressure)-node hexahedral solid elements based on the Lagrange multiplier method under fully incompressible assumption are used. Also, 8-node and 20-node hexahedral elements based on the penalty method are used for comparison.

2. Formulation for Hyper-Elastic Finite Element Analysis

2.1 Basic formulation for incompressible hyper-elastic material

In this paper the following notations are used:

$\mathbf{X}, \mathbf{x}$ undeformed and deformed position vectors of a material point

$\mathbf{F} = (\partial x_i / \partial X_j) \mathbf{e}_i \otimes \mathbf{e}_j$ deformation gradient tensor

$\mathbf{C} = \mathbf{F}^T \cdot \mathbf{F}$ right Cauchy-Green deformation tensor

$\mathbf{E} = (\mathbf{C} - \mathbf{I})/2$ Green-Lagrange strain tensor

I_c, II_c, III_c principal invariants of $\mathbf{C}$

$\mathbf{T}$ Cauchy stress tensor

$\mathbf{\Pi} = \mathbf{J}\mathbf{F}^{-1} \cdot \mathbf{T}$ first Piola-Kirchhoff stress tensor

$\mathbf{S} = \mathbf{J}\mathbf{F}^{-1} \cdot \mathbf{T} \cdot \mathbf{F}^{-T}$ second Piola-Kirchhoff stress tensor

where $\mathbf{e}_i$ denote the base vectors of an orthogonal Cartesian coordinate system, $\otimes$ a tensor product, and $J = \det \mathbf{F}$.

Hyper-elastic material is defined as a material model which has an elastic potential function W , which is independent of the deformation paths. A stress tensor is derived by differentiating W by the conjugate deformation gradient or strain tensors as

$$\Pi_{ji} = \partial W / \partial F_{ij} \quad \text{or} \quad S_{ij} = \partial W / \partial E_{ij}. \quad (1)$$

From Eq. (1), one can easily derive

$$S_{ij} = 2 \left(\left(\frac{\partial W}{\partial I_c} + \frac{\partial W}{\partial II_c} I_c \right) \delta_{ij} - \frac{\partial W}{\partial II_c} C_{ij} \right.$$

$$\left. + \frac{\partial W}{\partial III_c} III_c (C^{-1})_{ij} \right). \quad (2)$$

Equation (1) is reduced to the following under incompressible conditions:

$$S_{ij} = -p (C^{-1})_{ij} + 2 \left(\frac{\partial W}{\partial I_c} + \frac{\partial W}{\partial II_c} I_c \right) \delta_{ij} - 2 \frac{\partial W}{\partial II_c} C_{ij}, \quad (3)$$

where p denotes indeterminate hydrostatic pressure which can be evaluated by boundary conditions⁽¹²⁾. The total potential energy for the finite element formulation under the assumption of incompressibility is given as Eq. (4) or (5). Since the total Lagrange formulation is employed, the domain V denotes the initial configuration, and $\bar{\mathbf{t}}$ the nominal traction force vector. λ and α are the Lagrange multiplier and the penalty parameter, respectively. Body force is not taken into account. Equation (5) can also express slight compressibility under a suitable choice of the α value.

$$\bar{\Phi} = \int_V W dV - \int_S \bar{\mathbf{t}} \cdot \mathbf{u} dS + \int_V \lambda U(III_c) dV \quad (4)$$

$$\bar{\Phi} = \int_V W dV - \int_S \bar{\mathbf{t}} \cdot \mathbf{u} dS + \int_V \frac{\alpha}{2} \{U(III_c)\}^2 dV \quad (5)$$

U , the function of III_c , is defined so as to be $U=0$ and $dU/dIII_c=0$ for $III_c=1$, and $U=III_c-1$ or $U=\ln III_c$ is commonly used. The stationary condition of Eq. (4) is

$$\delta \bar{\Phi} = \int_V \left(\frac{\partial W}{\partial C_{ij}} + \lambda \frac{\partial U}{\partial C_{ij}} \right) \delta C_{ij} dV - \int_S \bar{t}_k \delta u_k dS + \int_V \delta \lambda U(III_c) dV = 0. \quad (6)$$

By applying the divergence theorem and equilibrium equation $\nabla_x \cdot \mathbf{\Pi} = 0$, the above equation yields

$$S_{ij} = 2 \frac{\partial W}{\partial C_{ij}} + 2\lambda \frac{\partial U}{\partial C_{ij}}. \quad (7)$$

Under the incompressible condition, Eq. (7) is reduced to Eq. (3) where $-p$ is replaced by 2λ . This indicates the well-known relation which holds between the Lagrange multiplier and the indeterminate hydrostatic pressure, i.e., $\lambda = -p/2$.

Now it is natural to split the second Piola-Kirchhoff stress tensor into the following two parts, i.e., $S^{(d)}_{ij}$ which is derived from the elastic potential W , and $S^{(p)}_{ij}$ which is derived from the Lagrange multiplier and the incompressible constraint function U .

$$S^{(d)}_{ij} \equiv 2 \frac{\partial W}{\partial C_{ij}}, \quad S^{(p)}_{ij} \equiv 2\lambda \frac{\partial U}{\partial C_{ij}} \quad (8)$$

Equation (6) is thus rewritten as

$$\int_V \frac{1}{2} (S^{(d)}_{ij} + S^{(p)}_{ij}) \delta C_{ij} dV - \int_S \bar{t}_k \delta u_k dS + \int_V \delta \lambda U(III_c) dV = 0. \quad (9)$$

The finite element stiffness equation is obtained from Eq. (9) through the incremental decomposition and

linearization.

Considering the stationary condition of Eq.(5), we have

$$\delta\bar{\Phi} = \int_V \left(\frac{\partial W}{\partial C_{ij}} + \alpha U \frac{\partial U}{\partial C_{ij}} \right) \delta C_{ij} dV - \int_S \bar{t}_k \delta u_k dS = 0. \quad (10)$$

Then the following equations are derived from Eq.(10) analogously to the case of the Lagrange multiplier method.

$$S_{ij} = 2 \frac{\partial W}{\partial C_{ij}} + 2\alpha U \frac{\partial U}{\partial C_{ij}} \quad (11)$$

$$S^{(d)}_{ij} \equiv 2 \frac{\partial W}{\partial C_{ij}}, \quad S^{(p)}_{ij} \equiv 2\alpha U \frac{\partial U}{\partial C_{ij}} \quad (12)$$

$$\int_V \frac{1}{2} (S^{(d)}_{ij} + S^{(p)}_{ij}) \delta C_{ij} dV - \int_S \bar{t}_k \delta u_k dS = 0 \quad (13)$$

Through the incremental decomposition and linearization, the stiffness equation is derived from Eq.(13). Note that there exists a well-known relation between αU in $S^{(p)}_{ij}$ in Eq.(12) and the Lagrange multiplier λ , i.e., $\lambda \approx \alpha U$. In finite element analyses it is conventional to interpolate the Lagrange multiplier by a shape function which is one order lower than that of the displacement. Furthermore, it is sometimes shown that this type of elements are equivalent to those based on the penalty method in which reduced integration is applied to the term corresponding to $S^{(p)}_{ij}$ in the stiffness equation derived from Eq.(13) (so-called selective integration)⁽¹³⁾. Especially for hexahedral solid elements, 20-node serendipity elements with selective integration are often used⁽⁸⁾. In this paper, hexahedral solid mixed elements based on the Lagrange multiplier method with eight displacement nodes and one pressure node (denoted by L8/1F for simplicity) are mainly used. For comparison, the following hexahedral solid elements based on the penalty methods are also used, i.e., 8-node elements with full integration (P8F), 8 node elements with selective integration (P8S), 20-node (serendipity) elements with full integration (P20F), and 20-node (serendipity) elements with selective integration (P20S).

2.2 Mooney-Rivlin model

In this paper, the Mooney-Rivlin material⁽¹⁴⁾, the elastic potential function of which is given by

$$W = c_1(I_c - 3) + c_2(II_c - 3), \quad (14)$$

is employed. In Eq.(14), c_1 and c_2 are the so-called Mooney-Rivlin constants which depend only on the material. With this W , $S^{(d)}_{ij}$ appearing in Eqs.(8) and (12) is derived as follows.

$$S^{(d)}_{ij} = 2 \frac{\partial W}{\partial I_c} \frac{\partial I_c}{\partial C_{ij}} + \frac{\partial W}{\partial II_c} \frac{\partial II_c}{\partial C_{ij}} \\ = (2c_1 + 2c_2 I_c) \delta_{ij} - 2c_2 C_{ij} \quad (15)$$

Since Eq.(15) has a drawback to yield the initial value of $S^{(d)}_{ij} = (2c_1 + 4c_2) \delta_{ij}$ for the undeformed state, the elastic potential W should be modified. In general

there are two methods to modify the function. The first method is to add a modification term to the original function⁽⁷⁾⁻⁽⁹⁾ as

$$W = c_1(I_c - 3) + c_2(II_c - 3) \\ - (2c_1 + 4c_2) G(III_c), \quad (16)$$

where $G(III_c)$ is a function defined such that $G=0$ and $dG/dIII_c=1$ for $III_c=1$. The second method is to define a new function $\bar{W}$ whose arguments are the reduced invariants $\bar{I}_c$ and $\bar{II}_c$ instead of I_c and II_c ^{(6),(10)}, i.e.,

$$\bar{I}_c = I_c / III_c^{1/3}, \quad \bar{II}_c = II_c / III_c^{2/3} \quad (17)$$

$$\bar{W} = c_1(\bar{I}_c - 3) + c_2(\bar{II}_c - 3). \quad (18)$$

In this paper the first method is employed with $G(III_c) = \ln III_c$.

2.3 Incremental analysis method for nonlinear FEM

In this study, the arc-length method (the Riks-Wempner method⁽¹⁵⁾) which is often used for a post-buckling analysis is adopted. In nonlinear analyses, there often exist bifurcation points on the solution path, and in general, the secondary paths can be found by conducting a bifurcation analysis in the vicinity of the buckling points. In this paper the scaled corrector method, which was developed by one of the authors⁽¹⁶⁾, is used to find the secondary paths in order to realize the kinking mode.

2.4 Incompressible constraint function

Several kinds of incompressible constraint function U have been proposed⁽⁵⁾⁻⁽⁹⁾. In this paper Eqs.(19•a) and (19•b) are employed for the Lagrange multiplier method and the penalty method respectively.

$$U = \ln III_c, \quad U = (\ln III_c)/2 \quad (19\bullet a, b)$$

The authors also examined $U = III_c - 1$ for the Lagrange multiplier method, and $U = III_c - 1$, $U = J - 1$, $U = \ln J$, $U^2 = \{(J-1)^2 + (\ln J)^2\}$ and $U^2 = \{(III_c - 1)^2 + (\ln III_c/2)^2\}$ for the penalty method. Although there was little difference for simple stretch and simple shear deformations, a significant difference was observed for torsional plus bending deformation which corresponds to the kinking modes, as shown later. The advantages of Eq.(19) are summarized as follows:

(i) Comparatively large increments can be taken.

(ii) Incremental calculation is fairly stable under large deformation.

2.5 Penalty parameter

In this paper authors followed the method of Peeken et al.⁽⁸⁾ to determine the penalty parameters. By substituting $U(III_c)$, $G(III_c)$ and Eq.(16) into Eq.(13) and differentiating by C_{k1} , the (1, 1, 1, 1) component of the constitutive tensor $D_{ijk1} \equiv 2 \partial S_{ij} / \partial C_{k1}$ of the Mooney-Rivlin material is derived as follows for infinitesimal deformation.

$$D_{1111} = \alpha + 4c_1 + 8c_2 \quad (20)$$

On the other hand the (1, 1, 1, 1) component of the constitutive tensor of Hooke's law, i.e., $D^{(L)}_{ijkl}$, is given by

$$D^{(L)}_{1111} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}. \quad (21)$$

By considering the infinitesimal simple stretch, Young's modulus E and the incompressible Mooney-Rivlin constants are related as $E=6(c_1+c_2)$. Therefore, the penalty parameter α is determined as follows.

$$\alpha = \frac{6(c_1+c_2)(1-\nu)}{(1+\nu)(1-2\nu)} - (4c_1+8c_2) \quad (22)$$

In the following numerical assessment, $\nu=0.48, 0.49, 0.4995$ and 0.4999 are assumed for Poisson's ratio. The corresponding penalty parameters are $\alpha=21.3, 175, 1000$ and 5000 , respectively, when $c_1=0.3$ and $c_2=0.15^{(10)}$ are employed. Although $\alpha=21.3$ seems to be small as a penalty parameter, it was used to assess the effect of the slightly compressibility on kinking.

3. Numerical Results

The hyper-elastic finite element analysis code was developed for the kinking simulation based on the above formulation with 8-node and 20-node hexahedral solid elements.

The soundness of each type of element is checked through analyses under static pressure, simple stretch and simple share. Since torsion is an important deformation mode in this study, it is also checked whether one element preserves its volume under large torsion, and satisfactory results are obtained.

Figure 1 shows the FE model of the rods with 1×1 square cross section and the lengths of $L=10, 20, 30, 40$ and 100 . The $L=20$ rod is mainly used in this study, and the other rods are used to assess the effect of length on kinking. Throughout the analysis $c_1=0.3$ and $c_2=0.15$ are used as the Mooney-Rivlin constants.

In order to describe the analysis result conveniently, a few terminologies are introduced in relation to the analysis process. All the analyses in this study consist of two processes, i.e., (i) the rod is twisted

first, and then (ii) axial compression is applied to the rod. In both (i) and (ii) the bottom of the rod is fixed and the top surface of the rod is prescribed to have the undeformed square shape. In process (i), the notation ϕ , which is the rotation angle normalized by 2π , is conveniently used. For example, a rod twisted up to 2π is called the ' $\phi=1$ rod'. In process (ii) 'compression ratio ζ ' expresses the vertical displacement of the top normalized by L . For example, no displacement at the top corresponds to $\zeta=0$ and contact of the top with the bottom corresponds to $\zeta=1.0$. On the other hand if a rod is stretched, the compression ratio ζ takes a negative value. Furthermore, 'torsional moment' denotes the torsional moment at the top of a rod.

3.1 Basic deformation mode of kinking

In this section the 'basic deformation mode' of kinking is simulated. The $L=20$ rod is discretized into 1 (in cross section) $\times$ 300 (in longitudinal direction) L8/1F elements. Hereafter the same elements and mesh are employed. The accuracy of this mesh will be discussed later (in section 3.5). First we analyze the $\phi=1$ case. The resulting torsional moment-compression ratio curve is shown as Path A in Fig. 2. A marked deformation occurs at $\zeta=0.67$, where a complete plane deformation mode is obtained. The torsional moment is completely released at this point by forming a loop, which is subjected to only bending. In actuality, however, contact occurs and the complete transfer from torsional mode into bending mode is unrealizable, and thus some torsional energy should remain. In this paper, contact is not considered in the analysis. Next we start stretching from $\zeta=1.0$. As shown in Fig. 3 (Path B) the diameter of the loop decreases and finally a knot is made. The torsional moment-compression ratio curve of this deformation is shown as Path B in Fig. 2. It is seen that the torsional moment remains zero during this deformation. Now it is clear that one knot absorbs one twist, and this sequence of deformation is most instructive to understand kinking. The authors thus call it the 'basic

Fig. 1 FE model

Fig. 2 Torsional moment-compression ratio curve ($\phi=1$)

Fig. 3 Basic deformation of kinking ($\phi=1$)

deformation mode of kinking'.

3.2 Kinking analysis under various initial rotations

Following the previous section, we next simulate kinking under various initial rotations.

The first case is that the initial rotation ϕ is less than 1. Figure 4 shows the compression of the $\phi=0$ rod. A bifurcation point appears at $\zeta=0.60$ (Fig. 4 (b)), and finally it becomes (c) or (d) at $\zeta=1.0$. However if we stretch the rod from $\zeta=1.0$, both (c) and (d) recover the original shape and there is no

Fig. 4 Axial compression of $\phi=0$ rod Fig. 5 Torsional moment-compression ratio curve ($\phi=2$)

deformation mode that leads to kinking. This kind of phenomenon is commonly observed for the rods with $0 \leq \phi < 1$; the deformations at $\zeta=1.0$ are similar to that in Fig. 4 (d), and no deformation mode leading to kinking is observed.

The second case is that ϕ is more than 1. Figure 5 shows torsional moment-compression ratio curve of the $\phi=2$ rod, and the deformation is shown in Fig. 6 (a). This analysis consists of three stages. First a rod is twisted up to $\phi=2$ while $\zeta=0$ is preserved (state 0), and then stretched in the longitudinal direction to find a suitable bifurcation point which leads to kinking. Utilizing the bifurcation analysis, an equilibrium point on the secondary path is found (state 1). Finally the rod is compressed by the Riks method. If we stretch an elastic string containing a knot by strong tension, the knot vanishes. The reverse process of this phenomenon seems to correspond to the above sequence of

operation. State 2 in Fig. 5 indicates the maximum compression ratio which can be obtained by the Riks method. After state 2, the deformation proceeds in

Fig. 6 Kinking of $\phi=2$ and $\phi=5$ rods

Fig. 7 Torsional moment-compression ratio curve

Fig. 8 Relationship between initial rotations and torsional moments

a different mode. If path a is chosen after an intermediate bifurcation point (see Fig. 5), a kink is finally formed at $\zeta=1.0$, as shown in Fig. 6 (state 3). As long as we trace the path which is found in state 1, no continuous solution exists in the range where ζ is more than the maximum compression ratio (state 2). A similar phenomenon has also been observed by Miyazaki and Kondo⁽⁴⁾.

The deformation modes are essentially the same for the $\phi \geq 2$ rods. Figure 6(b) shows the deformation of state 3 of the $\phi=5$ rod. Figure 7 shows torsional moment-compression curves of the $\phi \geq 1$ rods. Figure 8 indicates the relationship between the torsional moments at state 0 and state 3 versus various initial rotations ϕ , while Fig. 9 shows the relationship between initial rotation ϕ and maximum compression ratio (state 2). It is seen in Fig. 8 that one knot absorbs one twist. The broken lines connecting state 0 and state 3 are not perfectly parallel. This seems to be caused by (i) the cross section of the rod is rectangular, and (ii) the sizes of cross section are not negligible compared with the length of the rod. According to Fig. 9, the maximum compression ratio of the $\phi = 6$ rod takes a negative value, and state 3 cannot be determined because there is no continuous solution path which returns to $\zeta=0$ through states 1 and 2. This type of phenomenon seems to occur for $\phi \geq 5.5$, as shown in Fig. 9.

The above discussions imply the possibility of segregating the initial rotation into three types, i.e., the “non-kinking region” in which kinking must not occur, the “transition region” in which kinking does not occur without finite compression, and the “kinking region” in which kinking occurs without any compression.

3.3 Kinking analysis with preservation of initial height

If we continue twisting a rod while preserving the initial height, a bifurcation point appears soon after 2π . If branch-switching is done at this point and further twist is applied by the Riks method, the torsional moment-torsional angle history shown in

Fig. 9 Relationship between initial rotations and maximum compression ratios

Fig. 10 Torsional moment-rotation angle curve

Fig. 11 The relationship between various initial rotations and maximum compression ratios

Fig. 10 (solid line) is obtained. The broken lines in the figure are the same as those of Fig. 8 (the lines connecting state 0 or 3). It is seen that the torsional angle begins to decrease around 5.5 revolutions which corresponds to the lower limit of the "kinking region", and then the history follows the broken line of state 3, revealing some bifurcation points. On the other hand if we twist an actual elastic string, a knot will be formed around 5.5 revolutions for the reason discussed in section 3.2. Once a knot is formed, the torsional moment abruptly decreases, but if we continue twisting, the torsional moment will increase again. This kind of simulation, however, cannot be realized in this paper because the Riks method does not cope with a discontinuous solution. A new algorithm to search isolated paths must be developed.

3.4 Effect of rod length on kinking

The same analysis as in section 3.2 is also done for the $L=10, 30, 40$ and 100 rods. The same mesh as the case of $L=20$ is adopted. All results are found to be similar to the case of $L=20$. Figure 11 shows the relationship between initial rotation and maximum compression ratio. Also shown by the solid line is the solution to the elastica equation by Miyazaki and Kondo⁽⁴⁾. It is seen that the initial rotation corresponding to the zero-maximum compression ratio becomes large as the rod becomes longer. Since the present

Fig. 12 Solutions with finer mesh and higher order elements (Relationship between initial rotations and maximum compression ratios)

Fig. 13 Solution with full and selective integrations (Torsional moment-compression ratio curve ($\phi=1$))

solution tends to agree with the elastica solution in spite of the difference in analysis, kinking seems to be primarily governed by the geometrical nature of the model. It is noted, however, that the maximum compression ratio is not a negative value in the case of the elastica equation.

3.5 Discussions on mesh and elements

To confirm the validity of the analysis presented in the preceding sections, the same analysis as in section 3.2 is repeated with a finer mesh or higher order elements. With L8/1F elements the $L=20$ rod is divided into 4 (in cross section) $\times$ 300 (in longitudinal direction). Also, with P20S elements the same rod is divided into 1 (in cross section) $\times$ 150 (in longitudinal direction). The relationship between initial rotation and maximum compression ratio is shown in Fig. 12. These solutions seem to agree well with each other, though a slightly stiffer solution is obtained in the case of 1×300 L8/1F elements.

Next the authors compared L8/1F, P8F and P8S elements. Figure 13 shows the result of the same analysis as shown in section 3.1. In both P8S and P8F cases, four kinds of penalty parameters α corresponding to ν as mentioned above are used. The result of P8S mostly agrees with that of L8/1F. On the other hand, results of P8F are different even in the case of a small α value corresponding to $\nu=0.48$. It seems that locking occurred due to the full integration. The same

Fig. 14 Effect of penalty parameters on full and selective integration cases (Relationship between initial rotations and maximum compression ratios)

is true for the analysis in section 3.2. Figure 14 shows a comparison of the case of L8/1F with P8F (with only $\nu=0.48$ and 0.49) and P8S. The result of P8S mostly agrees with that of L8/1F for various penalty parameters. On the other hand the result of P8F shows significant difference. It is found from the above analysis that even a small penalty parameter that allows slight compressibility affects the solution slightly if the selective integration is applied.

Finally, the computational efficiencies are compared. For example, if L8/1F elements are used to analyze the axial compression as shown in section 3.1, it takes about 15 minutes using a 76 Mips (23 Mflops) EWS. On the other hand if P8S elements are used for the same analysis, it takes about 3 hours with a smaller penalty parameter and about 5 hours with an ordinary penalty parameter. This is because larger increments must be taken in the penalty method compared with the Lagrange multiplier method. In general the penalty method is thought to be more convenient than the Lagrange multiplier method because it is unnecessary to introduce additional unknowns, but it is computationally less efficient to analyze extremely large deformations such as kinking.

4. Conclusions

Kinking, which is known as an unstable behavior of a twisted slender rod, is simulated by the hyper-elastic finite element code developed by the authors. The mechanical characteristics of kinking are shown through simulation, and the condition under which

kinking occurs is discussed by dividing the initial rotation into three regions.

The computational efficiencies are compared between the Lagrange multiplier and the penalty methods, and the former method is shown to be more efficient.

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