Inverse Analysis of the Magnitude and Direction of Impact Force*

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This paper treats an inverse problem to estimate the magnitude and direction of impact force acting on a body of arbitrary shape. Three directional components of the impact force vector can be estimated from responses measured at more than three points of the body by means of deconvolution. It is pointed out that the impact force cannot always be estimated satisfactorily since the inverse problem is ill-posed or ill-conditioned in some situations. To improve the estimation accuracy, the least squares method based on the singular value decomposition is adopted. The validity of this method is demonstrated through an experiment in which the impact force acting on a simply supported beam is estimated from strain responses measured by strain gages.

Key Words: Inverse Problem, Impact Force, Experimental Mechanics, Transfer Function, Transient Response

1. Introduction

Accurate evaluation of impact force is necessary for examining the reliability and safety of structures subjected to impact. In experimental studies of structures under impact loading, accurate measurement of impact force is an important issue. However, unlike the static case, the time variation of the strain or displacement at any point of the impacted body is generally different from that of the impact force because of the propagation of stress waves within the body. Therefore, the measurement of the impact force is much more difficult than that of the static force. There has been no method appropriate for measuring impact force acting on bodies of arbitrary shape.

The development of computers and measuring instruments has made the numerical processing of

measured data much easier than before. Various methods based on numerical data processing have been proposed for the measurement of physical or mechanical quantities. Such methods are given attention also for the measurement of impact force⁽¹⁾⁻⁽⁶⁾. These methods are based on the estimation of impact force by means of inverse analysis with the knowledge of impact response of the body, such as strain, acceleration or displacement.

We also studied a method of inverse analysis to estimate the impact force acting on bodies of arbitrary shape, from the measured response of the body, and showed the validity of the method through both numerical simulations⁽⁷⁾ and experiments⁽⁸⁾⁻⁽¹⁰⁾. In addition, we applied the method to the measurement of impact force in instrumented impact testing of materials^{(11),(12)}. In these studies, we assumed that the direction of impact force was constant and prescribed. In practice, however, there are many cases in which the direction of the impact force is constant but unknown or changes with time during the impact event. Therefore, in order to measure the impact force under a variety of situations, we need a method to obtain not only the magnitude but also the direction

* Received 8th October, 1993. Paper No. 92-0934

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of the impact force.

In this paper, we study an inverse problem to estimate the magnitude and direction of impact force from the knowledge of impact responses of a body. First, we will show the basic scheme of the inverse analysis. Next, we will point out the ill-posed or ill-conditioned aspects of the inverse problem and present a method of regularization. Finally, we will demonstrate the validity of our method by applying it to the measurement of impact force acting on a simply supported beam.

2. Inverse Problem of Impact Force

Let us consider a body of arbitrary shape subjected to an impact force in an arbitrary direction, as shown in Fig. 1. We decompose the impact force vector into three arbitrary directions and designate the components by $f_i(t)$, $i=1, 2, 3$. In addition, we designate the impact responses (such as strains, displacements or accelerations) at arbitrary n -points ($n \geq 3$) of the body by $e_j(t)$, $j=1, 2, \dots, n$. If the responses linearly depend on the impact force, the relationship between the impact force and the responses may be expressed by

$$e_j(t) = \sum_{i=1}^3 \int_0^t h_{ji}(t-\tau) f_i(\tau) d\tau, \quad (j=1, 2, \dots, n), \quad (1)$$

where $h_{ji}(t)$, $i=1, 2, 3$ and $j=1, 2, \dots, n$, denote the impulse response functions of this linear system. We take the Fourier transforms of both sides of Eq. (1) to obtain, in the matrix form,

$$\mathbf{E} = \mathbf{H}\mathbf{F}, \quad (2)$$

where

$$\mathbf{E} \equiv \{E_1(\omega), E_2(\omega), \dots, E_n(\omega)\}^T, \quad (3)$$

$$\mathbf{H} \equiv \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & H_{13}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & H_{23}(\omega) \\ \vdots & \vdots & \vdots \\ H_{n1}(\omega) & H_{n2}(\omega) & H_{n3}(\omega) \end{bmatrix}, \quad (4)$$

$$\mathbf{F} \equiv \{F_1(\omega), F_2(\omega), F_3(\omega)\}^T. \quad (5)$$

The functions $E_j(\omega)$, $F_i(\omega)$ and $H_{ji}(\omega)$ denote the Fourier transforms of $e_j(t)$, $f_i(t)$ and $h_{ji}(t)$, respectively, and the superscript T indicates the transpose of the matrix. We call $H_{ji}(\omega)$ the transfer function.

Although it is difficult to measure the impact

Fig. 1 A body of arbitrary shape subjected to impact force

force directly, it is usually easy to measure the responses by using strain gages or accelerometers. Therefore, if we have identified the transfer functions in advance, we can estimate the directional components $f_i(t)$ of the impact force by

1. computing the Fourier transforms of the measured responses $e_j(t)$,
2. solving Eq. (2) numerically for $\mathbf{F}$, and
3. computing its inversion.

Once the directional components are estimated, it is easy to estimate the magnitude and direction of the impact force. The numerical Fourier transformation and its inversion can be easily and efficiently computed by using the fast Fourier transform (FFT)⁽¹³⁾. As we have shown in previous papers^{(7),(9),(14)}, use of an exponential function as the window function for the FFT reduces errors due to the numerical transformation and inversion. This corresponds to adopting the Laplace transform instead of the Fourier transform.

The transfer functions can be identified from data obtained by calibrations. If we apply any measurable impact force in each direction $i=1, 2, 3$, respectively, and measure the impact force and responses simultaneously, then we can calculate the transfer functions from the Fourier transforms of these data. If we repeat the calibration many times to obtain many sets of data, we can estimate the auto-spectral density functions of the responses and the cross-spectral density functions between the impact force and the responses. Then we can identify the transfer functions according to the spectral input-output relationship of the linear system. As we have shown in previous papers^{(7),(12)}, identification of the transfer functions by this scheme leads us to a regularization of the inverse problem based on the noncausal Wiener filter theory.

3. Ill-Posedness of the Inverse Problem and Regularization

The inverse problem solving Eq. (2) for $\mathbf{F}$ may be ill-posed or ill-conditioned in the following sense.

1. The algebraic equations (2) can be indeterminate or inconsistent depending on the combination of the locations to measure the response, which means that the uniqueness or existence of the solution are not satisfied.
2. In practice, the vector $\mathbf{E}$ and the matrix $\mathbf{H}$ cannot be free from errors such as errors due to limitations of the measuring instruments, measurement noises, quantization errors associated with digital sampling, and numerical errors of the discrete Fourier transformation. The uniqueness or existence of the solution might be violated due to such errors.

3. If the condition number $\text{cond}(\mathbf{H})$ is large enough, the stability of the solution may be poor because of expansion of the errors involved in $\mathbf{E}$ and $\mathbf{H}$.

In order to improve the first two of the ill-posedness listed above, we adopt the least squares method based on the singular value decomposition (SVD) to solve Eq. (2).

The SVD of the coefficient matrix $\mathbf{H}$ is given by⁽¹⁵⁾

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H, \quad (6)$$

where $\mathbf{U}$ and $\mathbf{V}$ are $n \times n$ and 3×3 unitary matrices, respectively, and the superscript H indicates the conjugate transpose of the matrix. In addition, $\mathbf{\Sigma}$ is an $n \times 3$ real matrix in the form

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

where $\sigma_i, i=1, 2, 3$ are the singular values of $\mathbf{H}$ such that

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0. \quad (8)$$

According to the theory of the SVD, the least squares solution $\tilde{\mathbf{F}}$ of Eq. (2), in the sense that both the Euclidean norms $\|\tilde{\mathbf{F}}\|_2$ and $\|\mathbf{H}\tilde{\mathbf{F}} - \mathbf{E}\|_2$ are minimum, is given by⁽¹⁶⁾

$$\tilde{\mathbf{F}} = \mathbf{H}^+ \mathbf{E}, \quad (9)$$

where $\mathbf{H}^+$ denotes the Moore-Penrose generalized inverse defined by

$$\mathbf{H}^+ = \mathbf{V} \mathbf{\Sigma}^+ \mathbf{U}^H, \quad (10)$$

$$\mathbf{\Sigma}^+ = \begin{bmatrix} \sigma_1^+ & 0 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^+ & 0 & 0 & \cdots & 0 \\ 0 & 0 & \sigma_3^+ & 0 & \cdots & 0 \end{bmatrix}, \quad (11)$$

$$\sigma_i^+ = \begin{cases} 1/\sigma_i & (\sigma_i \neq 0) \\ 0 & (\sigma_i = 0) \end{cases}. \quad (12)$$

The least squares solution (9) does exist uniquely, which means that the existence and uniqueness of the solution are satisfied by imposing two restrictions:

1. $\|\tilde{\mathbf{F}}\|_2$ is minimum, that is, the energy of the impact force is minimum and
2. $\|\mathbf{H}\tilde{\mathbf{F}} - \mathbf{E}\|_2$ is minimum, that is, the length of the residual vector is minimum.

Next, in order to improve the third ill-conditioned aspect, we adopt the method of rank decision⁽¹⁷⁾. The condition number of $\mathbf{H}$ is expressed in terms of the maximum singular value σ_1 and the minimum nonzero singular value σ_r (r is the rank of $\mathbf{H}$) as

$$\text{cond}(\mathbf{H}) = \sigma_1/\sigma_r. \quad (13)$$

If singular values smaller than a certain threshold value are replaced with zero, that is, if the rank of $\mathbf{H}$

is reduced, then the condition number becomes smaller, improving the stability of the solution. If the threshold value is too small, the solution will still be unstable. On the other hand, if the threshold value is too large, the solution will be over-smoothed. Therefore, we should choose an appropriate value referring to the condition number and the norm of the residual vector.

4. Impact of a Simply Supported Beam

To examine the validity of the method described in the previous section, we conducted an experiment to estimate the magnitude and direction of the impact force acting on a simply supported beam perpendicularly to its neutral axis.

4.1 Experimental setup

Figure 2 shows the scheme of the experiment. A brass rod (20 mm diameter by 2 m long) was supported with a 1.5 m span. The support was rotation-free but fastened with bolts and nuts to avoid the loss of contact due to the reaction. The impact force was applied at the center of its span with impactors falling freely along a guide tube. The impactors were a steel rod (10 mm diameter by 500 mm long) and a steel ball (25 mm diameter). The direction of impact force was changed by rotating the beam around its neutral axis by a specified angle θ . The strain responses of the beam were measured by strain gages (Kyowa Electronic Instruments, KSP-2-E 4) attached 100 mm away from the impact site and in parallel to the

Fig. 2 Impact of a simply supported beam

Fig. 3 The components of impact force and the location of the strain gages

neutral axis. Figure 3 shows the locations of the strain gages in the cross-sectional view of the beam. The directional components of the impact force $f_1(t)$ and $f_2(t)$ were defined as shown in Fig. 3.

We conducted the experiment in the following way.

- Step-1** Impact the beam with the steel rod in the direction $i=1$, and measure the impact force $f_1(t)$ and the strain responses $e_j(t)$, $j=1, 2, 3$. Then, identify the transfer functions $H_{j1}(\omega)$, $j=1, 2, 3$ from this set of data.
- Step-2** Impact the beam in the direction $i=2$ and identify the transfer functions $H_{j2}(\omega)$, $j=1, 2, 3$ in the same manner as in Step-1.
- Step-3** Impact the beam with the steel rod or the steel ball in an arbitrary direction θ and measure the strain responses $e_j(t)$, $j=1, 2, 3$.
- Step-4** Compute the Fourier transforms of the data obtained in Step-3 and calculate the least squares solution (9) by using the transfer functions identified in Steps-1 and -2. Then, compute its inversion to obtain the directional components $f_i(t)$, $i=1, 2$. In addition, calculate the magnitude and direction of the impact force.

The steel rod was instrumented with strain gages (Kyowa Electronic Instruments, KSP-2-E 4) attached 10 mm away from its impact end in order to measure the impact force acting on the beam. The time variation of the strain measured by this strain gage is slightly different from that of the impact force. We applied the inverse analysis again to estimate the impact force from the strain response of the rod. The

Fig. 4 Calibration of the impact rod

transfer function of the rod can be identified by longitudinally impacting two rods of same material and same diameter, as shown in Fig. 4. Let us designate the rod used for the impact of the beam as Rod-a and the other rod as Rod-b. In addition, let us designate the strain gages attached to these rods as Gage-1 and Gage-2, respectively. According to the theory of the one-dimensional longitudinal impact of rods, the time variation of the impact force and strain responses can be predicted as illustrated in Fig. 4. We understand that the initial pulse of the strain history measured by Gage-2 is the same as the impact force history except for a certain time delay. Therefore, we can measure the impact force with Gage-2 of Rod-b and the strain response with Gage-1 of Rod-a, which enables us to identify the transfer function of the rod. Note that the direction of the impact force is constant and known in this case.

The measurement system was as follows. The strain gages were connected to a DC strain amplifier. The output voltage signal of the amplifier was captured with a digital memory. Then, the data were transferred to a personal computer and numerically processed. The sampling rate Δt was $1 \mu\text{s}$ and the data length N was 1024. The Fourier transformation and its inversion were computed by using the FFT algorithm with the exponential window function $\exp(-\gamma t)$, $\gamma=5/(N\Delta t)^{(14)}$.

4.2 Results of the calibration

We impacted the beam with the steel rod to perform the calibration. The impact velocity was 1.98 m/s. Figure 5 shows a typical set of data obtained by the calibration in the direction $i=1$. Figure 5(a) shows the impact force $f_1(t)$. The duration of the impact is $200 \mu\text{s}$, which is equal to the time for the one-dimensional longitudinal stress wave in the steel rod to propagate down to the free end and back to the

Fig. 5 Result of the calibration of the beam

impact end. Figure 5(b) shows the strain responses $e_j(t)$, $j=1, 2, 3$, induced by $f_1(t)$. The strain response $e_1(t)$ has sign opposite to $e_2(t)$ and $e_3(t)$ but the time variations of these three are the same. We repeated the calibration 10 times to obtain 10 sets of data similar to that shown in Fig. 5. Then we used these 10 sets of data to identify the transfer functions $H_{j1}(\omega)$, $j=1, 2, 3$, by the method described in the previous paper⁽⁷⁾. We also performed the calibration in the direction $i=2$ and identified the transfer functions $H_{j2}(\omega)$, $j=1, 2, 3$, in the same manner.

4.3 Estimation of impact force by a rod

We impacted the beam with the steel rod in the direction $\theta=75^\circ$ and estimated the magnitude and direction of the impact force by the inverse analysis. The impact velocity was 1.98 m/s.

First, we estimated the impact force from two strain responses $e_1(t)$ and $e_2(t)$. The size of the matrix $\mathbf{H}$ is 2×2 in this case. Figure 6(a) shows the estimated magnitude of the impact force. In order to examine the estimation accuracy, we also measured the impact force in the same manner as in the calibration. Figure 6(a) also shows the measured magnitude as the thin curve. Figure 6(a) indicates that these results are in good agreement. Figure 6(b) shows the estimated direction. Since the direction of the impact force is indefinite when the magnitude is zero, we set $\theta=0^\circ$ when the magnitude was less than 5% of its maximum value. We applied this convention also in the following examples. Figure 6(b) indicates that the estimated direction is 75° during impact, which is equal to the actual value. Therefore, in this case, the magnitude and direction were accurately estimated.

Next, we estimated the same impact force from two strain responses $e_1(t)$ and $e_3(t)$. Figures 7(a) and (b) show the estimated magnitude and direction,

Fig. 6 Estimated impact force by the rod ($j=1, 2$)

respectively. The estimated magnitude is very noisy and tends to diverge with time. The estimated direction is not constant but oscillatory during impact. The estimate in this case was much poorer than that in the previous case.

In order to investigate the cause of the difference between the above two cases, we evaluated the condition number of the matrix $\mathbf{H}$ at every frequency for both cases. Figures 8(a) and (b) show the condition numbers for the cases of $j=1, 2$ and $j=1, 3$, respectively. The condition number for the case of $j=1, 3$ is larger than those for the case of $j=1, 2$ at frequencies lower than 100 kHz. At frequencies lower than 10 kHz, in particular, the condition number for the case of $j=1, 3$ is larger than 10, while that for the case of $j=1, 2$ is about 1. Therefore, the matrix $\mathbf{H}$ was much more ill-conditioned in the case of $j=1, 3$ than in the case of $j=1, 2$, which caused a poor estimation in the case of $j=1, 3$.

Fig. 7 Estimated impact force by the rod ($j=1, 3$)

Fig. 8 The condition numbers of the matrix $\mathbf{H}$
(a) $j=1, 2$, (b) $j=1, 3$

We applied the method of rank decision to the case of $j=1, 3$ to improve the ill-conditioned nature. The smallest singular value was replaced with zero at each frequency where the condition number was larger than 16. Figures 9(a) and (b) show the estimated magnitude and direction, respectively. The estimate shown in Fig. 9 is better than that shown in Fig. 7. Therefore, the method of rank decision is effective for improving the estimation accuracy of the impact force.

Next, we estimated the same impact force from three strain responses $e_1(t)$, $e_2(t)$ and $e_3(t)$ in order to examine the improvement of accuracy by estimation from more data (number of points for the strain response measurement) than the number of unknowns (number of components of the impact force vector). Figures 10(a) and (b) show the results of estimation. The magnitude of the impact force coincides very well with that obtained from the strain gages of

Fig. 9 Estimated impact force by the rod ($j=1, 3$, $\text{cond}(\mathbf{H}) \leq 16$)

Fig. 10 Estimated impact force by the rod ($j=1, 2, 3$)

the rod. The estimated direction is 75° , which is consistent with the actual direction. This result seems slightly better than that shown in Fig. 6.

4.4 Estimation of impact force by a ball

As an example of the application of our method, we estimated the impact force induced by collision of the steel ball. The beam was impacted at 3.13 m/s velocity in the direction $\theta=75^\circ$. Figure 11 shows the strain responses $e_1(t)$, $e_2(t)$ and $e_3(t)$ measured by strain gages of the beam. We estimated the impact force from these strain responses.

Figure 12 shows the impact force estimated from two strain responses $e_1(t)$ and $e_2(t)$. The broken curve shows the result predicted by an approximate analysis⁽¹⁸⁾ based on the Hertzian contact theory (contact of a sphere and a cylinder⁽¹⁹⁾). This approximate analysis has been proven to provide a satisfactory prediction of the impact force. Figure 12 indicates that the magnitude estimated by the inverse analysis agrees fairly well with the predicted one. The estimated direction is almost constant at 75° during impact. Therefore, we can say that the estimation accuracy is sufficiently satisfactory in this case.

Figure 13 shows the same impact force estimated from two strain responses $e_1(t)$ and $e_3(t)$. As in

Fig. 11 Strain responses of the beam induced by impact of the ball

Fig. 12 Estimated impact force by the ball ($j=1, 2$)

Fig. 13 Estimated impact force by the ball ($j=1, 3$) Fig. 14 Estimated impact force by the ball ($j=1, 3$, $\text{cond}(\mathbf{H}) \leq 10$)

impact with the rod (Fig. 7), the estimated magnitude is significantly noisy and different from the predicted one. The estimated direction changes monotonically from -30° to 110° during impact, which is certainly incorrect.

Next, we applied the method of rank decision to the case of $j=1, 3$ to improve the estimation accuracy. The smallest singular value was replaced with zero at each frequency where the condition number was larger than 10. Figure 14 shows the result. The magnitude becomes less noisy, and the direction becomes almost constant. However, the estimated magnitude is slightly smaller than the predicted one, and the estimated direction is not equal to the actual value of 75° .

Finally, we estimated the same impact force from three strain responses $e_1(t)$, e_2 and $e_3(t)$. Figure 15 shows the result. The estimation accuracy is good

Fig. 15 Estimated impact force by the ball ($j=1, 2, 3$)

enough and seems slightly better than in the case of $j=1, 2$ (Fig. 12).

5. Discussions

In this study, we adopted the method of rank decision for the regularization of the inverse problem. It is one of the simplest of many methods. Unfortunately, this method was not sufficiently effective in the example shown in Fig. 14. We consider that this is due to the discreteness of this regularization scheme. In the inverse analysis of impact force, there exist only three singular values, as indicated in Eq. (7). In the case shown in Fig. 14, in particular, there are only two. Therefore, further study is required on the regularization method for the inverse analysis of impact force. Tikhonov's method may be more appropriate because it has a continuous regularization parameter.

The condition number of $\mathbf{H}$ changes depending on the combination of the directions of the components of the impact force vector and the locations of the strain measurement. Therefore, if an appropriate combination such that the condition number is small enough is selected, as in the case shown in Figs. 6 and 12, no regularization would be required. However, for bodies of arbitrary shape, selection of such a combination might be difficult or require a great amount of numerical analysis. Therefore, it is worth considering regularization in order to improve the estimation accuracy.

6. Conclusions

In this paper, we proposed a method of inverse analysis of impact force to measure the magnitude and direction of impact force acting on a body of arbitrary shape. Our method was verified through experiments of impact of a simply supported beam.

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