Application of Nonlinear Mixed Integer Programming as Optimization Procedure*

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Optimum design analysis in engineering is generally formulated as a problem of nonlinear programming or nonlinear mixed integer programming including selection among alternatives. However, all working methods presently available for linear mixed integer programming are confined essentially to linear problems and lack versatility. To overcome this situation, a new procedure for nonlinear mixed integer programming with constraint functions was developed for solving general optimum design problems. In this paper, an application study on well design is presented as an example to verify the usefulness of the developed optimization procedure.

Key Words: Optimum Design, Nonlinear Problem, Rational Mechanics, Nonlinear Mixed Integer Programming, Optimization Technique, Well Design

1. Introduction

In the field of engineering, the problems of optimal design and optimal control are generally formulated as nonlinear programming problems consisting of continuous variables when physicochemical and economic constraints are taken into account. However in construction design, the problems are often formulated as nonlinear mixed integer programming including integer variables which represent selections among their alternatives. The selection of alternatives in integer programming implies general

difficulties in constructing a powerful solution algorithm; therefore at present, by utilizing, for instance, the peculiar structure of problems, a satisfactory solution of the local optimum is usually obtained with little time and cost in calculation^{(1),(2)}.

In view of this present status, the authors have proposed an approximate method for nonlinear mixed integer programming with inequality constraints^{(3),(4)}. In this paper we report the outline of the algorithm for the approximation of nonlinear mixed integer programming and its practical application to optimal design of oil well casing strings as an example for demonstrating the validity and optimality of the method.

2. Outlines of Nonlinear Mixed Integer Programming and Developed Algorithm for Approximation

The problem of nonlinear mixed integer programming consists of design vectors of continuous variables (X) and integer variables (Y) and minimizes the objective function $F(X, Y)$ under the constraints of $h_i(X, Y) \leq 0, i=1, \dots, m$ and $g_j(X, Y)=0, j=1, \dots, l$, where some of F, h_i and g_j are of nonlinear form.

* Received 11th January, 1995. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 60, No. 573, A (1994), pp. 1308-1314. (Received 20th July, 1993)

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The problem is simply represented by Eq. (1), where the integer variables Y_i are composed of y_{ij} (i =design variables and j =their alternatives) and alternatives are selected or rejected, as defined in Eq. (2).

$$\left. \begin{array}{l} \text{Minimization: } F(X, Y) \\ \text{Constraint: } h_i(X, Y) \leq 0, i=1, \dots, m \\ \quad \quad \quad g_j(X, Y) = 0, j=1, \dots, l \end{array} \right\} \quad (1)$$

$$\left. \begin{array}{l} y_{i1} + y_{i2} + \dots + y_{in} = 1 \quad (i=1, \dots, n_i) \\ y_{ij} = 1 \text{ (selection of alternative } j) \\ y_{ik} = 0 \text{ (rejection: } k \neq j) \end{array} \right\} \quad (2)$$

In the following, the fundamental concept and algorithm of approximate methods of solving the nonlinear mixed integer programming problem and the nonlinear pure integer programming problem to be used in the present report are described.

2.1 Approximate method of solving nonlinear mixed integer programming problem

This method is characterized by a combined procedure for conversion of continuous variables into integer variables and the balance between the search procedure of the global optimal solution and efficiency of computation time. The following is the basic logic of the algorithm developed⁽⁴⁾.

2.1.1 Method of mixing continuous variable and integer variable In general the conventional mixed integer program searches integer solutions after solving the linear programming relaxation problem by approximating integer variables as continuous variables. On the other hand continuous variables can be converted into integer variables by defining the maximum and minimum values and the number of interval divisions for continuous variables. The present approximate method uses the latter mixing method.

2.1.2 Two procedures of optimization by SUMT⁽⁵⁾ for continuous variables and determination of search range This procedure is an algorithm for deriving the maximum and minimum values for continuous variables. The range of search (maximum and minimum) is determined by considering the current solution with the two solutions obtained for each continuous variable after optimization by SUMT in terms of the continuous variables for conditions of both strictest and loosest constraint of the integer variable group. Based on this algorithm the balance between improvement of optimality of iterative solutions and efficiency of computation is considered.

2.1.3 Transformation to pure integer programming problem with conversion of continuous variables into integer variables and application of approximate method of solving nonlinear pure integer programming problem By the above procedures 2.1.1 and 2.1.2, the mixed integer problem is

transformed into a pure integer problem and the approximation of nonlinear pure integer programming is applicable, which will be explained in the next section.

Figure 1 shows a flow chart of the algorithm of the approximation of nonlinear mixed integer programming.

2.2 Approximate method of solving nonlinear pure integer programming problem

The approximation procedure is basically the algorithm of iterative improvement of the objective function value by the interior path method. The algorithm is characterized by the introduction of a virtual coordinate for integer variables to approximate the differentiability of the objective function and to determine the vector of the search direction by the steepest-descent method and thus by processability of integer variables in different dimensions through a common procedure.

In addition, local optimal solutions of linear programming problems are at the extreme points of the set of feasible solutions, while those of nonlinear programming problems are usually near the constraint

Fig. 1 Algorithm of approximation of nonlinear mixed integer programming

equations. This knowledge permits us to assume the existence of a local optimal solution in the vicinity of the boundary (constraint equation) of the solution set and make the correction to the search vector in the vicinity of the boundary. The three logical concepts constituting the algorithm are as below⁽³⁾.

2.2.1 Determination of search direction vector

Application of the steepest-descent method to integer variables includes problems in determining the search direction vector (in changing the integer solutions). The one need is for providing a coordinate to approximate the differentiability and another for formulating a simple polynomial objective function. These problems can be solved approximately by using the following two algorithms.

(1) Providing a virtual coordinate (L value) for an alternative by sequencing in terms of the objective function value

Based on the previously feasible solution Y_0 , a virtual coordinate (L value) is provided by sequencing the currently feasible solutions (alternatives) in the vicinity of $N_R[N_R(Y_i), Y_j = Y_0(j), i \neq j]$ of Y_i for each integer variable i in terms of the objective function value in ascending order, and by assigning the level variables, $L_i = 1, 2, \dots$, to the currently executable solutions.

(2) Derivation of simple polynomial objective function by regression

Since the currently feasible solutions for each integer variable are arranged in order as described in section 2.2.1. (1) and expressed on the L coordinate of the level variable L , a polynomial objective function can be obtained and treated as a set of continuous variables. The highest order is the second-order polynomial.

2.2.2 Correction of search vector in vicinity of boundary (constraint inequality)

Since the changes of solutions for integer variables may sometimes have large effects on constraint equations, i.e., whether they are feasible or not, the same search procedure, without violation of the constraints in the search as continuous variables, may be difficult to apply to integer variables. In such a case, it may be effective to apply an algorithm to correct the search direction, which yields the direction to return to the previously feasible solution if the current solution violates the constraint.

In the search process, if new integer solutions obtained by changing all the level variables simultaneously violate the constraints, the current solutions must return to permissible solutions. In this case the search vectors are corrected by using the derivative values of the objective function. Iterative corrections will be continued by returning to the levels of previous

solutions of the variables according to the ascending order of derivative values of the objective function to obtain feasible solutions with minimal interferences with the optimality of current solutions, and to continue the iterative search by the interior path method.

2.2.3 Determination of move vectors along the boundary from local optimal solution

The local optimal solution is considered to be attained if the current solutions are equal to the previous solutions or the objective function value is not improved. In this case the current solutions are moved along the boundary (constraint equation) to continue searching in a new range for improvement of the current solutions of the local optimum. Since the existence of local optimal solutions is assumed in the vicinity of the boundary, the strictest-constraint equation was selected against the current solutions and a linear approximate plane corresponding to the curved original constraint equation was formulated using the integer variables having influences on the constraint equation. Thus the vector of movement (change in solution) was determined along the boundary of local optimal solutions. The algorithm of the approximation of nonlinear pure

Fig. 2 Algorithm of approximation of nonlinear pure integer programming

integer programming is shown in Fig. 2.

3. Outline and Formulation of Optimal Design for Oil Well Casing Strings

Oil wells usually consist of strings of oil well casings with different diameters, as shown in Fig. 3, however, the design procedures are generally commonly applicable. A single string is composed of sections ($L_1, L_2, \dots$) with a casing of the same grade (yield strength) and wall thickness; therefore the problem is formulated as nonlinear mixed integer programming involving the selection of casings as integer variables and the determination of section lengths as continuous variables with constraint inequalities.

With reference to the distribution of acting loads in the typical well shown in Fig. 4, the principal constraint conditions in design are the external pressure on the well bottom and the axial stress (stress in the axial direction, i.e., tension), caused by the self-weight of oil well casings below, on the well head. The conventional design procedure is to stack individual optimizations for each section with repetition from the lower sections where high external pressure is in effect. Consequently, optimization is not satisfactory on the whole, because the selection of casings in the upper sections is restricted by the weights of casings in the lower parts already determined.

The basic formulation of oil well string design is as follows. API formulas are usually applied to the calculation of the collapse and burst strengths of oil well casings; the weights of casings in the lower sections are variables in the strength formulas and constrain the selection conditions of casings in the upper parts.

- (1) Design specifications [example in numerical representation]

Depth of well : 5 182 m (17 000 ft)

Fig. 3 Oil well casing strings

Mud density : 1.80 gf/cm³ (15 ppg)

Designed external pressure : 931.9 kgf/cm²
(13 256 psi) at the well bottom

Designed internal pressure : 621.0 kgf/cm² (8 845 psi) throughout the whole well

Designed safety factors :

Collapse strength=1.125

Burst strength=1.0

Joint strength=1.60

Number of sections : $N=3$

- (2) Variables in designing

Continuous variables : length of each section L_i
($i=1, \dots, N$)

Integer variables : selection of oil well casing
(grade & wall thickness) for each section

- (3) Objective function F : weight of oil well casing

- (4) Constraint h : mechanical strength of casing > external load in design ($i=1, \dots, N$ upper and lower of section)

(collapse strength) $_i$ > (external pressure in design) $_i \times$ (safety factor for collapse)

(burst strength) $_i$ > (internal pressure in design) $_i \times$ (safety factor for burst)

(joint strength) $_i$ > (axial stress in design) $_i \times$ (safety factor for joint)

- (5) Collapse strength formula of API⁽⁶⁾

$$P_{ca} = P_{co} [\{ 1 - 0.75 (\sigma_z / \sigma_Y)^2 \}^{1/2} - 0.5 (\sigma_z / \sigma_Y)]$$

P_{ca} : collapse strength under axial load

P_{co} : collapse strength in the specifications (guaranteed collapse strength)

σ_Y : minimum yielding strength in the specifications (yielding load of a casing)

σ_z : axial stress (stress caused by weight of lower oil well casings to be supported)

4. Examples of Optimal Design of Oil Well Casing Strings

The algorithm for the approximation of nonlinear mixed integer programming is elaborated in the following, together with examples as verification of the

Fig. 4 Distribution of loads acting on casing string along well depth (buoyancy is neglected)

optimality; the examples are based on the computer program for oil well design developed by a major oil company in the USA⁽⁷⁾ and the CIS oil well design manual (the version before revision in 1989)⁽⁸⁾.

4.1 Examples of design by the conventional procedure

Table 1 shows examples of design by the conventional procedure. The objective function weights of casings for the oil well are 604 000 lb for design example 1 and 181 091 kg for example 2.

4.2 Converged solutions of design example 1 obtained by the present approximate method

For evaluation of the optimality and convergence behavior of the approximate method, effects of optimization are explained with reference to the case of the heaviest oil well casings for the initial values for integer variables (oil well casing). Table 2 shows the performances of seventeen types of oil well casings which are adopted as the objectives in the conventional design. The table includes the collapse strength and joint strength which are the principal constraints in the design safety factors.

Figure 5 shows the converged solutions (design plan) obtained by the nonlinear mixed integer programming proposed and Table 3 shows the safety factors for collapse strength, burst strength and joint strength. The following is the evaluation of the optimization of converged solutions.

4.2.1 Optimality (Fig. 5) The weight of the oil well casing is optimized (-13.4%) from the initial value of 646 000 lb at the extreme to the converged value of 559 693 lb, which implies the improved optimality of 7.3% in comparison with the value of 604 000 lb obtained by the conventional procedure. It

Table 1 Well design by conventional procedures

Section No.	Solutions of Example 1			Solutions of Example 2		
	Weight lb/ft	Grade	Section length	Weight kg/m	Grade	Section length
1	38	P-110	3000 ft	35.6	M	300 m
2	35	P-110	5000	32.8	JT	1360
3	35	N-80	3800	29.8	JT	780
4	35	P-110	5200	29.8	E	822
5				32.8	E	352
6				35.6	E	388
7				35.6	JT	627
8				35.6	M	648
9				32.8	P	223
Weight	604,000 lb			181,091 kg		

Table 2 Collapse strength and joint strength of casing (7 inch)

Weight lb/ft	Wall thick. inch	Collapse strength, psi			Joint strength, kip		
		N-80	C-95	P-110	N-80	C-95	P-110
23.0	0.317	3,830	4,150	---	588	636	---
26.0	0.362	5,410	5,870	6,210	667	722	853
29.0	0.408	7,020	7,820	8,510	746	806	955
32.0	0.453	8,600	9,730	10,760	823	891	1,053
35.0	0.498	10,180	11,640	13,010	876	920	1,096
38.0	0.540	11,390	13,420	15,110	876	920	1,096

is concluded that the converged solution obtained by the present approximate method represents the optimal solution in target, as will be explained in section 4.2.3.

4.2.2 Evaluation of selecting oil well casings (Table 3) The oil well casing at the first section in the lowest part of the well is usable if the collapse strength is ensured at the bottom. The oil well casing, having a collapse strength above the external pressure ($13\,256 \times 1.125 = 14\,913$ psi), taking into account the safety factor in design, is found to be P-110 $\times$ 38 lb/ft from Table 2. For the second and third levels, the optimal solution is said to be obtained if any one of the safety factors of collapse strength, burst strength and joint strength approaches the safety factor in design. The converged value can be satisfactory since the safety factor in design, 1.125, is attained for the safety factor of the collapse strength of thinner casings under the second and third sections.

4.2.3 Optimal solution in target (Fig. 5) In this case, since there are 17 types of objective casings and the number of combinations of alternatives for three sections is 4 913 ($17^3 = 4\,913$) and that of section length is 518 (unit of length = 10 m: $5\,182/10 = 518$), combinatorial computation is executable ($N \sim 2.5 \times 10^6$) and the optimal solution in the target is calculated to be 559 644 lb which is considered to be a global optimal solution, as shown in Fig. 5 and Table 4. The converged solution obtained by the present method is 559 693 lb, which is $+0.009\%$ relative to the optimal solution in the target. In addition, the converged solution is said to represent the optimal solution in the

Table 3 Safety factor of converged solution

Section Total	No.	Wall thick. inch	Collapse		Burst		Joint strength	
			Upper	Lower	Upper	Lower	Upper	Lower
3	1	0.540	1.313	1.140	1.316	1.316	9.682	10.000
	2	0.498	1.830	1.125	1.316	1.316	3.169	9.682
	3	0.408	10.000	1.125	1.268	1.268	1.706	2.761

Remarks : Safety factor of 10 indicates a value equal to or greater than 10.

Fig. 5 Comparison of values of objective function and design variables between converged solution and optimal solution in target (example 1)

Table 4 Best 5 solutions obtained by combinatorial calculation of casing and section length

Case	First section			Second section			Third section			Total weight lb
	Length ft	Grade	Weight lb/ft	Length ft	Grade	Weight lb/ft	Length ft	Grade	Weight lb/ft	
1	2976	P-110	38.0	6644	P-110	35.0	7381	P-110	29.0	559,644
2	6474	P-110	38.0	3157	P-110	32.0	7369	P-110	29.0	560,739
3	5348	P-110	38.0	4355	C-95	35.0	7297	P-110	29.0	567,265
4	2976	P-110	38.0	3380	P-110	35.0	10645	P-110	32.0	571,993
5	8501	P-110	38.0	1270	C-95	32.0	7229	P-110	29.0	573,318

target because the selection of the integer variable (oil well casing) is accurate and the solution of the continuous variable (section length) is within an acceptable difference of several feet. Table 4 lists the five probable solutions derived from combinatorial computation; the difficulty of oil well string design is suggested by the small number of executable solutions, 45, for the structure having one to three sections, in contrast to the total number of combinations of 4 913.

4.2.4 Effectiveness of algorithm developed (Fig. 6) Figure 6 shows the convergence behavior of the total weight of oil well casings as the objective function value, and Table 5 shows the sequence of solution points in the main iteration steps. In Fig. 6 and Table 5, SUMT-1 represents the results of optimization using the continuous variables (section lengths) only for the use of integer variables (casings) under the strictest constraint condition, and SUMT-2, those under the loosest conditions, and mixed integer programming represents the result of optimization of all the variables by the approximate method. Each column of the total weight shows the total weight of oil well casings and the figure in parenthesis denotes the ratio to the initial value. Iteration step 11 is an extra step to compensate the deviation from discrete values due to changes of the section lengths from continuous values to integers, and its solution gives the converged solution⁽⁴⁾.

For the total weights of casings in Fig. 6 and Table 5, the difference in the values between SUMT-1 and mixed integer programming is about 1 to 3% against the initial value which is slight. However, as is evident from Table 5, the casings in the second section differ, suggesting the effective performance of the algorithm. As seen in Fig. 6, iterative solutions in each iteration step are in the order of (mixed integer programming: ●) < (SUMT-1: ○) < (SUMT-2: △), which coincides with the algorithm.

4.2.5 Evaluation of iterative solutions (Table 5) For the iterative solutions of mixed integer programming shown in Table 5, the solution already approaches the second optimal solution (Table 4) in the second iteration step and continues until the seventh iteration step except the third step where the

Table 5 Iterative and converged solutions

Iteration No.	Section	SUMT-1			SUMT-2			Mixed integer prog.		
		Length ft	Weight lb/ft	Grade	Length ft	Weight lb/ft	Grade	Length ft	Weight lb/ft	Grade
Total	No.									
Initial	1	5667	P-110	38.0	5667	P-110	38.0			
value	2	5667	P-110	38.0	5667	P-110	38.0			
	3	5667	P-110	38.0	5667	P-110	38.0			
Total weight		646,000	(1.000)		646,000	(1.000)				
1	1	10809	P-110	38.0	11213	P-110	38.0	10809	P-110	38.0
	2	440	C-95	38.0	5562	P-110	35.0	724	P-110	29.0
	3	5751	P-110	29.0	224	P-110	35.0	5467	P-110	29.0
Total weight		594,245	(0.920)		628,640	(0.973)		590,282	(0.914)	
2	1	9814	P-110	38.0	5358	P-110	38.0	9357	P-110	38.0
	2	2824	C-95	32.0	6216	P-110	35.0	3281	P-110	32.0
	3	4362	P-110	29.0	5426	P-110	35.0	4362	P-110	29.0
Total weight		589,800	(0.913)		611,074	(0.946)		587,058	(0.909)	
3	1	9114	P-110	38.0	7016	P-110	38.0	8899	P-110	38.0
	2	1435	C-95	32.0	9884	P-110	32.0	1976	C-95	32.0
	3	6451	P-110	29.0	100	P-110	35.0	6125	P-110	29.0
Total weight		579,334	(0.897)		586,396	(0.908)		579,019	(0.896)	
4 Weight		575,175	(0.890)		604,623	(0.936)		571,946	(0.885)	
5 Weight		582,271	(0.901)		617,178	(0.955)		570,750	(0.884)	
6 Weight		581,975	(0.901)		618,511	(0.957)		570,553	(0.883)	
7	1	7981	P-110	38.0	2995	P-110	38.0	4529	P-110	38.0
	2	1898	C-95	38.0	13902	P-110	35.0	5710	P-110	35.0
	3	7121	P-110	29.0	103	P-110	35.0	6761	P-110	29.0
Total weight		581,912	(0.901)		603,985	(0.935)		568,022	(0.879)	
8 Weight		581,789	(0.901)		603,943	(0.935)		565,256	(0.875)	
9 Weight		581,784	(0.901)		606,862	(0.939)		565,129	(0.875)	
10	1	4313	P-110	38.0	3557	P-110	38.0	3596	P-110	38.0
	2	5552	C-95	38.0	101	P-110	35.0	6269	P-110	35.0
	3	7135	P-110	29.0	13342	P-110	35.0	7135	P-110	29.0
Total weight		581,782	(0.901)		605,670	(0.938)		562,975	(0.871)	
11	1							2979	P-110	38.0
	2							6647	P-110	35.0
	3							7374	P-110	29.0
Total weight								559,693	(0.866)

Fig. 6 Convergence of values of objective function for well design problem

solution is the fifth optimal solution. And then, the solution changes to the optimal solution in the seventh iteration step. In this case, the second optimal solution in the target can be used without practical problems as the converged solution, since its difference compared to the value of the objective function is $(560\,739 - 559\,644) / 559\,644 = +0.2\%$.

4.3 Converged solutions of approximate method for design example 2

Optimality in the application to the multivariable

problems was evaluated as an example of designing a structure with nine sections, in accordance with the CIS manual for oil well string design. It is noted that the pressure distribution is used in designing varies at the depth of 3 500 m from the well bottom. The design specifications are as follows.

(1) Design specifications [example in numerical representation]

Depth of well : 5 500 m (18 045 ft)

Designed external pressure : 634.0 kgf/cm² (9 019 psi) at the well bottom

390.0 kgf/cm² (5 548 psi) 3 500 m from the bottom

0 kgf/cm² (0 psi) at the well head

Designed internal pressure : 9.0 kgf/cm² (128 psi) at the well bottom

415.0 kgf/cm² (5 903 psi) 3 500 m from the bottom

605.0 kgf/cm² (8 606 psi) at the well head

Designed safety factors :

Collapse strength=1.20

Burst strength=1.15

Joint strength=1.30

Number of sections : $N=9$

(2) CIS collapse strength formula⁽⁸⁾

$$P_{ca} = P_{co} [1.0 - 0.3 (\sigma_z / \sigma_Y)]$$

For evaluation of the effectiveness of optimization by the present method, design was carried out in the same frame of nine sections as in the design example in the manual. Table 6 shows the performances in the use (P grade of the highest strength adopted in the design plan) of a part of the thirty-nine types of CIS oil well casings as the candidates in designing.

The computation result is as follows. Figure 7 shows a converged solution obtained for the problem of nonlinear mixed integer programming. All the design safety factors are verified to be satisfied. The converged solutions obtained by the present method are actually for the structure of four sections, because the same types of casings in terms of grade and thickness are used in consecutive sections. The converged solution for the weight of oil well casings is 168 034 kg in the present result, indicating an improvement of 7.2% in optimality in comparison with the design example in the manual shown in Table 1.

When the grade and thickness of oil well casings are given for each section, such as wall thickness variation from the bottom, 10 → 9 → 8 → 9 mm, the optimal solution is obtained analytically in the partial set of solutions of integer variables (only oil well casings) ; the optimal solution obtained is 167 992 kg, as shown in Fig. 7. The present method gave a converged solution of 168 034 kg which is +0.025% relative to

Table 6 Collapse strength and joint strength of casing (5.5 inch made by CIS)

No.	Wall thick. mm	Weight kgf/m	Collapse strength kgf/cm ²	Burst strength kgf/cm ²	Pipe yield st tonf	Joint strength tonf
1	6.0	20.5	229	714	120.0	134.0
2	7.0	23.6	347	833	137.0	168.0
3	8.0	26.7	487	952	157.0	198.0
4	9.0	29.8	644	1071	175.0	231.0
5	10.0	32.8	808	1190	192.0	258.0
6	11.0	35.6	973	1309	210.0	287.0

Fig. 7 Comparison of values of objective function and design variables between converged solution and optimal solution in target (example 2)

the optimal solution in the target, indicating its acceptability as an optimal solution. It is possible to obtain an optimal solution in the target accurate enough to be considered as a global optimal solution by the same computational procedure for the combination of objective oil well casings and section lengths, as described in the previous section. However, the number of combinations amounts to as much as $N = (39)^9 (5\,500/10) \sim 10^{17}$ which is difficult to calculate in practice.

5. Conclusion

The approximate method of nonlinear mixed integer programming is applied to the design procedures in the USA and in the CIS. The focus is on the verification of the algorithm for nonlinear mixed integer programming and the optimality of converged solutions in the former example, and on the verification of optimality and applicability in practical design for oil well strings in the latter example. The effectiveness for oil well design is evaluated.

In both design procedures, the present method is proven to give converged solutions which can be qualified as optimal solutions in the target; for instance the selection of integer variables (casings) is accurate and the solution of continuous solutions (section lengths) is within a difference of less than several feet. Consequently, it is concluded that the effectiveness of the proposed algorithm for nonlinear mixed integer programming and the optimality of the

converged solutions by the method are verified in practical application.

References

- (1) Research Committee on Integer Programming, Notes on Integer/Combinatorial Programming, Operations Research, (in Japanese), No. 11 (1978), p. 709, No. 12 (1978), p. 780, No. 1 (1979), p. 35.
- (2) Konno, H. and Suzuki, H., Integer Programming and Combinatorial Optimization, (in Japanese), (1982), p. 15, Nikka-Giren.
- (3) Mimaki, T. and Yanagimoto, S, Basic Studies on Optimum Design System for Large Scale Structure (Part-1 Approximate Solver for Nonlinear 0-1 Integer Problem), The Proceedings of the 28th Japanese Joint Conference for Material Science, (in Japanese), (1984-9), p. 145.
- (4) Mimaki, T. and Yanagimoto, S, Basic Studies on Optimum Design System for Large Scale Structure (Part-2 Approximate Solver for Nonlinear 0-1 Mixed Integer Problem), The Proceedings of the 35th Japanese Joint Conference for the Technology of Plasticity, (in Japanese), (1984-10), p. 351.
- (5) JSSC, Technical Committee, Research Group of Optimization Technique, Application of SUMT to Optimum Structural Design, JSSC, (in Japanese), Vol. 7 No. 66 (1971), p. 47.
- (6) American Petroleum Institute, API Bull. 5 C 3, 2nd Edition, (1974), p. 7.
- (7) Nippon Steel Corp., Nippon Steel's Casing Design Quick Charts, 2nd Edition, (in Japanese), (1985).
- (8) VNITTneft, Instructions for Calculation of Casing Strings for Oil & Gas Wells, (in Russian), (1989).