Dynamic Stress Analysis of Fracture Callus*

Tsuneo HIRAI**, Tsutao KATAYAMA***
and Hidetake YAMAMOTO****

A fracture callus changes in structural characteristics in response to changes in the dynamic loading condition during the fracture healing process. The purpose of this research is to understand the behavior of the intelligent material system during the process. In this paper, the stress wave propagation behavior of the fracture callus is investigated using finite-element analysis to estimate the structural change. It was concluded from the numerical result that the structural changes of the fracture callus relate closely to the differences in mechanical impedances between the constitutive materials in the callus. The result was experimentally verified by means of the fracture model using the femur of a rat.

Key Words: Biomechanics, Biomaterials, Composite Material, Intelligent Material, Stress Wave, Mechanical Impedance

1. Introduction

Living bone is an intelligent material which remodels itself to adapt to the mechanical surroundings. The fracture callus which is formed during the fracture healing process changes in structure with the healing process. The growth and remodeling of the bone relate closely to the stress conditions imposed by the mechanical surroundings, and there are reports that loading by suitable exercises accelerates bone formation during fracture healing⁽¹⁾⁻⁽³⁾. The diagnoses of recovery from the fracture are carried out using X-ray images and histological criteria, but prediction of

the mechanical strength of the fracture callus is essential in determining the healing level and when to remove the fixation instruments such as the plaster cast internal fixation instruments or when to start the physical therapy such as the rehabilitation. However, understanding the mechanical behaviors of the fracture callus⁽⁴⁾ is difficult because the internal structure changes with time heterogeneously and the callus is a composite structure which consists of the fracture callus trabeculae and the bone marrow⁽⁵⁾. Moreover, the loadings that are applied to living bodies are always dynamic, so that in clarifying the bone structural changes during fracture healing, it is reasonable to think that dynamic stress conditions of the fracture callus should be taken into consideration. Currently, by various experimental methods of applying dynamic loads to living bone, the effects of dynamic loading on bone formation are investigated⁽⁶⁾.

The purpose of this study is to clarify the behavior of the fracture callus as an intelligent material system by applying the numerical modeling technique to the response of the callus formation to dynamic loading during the healing process. In this study, the stress wave propagation behavior of the fracture

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** Intech Hirai Ltd., Tanabe-cho, Tsuzuki-gun, Kyoto-fu 610-03, Japan

*** Department of Mechanical Engineering, Doshisha University, Tanabe-cho, Tsuzuki-gun, Kyoto-fu 610-03, Japan

**** Faculty of Mechanical Engineering, Kobe City College of Technology, Gakuenhigashi-cho, Nishi-ku, Kobe 651-21, Japan

callus is analyzed by the finite-element technique, and efforts are made to predict the structural change of the fracture callus from the characteristics of stress wave propagation. In order to verify the numerical result, experimental fracture models are made using rat femurs. The volume fractions of the fracture callus are measured from the soft X-ray images, and the structural styles of the fracture callus are verified.

2. Numerical Methods

In this study, the stress wave propagation behaviors of the fracture callus are analyzed by representing the simplified structural changes of the fracture callus in the numerical models sequentially.

2.1 Numerical modeling

In the case of the diaphyseal fracture⁽⁷⁾, the structural changes of the fracture callus are classified into the following four stages from the histological viewpoint as shown in Fig. 1.

(1) Inflammation stage: hematoma is formed over the fracture area.

(2) Intermediate stage: cartilagenous tissue is formed and replaces the hematoma, and then woven bone is formed in the periosteal area.

(3) Formation stage: the bridging callus is formed by the woven bone and endochondral ossification occurs.

(4) Remodeling stage: the cancellous bone formed by the endo-chondral ossification becomes the cortical bone.

It is natural to assume that three-dimensional analyses are suitable for examining changes in the bone structure, but analyses of local changes in the struc-

Fig. 1 Callus structure during fracture healing process

ture require vast amounts of calculation time or capacity. Therefore, in order to decrease analytical complexity due to strict modeling of the callus structure, by assuming that the sequential structural changes correspond to changes in volume fraction of the bone tissue, finite-element models of the fracture callus were devised for the four stages as shown in Fig. 1. To simplify the treatment, the femoral diaphysis is assumed to be axisymmetric. In addition, to deal with the in-plane heterogeneous distribution of the bone tissue in the fracture callus, plane stress elements were applied. Figure 2 shows the four models for the inflammation, intermediate, formation and remodeling stages of the fracture callus. In this study, it was attempted to express the structure of the fracture callus by renewing mesh divisions in response to the geometrical changes and varying the local material properties sequentially. Lines A-C and C-D were fixed in the X and Y directions, respectively, and uniform forced velocity was applied to line A-B in the Y direction to induce stress wave propagations through the bone junction from the distal to the proximal fragment. Figure 3 shows an enlarged view of the area near the fracture line surrounded by the nodal points E, F, G and H of the inflammation stage model. Taking into consideration that the actual fracture line is not uniformly continuous, the minimum material properties within convergency of the numerical solution were applied to the bone junction

Fig. 2 Finite-element models for the fracture callus

Fig. 3 Enlarged view near the fracture line of the finite element model

to satisfy the assumption that the numerical model is a continuum.

2.2 Fundamental equations

When numerical modeling is carried out as described above, the governing equation is formulated on the principle of virtual work⁽⁸⁾ that exists in each constituent element equilibrating at an arbitrary deformation step. Infinitesimal stress increment $\Delta\sigma_{ij}$ and strain rate increment $\Delta\dot{E}_{kl}$ are usually coaxial under high-speed deformation, but the following constitutive equation was assumed in the region of strain speed treated in this study.

$$\Delta\sigma_{ij} = C_{ijkl}\Delta E_{kl} \quad (1)$$

Here, C_{ijkl} represents the modulus of the callus structure. When transforming the principle using Eq. (1) and digitizing it using inertial force acting on the center of gravity of each element as body force increment, elemental equilibrium equation (3)⁽⁹⁾ is obtained, in which the element displacement increment Δu_k is expressed using form function ϕ_{ki} and nodal displacement increment Δr_i .

$$\Delta u_k = \phi_{ki}\Delta r_i^{(e)} \quad (2)$$

$$(K_{\sigma ij}^{(e)} + K_{E ij}^{(e)} + K_{L ij}^{(e)})\Delta r_j^{(e)} + M_{ij}^{(e)}\Delta \dot{r}_j^{(e)} = \Delta f_i^{(e)} \quad (3)$$

Here, $K_{\sigma ij}^{(e)}$, $K_{E ij}^{(e)}$, $K_{L ij}^{(e)}$ and $M_{ij}^{(e)}$ represent the geometrical matrix, linear elastic matrix, large displacement matrix and mass matrix, respectively. Thus the equilibrium equation (Eq. (4)) of the callus structure modeled using finite elements is obtained by summing up Eq. (3) for the entire structure.

$$[M]\{\dot{r}(t + \Delta t)\} + (1 + \alpha)[K]\{r(t + \Delta t)\} - \alpha[K]\{r(t)\} = \{f(t + \Delta t)\} \quad (4)$$

Here, α is a parameter which determines the character of the integral formula, $\alpha = -0.1$ was used in Eq. (4) to obtain unconditional stability.

$$[K] = \sum_{e=1}^m (K_{\sigma ij}^{(e)} + K_{E ij}^{(e)} + K_{L ij}^{(e)}),$$

$$[M] = \sum_{e=1}^m M_{ij}^{(e)},$$

$$\begin{aligned} \{\Delta r\} &= \sum_{e=1}^m \Delta r_j^{(e)}, \\ \{\Delta \dot{r}\} &= \sum_{e=1}^m \Delta \dot{r}_j^{(e)}, \\ \{\Delta f\} &= \sum_{e=1}^m \Delta f_i^{(e)}. \end{aligned} \quad (5)$$

Here, m represents the number of elements. The HHT method (Hilber-Hughes-Taylor method)⁽¹⁰⁾ is applied to numerical integration of Eq. (4). When velocity $\{\dot{r}\}$ and acceleration $\{\ddot{r}\}$ are known at time t , displacement $\{r\}$ and velocity $\{\dot{r}\}$ at time $t + \Delta t$ after infinitesimal time increment Δt are expressed by the following equation.

$$\begin{aligned} \{\Delta r\} &= \{\dot{r}(t)\}\Delta t \\ &+ \left[\left(\frac{1}{2} - \beta \right) \{\dot{r}(t)\} + \beta \{\dot{r}(t - \Delta t)\} \right] \Delta t^2, \\ \{\Delta \dot{r}\} &= [(1 - \gamma)\{\dot{r}(t)\} + \gamma\{\dot{r}(t + \Delta t)\}]\Delta t \quad (6) \end{aligned}$$

Here, β and γ are parameters which determine the character of the integral formula, $\beta = 0.3025$ and $\gamma = 0.6$ were used in Eq. (6) to obtain unconditional stability. The following equation is obtained by transforming Eq. (4) using Eq. (6).

$$\begin{aligned} &\left((1 + \alpha)[K] + \frac{[M]}{\beta\Delta t^2} \{\Delta \dot{r}\} \right) \\ &= \frac{\{\Delta f\}}{\beta\Delta t^2} - \frac{(1 - \alpha)[K]}{\beta} \left(\frac{2\{\dot{r}(t)\}}{\Delta t} + \{\dot{r}(t)\} \right) \quad (7) \end{aligned}$$

When solving Eq. (7) to obtain displacement increment $\{\Delta r\}$, velocity and acceleration increments can be determined by Eq. (6). Infinitesimal time increment Δt was chosen on the condition that eigenvalue λ is $|\lambda| \leq 1$.

3. Results and Discussion

In this study, femoral fracture models are made experimentally, and sequential changes of the numerical models are verified.

3.1 Quantification of structural property

Volume fractions of the fracture callus were quantified to estimate the structural property. Femoral diaphyseal fracture models made using 7-week-old male Wistar rats were removed 2, 3, 5 and 8 weeks after the fracture (5 rats in each group), and then were sliced to 2 mm thickness. Soft X-ray images of the specimens were taken by a soft X-ray device (RF 100 GSB). The transillumination image was recorded by a charge-coupled device camera (NEC VC 1-B 106) and was digitized with a resolution of 256×256 pixels and 64 gray levels in an image memory device (IFM/PC). The volume fraction of each objective region V_f was expressed by the following equation.

$$V_f = \frac{\sum \sum G_{cal}(x, y)}{\sum \sum G_{cor}(x, y)} \quad (8)$$

Here, $\sum \sum G_{cal}(x, y)$ represents summation of the gray levels of the fracture callus pixels and $\sum \sum G_{cor}(x, y)$ represents that of femoral cortical bone pixels.

From the structural viewpoint, 4 objective regions of I (internal), E (external), P (peripheral) and J (junctional) were identified in the callus and their volume fractions were measured (see Fig. 4). Figure 5 shows the relationship between the volume fractions in the callus and time after fracture. Measurements of the volume fraction were carried out 3 times per specimen and the results were treated with two-way anova. From the figure, it was observed that the volume fractions of regions I and E barely increased up to 3 weeks and remained the same. On the other hand, those of regions P and J increased markedly after 3 weeks. From these results, it was considered that the callus structure of the fracture models changed mainly in regions I and E at the first stage and in regions P and J after 3 weeks. Figure 6 shows examples of the soft X-ray images of the callus specimens 2, 3, 5 and 8 weeks after fracture. Specimens with a clear fracture line were selected to be photographed from among the specimens. In the figure, the left-hand side of the specimens is the proximal end and the right is the distal end. Two weeks after fracture, loss of bone tissue could be seen in areas on both sides of the fracture line, which were connected to each other by the external callus which had a small volume fraction. Three weeks after fracture, an incomplete

Fig. 4 Input data of the image-processing system

Fig. 5 Relationship between volume fractions in fracture callus and time after fracture

bridging callus had formed, and this was completely formed 5 weeks after fracture. Eight weeks after fracture, the peripheral callus had increased in thickness and volume fraction, and the internal callus had undergone considerable absorption. Therefore, from the structural viewpoint, it was thought that the specimens taken 2, 3, 5 and 8 weeks after fracture corresponded to the structures at the inflammation, intermediate, formation and remodeling stages, respectively. Figure 7 shows fine distributions of the volume fraction of the above specimens that were measured in order to input the data to the numerical models. Three weeks after fracture, the volume fraction of the bone junction surrounding the fracture line was much

Fig. 6 Soft X-ray images of rat femur fracture model

Fig. 7 Distributions of volume fractions of fracture callus

smaller than that of the cortical bone. Compared with the distribution 3 weeks after fracture, 5 weeks after fracture the volume fraction of the bone junction was larger, and those of the nearby cortical bone and the center of the junction were smaller because of bone absorption. Eight weeks after fracture, bone absorption in the internal part and bone formation in the peripheral part had progressed further.

3.2 Measurement of mechanical characteristics

In order to understand the structural changes of the fracture callus from the mechanical viewpoint, static tensile tests were carried out, and stiffness and strength of the callus were measured. Femoral diaphyseal fracture models were removed 2, 3, 4, 5, 6, 7 and 8 weeks after fracture (10 rats in each group). Both ends of the specimens were covered with poly methyl methacrylate and were gripped by the chucks of the testing machine (Instron model 1122). Tensile tests were carried out at cross head speed of 0.1 mm/min and nominal stress-strain curve of the specimens was obtained by measuring the longitudinal strain in an area 10 mm long including the callus. Figures 8 and 9 show the relationship between stiffness of the callus and time after fracture and that between ultimate strength of the callus and time after fracture, respec-

Fig. 8 Relationship between stiffness of fracture callus and time after fracture

Fig. 9 Relationship between ultimate strength of fracture callus and time after fracture

tively. In this study, the ultimate strength is obtained as maximum stress on the nominal stress-strain curve. The results in both figures were treated with two-way anova. In Fig. 8, it was observed that the stiffness began to increase from 3 weeks, plateaued between 5 and 7 weeks and increased greatly from 7 weeks. The ultimate strength showed a trend of changes over time similar to that of stiffness, as shown in Fig. 9. Thus, from the viewpoint of the stiffness and the strength, it is reasonable to consider that the specimens taken 3, 5 and 8 weeks after fracture represent different structural stages. In addition, taking also into consideration the above result (Fig. 5), the following tentative conclusions were reached. The structural change occurring between 3 and 5 weeks resulted in increases in the volume fractions in the peripheral and junctional areas and in the stiffness and strength of the callus, and that occurring after 7 weeks resulted in a further increase of the volume fraction in the junctional area and a great increase in callus stiffness and strength.

3.3 Comparison between numerical and experimental results

Material properties of the cortical bone are presented in Table 1. Basic properties of the numerical models in this study are derived using these. Tensile and distortional tests of the femurs of 7-week-old male Wistar rats (20 rats for each test) were performed to obtain Young's and shearing moduli. Poisson's ratio was calculated on the assumption of a homogeneous and isotropic body. Mass and stiffness were assumed to be in proportion to the volume fraction, and the material properties were estimated sequentially by the following equations.

$$E = E_c V_f, G = G_c V_f, \nu = \nu_c \quad (9)$$

Here, E , G and ν are the material properties of the callus, E_c , G_c and ν_c are those of the cortical bone and V_f is the volume fraction of each callus element. Here, the material properties were provided for the numerical model of the intermediate, formation and remodeling stages by using the distributions of the volume fractions obtained from the specimens taken at 3, 5 and 8 weeks, respectively. Input velocity applied to this dynamic analysis is presented in Fig. 10. It is assumed to be a deformation speed resulting from activity in daily life. When dynamic loadings are

Table 1 Material properties of rat femur cortical bone

	Young's modulus E_c , MPa	Shearing modulus G_c , MPa	Poisson's ratio ν_c
Cortical bone	10000	3880	0.290

Fig. 10 Input velocity applied to the finite-element analysis

Fig. 11 Dynamic stress conditions of the fracture callus

transferred from the tibia to the femur through the knee joint, it is considered as the effect of the stress wave reflecting at the proximal end of the femur because actual bones have a finite length. However, in this analysis, this effect was disregarded to simplify the treatment.

Figure 11 shows the dynamic stress condition of the numerical model for each fracture healing stage. Each numerical result is the output $15.0 \mu\text{s}$ after applying the input velocity; the figure shows the flow front of the stress wave just passing through the callus formation area. In the case of the inflammation stage model, when the stress wave reached the junction, stress imbalance was observed on both sides of the junction. However, this phenomenon would occur in minimal area because actual bone tissue is heterogeneous. In the case of the intermediate stage model, the stress imbalance was observed in the peripheral

area of the junction. But compared with the above result, the imbalance was smaller. And in the cases of the formation and the remodeling stage models, it was observed that the imbalance was smaller and that the stress wave propagated through the new bone. Thus it was found that the disordered flow front of the inflammation stage was made more even as the model stage progressed. The stress imbalance of the inflammation stage model is due to the difference in mechanical impedance between the cortical bone and the junction area. Therefore, it was concluded that the decrease in imbalance with progression of fracture healing was due to a decrease in the difference in mechanical impedance. From the relationship between the stress wave propagation and the distribution of the volume fraction, the inflammation stage model showed the stress imbalance in the junction area, where the specimen taken 3 weeks after fracture showed a change in distribution of the volume fraction. The intermediate stage model showed the stress imbalance in the peripheral area of the junction, where the specimen taken 5 weeks after fracture also showed a change in distribution. Moreover, as mentioned above, the imbalance decreased in the intermediate stage model more than the inflammation stage one. The above results were understood to represent the intelligent behavior that the cortical bone exhibited. The dynamic stress concentration at the fracture interface results from the difference in mechanical impedance to that in the callus surroundings. Consequently, it was concluded that the mechanical behavior of the cortical bone during fracture healing represented a sequential change in the volume fraction and stiffness in response to the dynamic stress condition.

3.4 Discussion on numerical modeling

The geometrical structure of actual long bones is asymmetric due to the presence of surrounding fibrous tissues such as strong ligaments. Furthermore, diaphyseal fractures are generally spiral in nature. From the experimental results (Figs. 6 and 7), it was also observed that the fracture callus had an asymmetrically distributed structure. Taking these factors into consideration, it seemed to be difficult to assume that the fracture callus is geometrically symmetric from the beginning of formation. Therefore, in this analysis, plane stress field was assumed as a first approximation to modeling of the callus structure. The numerical models were constructed on the assumption that the callus represents the mechanical behavior symmetric to the axis of the long bone in the objective plane. Unfortunately, there are no experimental results yet that directly verify this numerical modeling based on the above assumption. However,

these numerical results allowed clarification of the mechanical behavior of the callus when considering the bone junction as a macro-interface. It was qualitatively shown that this dynamic stress analysis is applicable to understanding the style of sequential structural changes that the fracture callus undergoes during the healing process. Here, by progressive improvement of the numerical modeling and experimental elucidation of the callus material properties, it will be possible to predict the structural changes of the callus quantitatively.

4. Conclusions

In this study, to clarify the sequential structural changes of the fracture callus in response to dynamic loadings, diaphyseal fracture models were made experimentally using rat femur, and the stress wave propagations through the callus formation area were analyzed by the numerical modeling technique. The following conclusions were reached.

(1) Structural changes of fracture callus are closely related to differences in mechanical impedance between the constitutive materials. The changes can be estimated from characteristics of stress wave propagation through the callus structures.

(2) Frame structures of fracture callus can be quantitatively evaluated by using the volume fractions that are measured from soft X-ray images of the callus.

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