Analytical Evaluation of Residual Stress Singularity with Interfacial Slip in Dissimilar Material Bonded Joints*

Masato MURATA** and Yoshihiko MUKAI**

It is important to obtain information on the effect of interfacial slip on the interfacial stress field, which is induced by the thermal activation processes in the bonding process of dissimilar materials, for performing interfacial fracture analysis. In this paper, the effect of the interfacial slip occurring in the bonding processes of dissimilar materials on the stress singularity near the free edge was studied by thermoelastic finite-element method (FEM), and the characteristics of interfacial mechanics were simulated by modified bond-element method, of which fundamental types were often used for contact problems. The results are as follows. When the slip occurred near the bonded free edge, another singular stress field was produced at the slip tip, although the interfacial slip relieved the singular stress at the free edge, and the order of the singular term was found to decrease as the slip region extended. For this reason, if the interfacial slip did not accompany interfacial peel, the interfacial slip occurring in bonding processes would be effective in preventing interfacial fracture in cooling periods of bonding processes. In addition, since the stress field was found to resemble a crack problem, the singular stress field was estimated by stress intensity factor K .

Key Words: Bond Element, Interfacial Slip, Stress Singularity, Thermal Stress, Dissimilar Material Wedge, FEM

1. Introduction

Many kinds of interfacial peels occur due to residual stresses induced in bonding processes in bonded joints of dissimilar materials. The interfacial mechanics have been discussed up to now by analyzing the stress singularity at the free edge. It would be one of the most interesting problems to fabricate actual joints to study the effects of interfacial slip⁽¹⁾ occurring in the high-temperature bonding process on the singular stress field.

In this paper, analyses of interfacial fracture, occurring in the bonded joints of two different fine ceramics during the bonding process and accompanied by interfacial slip in high-temperature conditions, were performed by numerical analysis (finite element method, FEM). As the analytical procedure, the stress singularity at the bonded free edge was estimated by regarding the interfacial slip as a sort of contact problem, and the procedure classified as one of the

following two representative types. (i) Friction method: Applying the equilibrium relations between displacements and nodal forces of corresponding nodes at bonded interfaces to the FEM stiffness matrix^{(2),(3)}, (ii) Bond-element method: Inserting various types of bond-elements into the contact planes^{(4),(5)}. In this study, FEM with bond elements was adopted to perform the fracture analysis and to simulate the interfacial slip under high-temperature process conditions.

2. Calculation Method

2.1 Isoparametric elements

In this study, thermal elastic analyses were performed by FEM of which normal elements and bond-elements were isoparametric ones with 8 nodes each. The displacements in normal isoparametric elements and their node displacements are shown in the following equations.

$$\{S\} = \begin{Bmatrix} u \\ v \end{Bmatrix}, \quad \{d\} = \begin{Bmatrix} u_1 \\ v_1 \\ \vdots \\ u_8 \\ v_8 \end{Bmatrix} \quad (1)$$

* Received 18th October, 1993. Paper No. 92-0529

** Faculty of Engineering, Osaka University, 2-1 Yamada Oka, Suita City, Osaka 565, Japan

The mapped locations in the elements are shown by shape function N .

$$x = \sum_{i=1}^8 N_i x_i, \quad y = \sum_{i=1}^8 N_i y_i \quad (2)$$

The displacements in the elements are also given by N .

$$u = \sum_{i=1}^8 N_i u_i, \quad v = \sum_{i=1}^8 N_i v_i \quad (3)$$

The relationship between $\{S\}$ and $\{d\}$ is shown by the following equation.

$$\{S\} = [N]\{d\} \quad (4)$$

$[N]$: Matrix of the shape function

$$[N] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & \cdots & N_8 & 0 \\ 0 & N_1 & 0 & N_2 & \cdots & 0 & N_8 \end{bmatrix} \quad (5)$$

Strain in the elements, $\{\epsilon\}$, is given by the following equations.

$$\{\epsilon\} = [A][N]\{d\} = [B]\{d\} \quad (6)$$

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (7)$$

$$[A] = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial x & \partial/\partial y \end{bmatrix} \quad (8)$$

In thermal elastic problems, $\{\epsilon\}$ is given as the sum of elastic strain $\{\epsilon_e\}$ and thermal strain $\{\epsilon_t\}$.

$$\{\epsilon\} = \{\epsilon_e\} + \{\epsilon_t\} \quad (9)$$

Stressed are related to elastic strains by stress-strain matrix $[D]$ as follows.

$$\{\sigma\} = [D]\{\epsilon\} = [D](\{\epsilon\} - \{\epsilon_t\}) \quad (10)$$

$[D]$ and thermal strain under plane stress condition are shown by the following equations.

$$[D] = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (11)$$

$$\{\epsilon_t\} = \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix} \quad (12)$$

E : Young's modulus

ν : Poisson's ratio

α : Thermal expansion coefficient

ΔT : Increment of temperature

The relationships between load and nodal displacements in elements are individually calculated based on principle of virtual work.

$$\begin{aligned} \{f_p\} &= \int [B]^T [D] \{\epsilon\} dV - \int [B]^T [D] \{\epsilon_t\} dV \\ &= \left\{ \int [B]^T [D] [B] dV \right\} \{d\} - \int [B]^T [D] \{\epsilon_t\} dV \\ &= [k] \{d\} - \{f_t\} \end{aligned} \quad (13)$$

The element stiffness matrix is shown by the following equation.

$$\{f_p\} + \{f_t\} = [k] \{d\} \quad (14)$$

Element stiffness matrix :

$$[k] = \int [B]^T [D] [B] dV \quad (15)$$

Nodal force of thermal strain :

$$\{f_t\} = \int [B]^T [D] \{\epsilon_t\} dV \quad (16)$$

From these equations, the overall stiffness equation is shown as below.

$$\Sigma \{ \{f_p\} + \{f_t\} \} = [K] \Sigma \{d\} \quad (17)$$

$[K]$: Overall stiffness matrix

$[K] = \Sigma [k]$

2.2 Bond-element

The transfer of forces between two dissimilar materials were calculated by the following two analytical methods.

(i) Friction method: Applying the equilibrium relations between displacements and nodal forces of corresponding nodes at bonded interfaces to the FEM stiffness matrix^{(2),(3)}

(ii) Bond-element method: Inserting various types of bond-elements into the contact planes^{(4),(5)}

In the latter, the construction of stiffness matrix in a bond-element is the same as for one of a normal isoparametric element, because the conditions of contact between the dissimilar materials depend on the change of the stiffness of a bond-element. On the other hand, in the former, conditions of displacements, forces and their continuities between the dissimilar materials are imported to the stiffness equation. In this process, since the overall stiffness matrix does not become a square one, Gauss' elimination method must be used to solve the simultaneous equations instead of the conjugate gradient method.

In this study, a thermal elastic computer program with the bond-element method was adopted for considering the CPU time, capacity of memory and for convenience for practical use. In addition, the bond element was optimized to represent the features of interface mechanics.

The two-dimensional bond-element used in this work is shown in Fig.1. This element is an isoparametric one with 8 nodes. The horizontal and vertical directions of the contact plane are expressed as H and V respectively. The parameters are

p : contact pressure

f : friction force

μ : coefficient of friction

The bond-element was assumed to be a rectangular nonisotropic material of which the principal axis was

Fig. 1 Illustration of bond-element

in the H - V direction. The rotational angle θ between X - Y coordinate and H - V axes is given by the following equations.

$$\cos \theta = \frac{\partial x / \partial \xi}{\sqrt{(\partial x / \partial \xi)^2 + (\partial y / \partial \xi)^2}} \quad (18)$$

$$\sin \theta = \frac{\partial y / \partial \xi}{\sqrt{(\partial x / \partial \xi)^2 + (\partial y / \partial \xi)^2}} \quad (19)$$

From these equations,

$$\begin{aligned} & \begin{Bmatrix} \sigma_H \\ \sigma_V \\ \tau_{HV} \end{Bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \\ & \times \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad (20) \end{aligned}$$

$[T]$: Stress matrix of a coordinate transformation

The material constants in the bond-element, E_H , E_V , G_{HV} , ν_H and ν_V corresponded to the directions of H and V and matrix of elastic coefficients in the bond-element. $[D]$, shown below, was similar to that of a normal rectangular nonisotropic material.

$$[D] = \frac{1}{(1 - \nu_H \nu_V)} \begin{bmatrix} E_H & \nu_V E_H & 0 \\ \nu_H E_V & E_V & 0 \\ 0 & 0 & G_{HV}(1 - \nu_H \nu_V) \end{bmatrix} \quad (21)$$

The isotropic elastic constants of materials A and B were E_A , G_A , ν_A , E_B , G_B and ν_B ($G = E/2(1 + \nu)$). The stiffnesses⁽⁴⁾ of bond elements, which corresponded to the conditions of interfacial contact were obtained by Coulomb's friction law which states that the friction forces are proportional to the components of vertical direction of contact forces. Since stress components σ_y and τ_{xy} are in continuum through the bonded interface of dissimilar materials but σ_x cannot be defined, the bond-element should transmit only σ_y and τ_{xy} components. Though the programming would become easier by adopting a rectangular nonisotropic material as a bond-element, the stress component σ_x was calculated. This shows that the normal rectangular nonisotropic material cannot represent the feature of the bonded interface. For this reason, the material constants in bond-elements were established to be the same as those of one-sided material and when a slip or peel occurred on the interface plane, material constants corresponding to them were varied so as to indicate the phenomena. By this method, the effects of thickness of bond element were decreased.

(i) $|f| < \mu p$ (adhesion)

The displacements of materials A and B through the interface maintain continuum conditions in both H and V directions. In this case, the material constants of bond elements are regarded as the same as

Fig. 2 Illustration of nodal force under slip condition

those of material A (or B).

$$E_V = E_H = E_A, \quad G_{HV} = G_A, \quad \nu_{HV} = \nu_A$$

$$[D] = \frac{1}{(1 - \nu_A^2)} \begin{bmatrix} E_A & \nu_A E_A & 0 \\ \nu_A E_A & E_A & 0 \\ 0 & 0 & G_A \end{bmatrix} \quad (22)$$

(ii) $|f| = \mu p$ (slip)

The displacement of V direction maintains continuity but that of H direction is in discontinuum condition due to interfacial slip.

$$E_V = E_A, \quad E_H = G_{HV} \neq 0, \quad \nu_{HV} = 0$$

$$[D] = \frac{1}{(1 - \nu_A^2)} \begin{bmatrix} m & 0 & 0 \\ 0 & E_A & 0 \\ 0 & 0 & m \end{bmatrix} \quad (m \neq 0) \quad (23)$$

In slip condition, nodal forces corresponding to friction forces acting on the interface were applied to bond-elements as shown in Fig. 2. The methods of calculation of equivalent nodal forces to friction forces are as follows. Stress components of V directions (σ_V) were regarded as contact pressures acting on interfacial planes. Then, $\mu \sigma_V$ was regarded as friction stress due to friction force. The nodal forces $\{F_l\}$ which cause the friction stresses in bond-elements are obtained as below from Eq. (16).

$$\begin{aligned} \{F_l\} &= [k] \{d\} = q(\gamma_{HV}) \int [B]^T [D] [B] \{d\} dV \\ &= q(\gamma_{HV}) \int [B]^T [T]^{-1} \begin{Bmatrix} 0 \\ 0 \\ \mu \sigma_V \end{Bmatrix} dV \quad (24) \end{aligned}$$

γ_{HV} : Shear strain (H - V coordinate)

$$\begin{cases} q(\gamma_{HV}) = 1 & (\gamma_{HV} > 0) \\ = -1 & (\gamma_{HV} < 0) \end{cases}$$

(iii) $p = 0$ (opening)

There are no contacts between materials A and B. The displacements disappeared through the interface. Consequently,

$$E_V = E_H = G_{HV} \neq 0, \quad \nu_{HV} = 0$$

$$[D] = \frac{1}{(1 - \nu^2)} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \quad (m \neq 0) \quad (25)$$

If the conditions of contacts between materials A and B were previously known and they did not vary, the solutions could be obtained by calculating the stiffness matrix which was integrated with the stiffness matrix of bond elements as well as conventional FEM. However, the contact conditions were often unknown. In this case, at first, the stresses of all elements are

calculated by assuming that bond-elements are in adhesion condition. The conditions of interfacial contacts were judged by the stresses of bond elements which were obtained provisionally in this process, and the stiffnesses corresponding to contact conditions were assigned to bond-elements. After this process, true stresses for all elements were calculated by using the stiffnesses. The contact condition and stiffness of a bond-element were defined by the following equations.

(i) Adhesion: $\sigma_V \leq \sigma_B, |\tau_{HV}| \leq -\mu(\sigma_V - \sigma_B)$

(ii) Slip: $\sigma_V \leq \sigma_B, |\tau_{HV}| > -\mu(\sigma_V - \sigma_B)$

(iii) Opening: $\sigma_V > \sigma_B$

In these equations, σ_B (>0) shows interfacial peel strength of V direction as a compulsory pressure ($-\sigma_B$). In this study, a conventional value (20 MPa) of σ_B was used, which was reported as the adhesion strength of ZnS-C bonded interface. The representative values of σ_V and τ_{HV} , which are used for determining the contact conditions, are mean values of σ_V and τ_{HV} obtained from the 9 integral points in isoparametric element. The flow chart of calculation in this study is shown in Fig. 3.

2.3 Modeling for analysis

Figure 4 shows the illustrates the bonded joint and its mesh model for FEM analyses. The bonded joint consisted of graphite (substitute) and ZnS (coating material). A step was formed near its bonded free edge for a restrictive condition in its plating

process. In this study, the effects of the step on the stress distribution were analyzed in consideration of a slip deformation occurring near the free edge, and plane stress condition was assumed for simplification of the problem. The thickness of the bond elements was set to $10\text{ }\mu\text{m}$ and the material constants were selected to coincide with these of ZnS before setting the contact conditions. In addition, as thermal condition, the distribution of temperature was assumed to be uniform in the specimen because it was sufficiently thin to allow this assumption. Furthermore, as the thermal cycle in the bonding process, uniform cooling was assumed from 873 K (initial temperature) to 293 K (final temperature). Material constants in these FEM analyses are shown in Table 1. Since both materials were ceramics, temperature dependences were neglected.

(a) Dimensions of test model

(b) Mesh division of step model

(c) Mesh division of nonstep model

Fig. 3 Flow chart of thermoelastic analysis of contact problem

Fig. 4 Test model and mesh division

Table 1 Material constants

	E (GPa)	ν	$\alpha (\times 10^{-6}/K)$
ZnS	74.5	0.3	7.3
Graphite	14.7	0.236	6.5

Fig. 5 Effect of friction on stress singularities in bond elements

3. Results and Discussion

3.1 Effects of shape of edge and interfacial slip on stress singularity near free edge

The results of thermal elastic analyses in the case of accompanying slip deformation are shown in Fig. 5. In these analyses, the thickness of graphite was set to be 500 μm . Although τ_{xy} distribution was negative based on the definition under the condition of adhesion ($\mu = \infty$), the distribution of τ_{xy} was regarded to have a stress singularity. On the other hand, the distribution of σ_y was not a factor for peel of the interface, because it showed a non singular stress.

In contrast, in the case of accompanying interfacial slip ($\mu = 0.2$), a slight tensile stress acted at the tip of the slip. In general, stress component σ_y was induced on the interface in bonded dissimilar materials, though σ_y did not exist on the x axis in a similar material (homogeneous body) in mode II. However, σ_y was small enough compared with τ_{xy} so that interfacial peel did not occur under these contact condi-

(a) Nonslip model

(b) Slip model

Fig. 6 Effect of friction coefficient on stress distribution in bond element

tions. In the profile of σ_y distribution, compressive stresses act on the interfacial slip planes. In addition, it was clarified that the level of stresses decreased as the region of slip extended. Moreover, the stagnation of slip was estimated, because the driving force for producing additional slip would be diminished by this stress relief. In Fig. 5(b), the stress distribution was calculated in the case of nonstep model, of which $|\tau_{xy}|$ in adhesive condition was found to decrease slightly. In this case, the singular stress field the near bonded free edge was generally described by the following equation^{(7),(8)}.

$$S(r) = Kr^{-\lambda} \quad (26)$$

r : Distance from the singular point

$S(r)$: Stress

K : Stress intensity of singular stress field

λ : Order of singularity

The stress distributions near the tip of slip on the interfaces are shown in Fig. 6(a). In the case of the nonslip model, greater interfacial peel tend to occur in the step model than the nonstep one so that the stress

Fig. 7 Effect of thickness of substrate on stress singularity (Step model)

level and the order of singular term λ in the step models was found to be larger than those in the nonstep models. In the case of slip occurring, the stress distributions σ_y and τ_{xy} ($\mu=0.2$) ahead of the slip elements are shown in Fig. 6(b). The value of λ equals approximately 0.5 in both the cases of step and nonstep model as well as in the case of crack⁽⁸⁾. For this reason, the stress field accompanying slip deformation could be estimated by a stress intensity factor, K . The singular stress field moved to the tip of the slip and τ_{xy} distribution became unaffected by the shape of the bonded free edge. In addition, the gradient of distribution in the logarithmic axis approached 0.5, which was a similar value to that in the case of a crack problem. However, in the strict sense, the λ value slightly varied with the shapes of models. This was attributed to the fact that the stress field near the slip tip depended on the compressive stress on the slip plane which was affected by the shape of the free edge. Although it was reported that the stress singularity of the free edge disappeared when a slip occur-

red on the interface near the free edges^{(9),(10)}, it was found that a slight stress singularity near the tip of the slip occurred in the case of a threshold shearing stress existing in the interfacial slip.

3.2 Effect of the thickness of ZnS

The variations of stress distribution and initiation of interfacial slip with the ZnS thickness t_f were studied. In this study, the thicknesses of ZnS were set at 300 μm and 500 μm . The thickness of the graphite was set at 500 μm . The stresses of the bond elements obtained in the cases of $\mu=\infty$ and 0.2 (step model) were shown in Fig. 7. Although, no differences in τ_{xy} distribution under adhesive and slip conditions were found with varying ZnS thickness, the σ_y in slip condition decreased with the thickness of the substrate, which would be caused by the bending deformation of the bonded joint. But, in practical use, σ_y would be sufficiently low compared with τ_{xy} and could be neglected.

Consequently, the bonded residual stress near the bonded free edge was relieved by the interfacial slip and its order of singular term λ tended to decrease. The interfacial slip, with no accompanying peel, would be expected to be a factor in preventing the interfacial fracture.

4. Conclusion

An application of a bond-element method in thermoelastic FEM to the thermal stress problem accompanying an interfacial slip in the bonding process of dissimilar materials was studied. In this calculation, the conditions of bonding were set by establishing the critical value of slip initiation. The results are as follows.

- (1) The singular stress field near the bonded free edge of dissimilar materials, induced in the bonding process, was satisfactorily estimated by thermoelastic FEM with bond elements which were used in contact and friction problems.
- (2) The singularity of thermal stress at the free edge disappeared due to an interfacial slip in the case of the slip occurring easily. However, another singular stress was found to be induced near the slip tip.
- (3) Since the order of singular term near the interfacial slip tip was approximately equal to 0.5, similarly to the case in a crack problem, the stress field was estimated by stress intensity factor K .
- (4) The stress singularity near the interfacial slip decreased as the slip region extended until the slip was finally arrested.
- (5) The singular stress in the step model was found to be higher than that in the nonstep model.
- (6) The stresses near the free edge were relieved by interfacial slip at the free edge and the singular

stress field near the tip of the interfacial slip tended to become lower than for the nonslip free edge case. This seemed to suggest that interfacial slip occurring in the bonding process prevented peel of the bonded interface.

Acknowledgement

The authors would like to thank Japan Steel Works, Ltd. for providing samples and S. Hamabe (graduate student of Osaka Univ.) for his help.

References

- (1) M. Murata, Evaluation of Fracture in Bonded Dissimilar Materials, The Society of Materials Science, Japan, The 79th Seminar, (1991), p. 1.
- (2) T. Ote, Trans. Jpn. Soc. Mech. Eng., Vol. 38, No. 313(1972), p. 2210.
- (3) N. Tsuta and S. Yamaji, J. Jpn. Soc. Mech. Eng., Vol. 76, No. 651(1973), p. 348.
- (4) H. Marui, Japanese Society of Steel Construction, The 5th National Symposium on The Finite Element Method, (1977), p. 253.
- (5) Sharma, H. D., Nayak, G. C. and Maheshwari, J. B., ASCE, (1976), p. 662.
- (6) M. Shimonoseki and H. Fujimura, Programming of Finite Element Method/Non Stationary Thermal Stress, (1988), the Nikkan Kogyo Shinbun, Ltd.
- (7) D. B. Bogy, J. Appl. Mech., (1971), p. 376.
- (8) T. Hattori and S. Sakata, Trans. Jpn. Soc. Mech. Eng., Vol. 54, No. 499(1988), p. 597.
- (9) M. Murata and Y. Mukai, Trans. Jpn. Soc. Mech. Eng., The 4th Computational Mechanics Conference, Vol. 910, No. 79(1991), p. 235.
- (10) M. Murata, Y. Mukai and H. Hiramatsu, Japan Welding Society, The Symposium of Welding Construction'91, (1991), p. 62.