Scanning Stress Measurement Method by Laser Photoelasticity* (1st Report : Stress Measurement by Synthesized Method)

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Optical birefringence is measured by a high-frequency modulation method using a photoelastic modulator and polarized laser. A modified synthesized method was developed and applied for the measurement of stress distributions of a glass plate. The distributions of the differences of principal stresses and their directions were obtained directly by this method. The stress distributions under tensile load of a pulled rectangular glass plate with a hole at its center were measured. The results of stress distributions agreed with the analytical results. It was confirmed experimentally that the spatial resolution of stress measurement was the same as the diameter of the laser light. The stress at many points could be obtained quickly by the synthesized method and scanning stress distribution measurement was realized.

Key Words : Experimental Stress Analysis, Polarized Laser, Synthesized Method, Scanning Photoelasticity, Photoelastic Modulator

1. Introduction

Recently, the manufacturing technology of precision devices such as VLSIs, sensors and optical instruments is making remarkable progress and is widely used for machinery or electric equipment. These electronic devices are composed of various materials such as ceramics, plastics and metals. The strength and stress evaluation of such fine structures have become of major interest and progress of evaluation technology of stress in complex micro devices is anticipated. We have been developing a high-precision stress measuring method using a transmission polarized laser. With the equipment, the two-dimensional distribution of the principal stress difference and principal stress direction in plane stress can be measured nondestructively and quantitatively⁽¹⁾⁻⁽⁵⁾. This method excels the conventional photoelastic

experiment in sensitivity and in the simultaneous measurement of stress magnitude and direction. Because of the high sensitivity of our method, it is possible to measure the stress on materials with small photoelastic constants as well as weak stress such as residual stress distribution produced during a bonding process.

In this paper, a new synthesized method is developed by improving the conventional synthesized method⁽⁶⁾ of the photoelastic experiment. Furthermore, by measuring the stress distribution of a glass plate having a small round hole at the center, comparison with a theoretical solution was made. As a result, it is confirmed that the stress distribution can be measured by means of the synthesized method.

2. Experimental Apparatus and Photoelastic Constant of Glass

2.1 Relation between transmitted light intensity and stress

We have demonstrated stress measurement by the high-frequency modulation method using a polarized laser and a photoelastic modulator (PEM)⁽¹⁾. Figure 1 shows the construction of the birefringence

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Fig. 1 Schematic of the optical birefringence measuring system

measuring system. Laser light passes through a polarizer, a PEM, a stressed specimen and an analyzer, where upon it is detected by a phototransistor. The PEM is the active optical device which converts linearly polarized light into oscillating elliptic light at its 50 kHz resonant frequency. The stressed specimen is placed on an X - Y - θ stage as shown in Fig. 1. The direction of the polarizer is inclined 45° from the axis of the PEM, and the analyzer is inclined 90° from the PEM. A semiconductor laser of 1.5 mW with central wavelength 670 nm is used as the light source. The laser beam is focused on the test specimen and its shape is an ellipse of $50 \times 170 \mu\text{m}$.

The laser light passes through the stressed specimen and optical devices and reaches the photodetector. The light intensity I at the photodetector is given by the following equation, where the phase difference (retardation) of the PEM and the stressed specimen denoted by γ and δ , respectively.

$$I = \frac{1}{2} \alpha \cdot I_0 \cdot \left(1 + \frac{\sin 4\theta \cdot (\cos \gamma - 1) \cdot \cos \delta}{2} - \sin 2\theta \cdot \sin \delta \cdot \sin \gamma \right) \quad (1)$$

where α is the transparency of the test piece, I_0 is the strength of the incident ray and θ is the angle between the optical birefringence of the specimen and the PEM axis. The angle θ is positive in counterclockwise

from the incident side. According to Brewster's law, the retardation of the specimen γ is proportional to the principal stress difference $(\sigma_1 - \sigma_2)$. The PEM retardation δ is modulated with the resonance frequency ω of approximately 50 kHz. The retardations γ and δ are given by following equations,

$$\gamma = 2\pi C \cdot (\sigma_1 - \sigma_2) \cdot d / \lambda \quad (2)$$

$$\delta = \delta_0 \sin \omega t \quad (3)$$

where C is the Brewster's constant (photoelastic constant), d is the thickness of the test piece (distance of the passage of the ray), and λ is the wavelength of the laser ray. σ_1 and σ_2 are the maximum and minimum principal stress, respectively. On the other hand, δ_0 is proportional to the amplitude of the PEM modulation signal (voltage). A direct component I_{DC} and a modulation frequency component (alternating component) I_{AC} are obtained⁽¹⁾ by Fourier expansion of Eq. (1) for ωt ,

$$I_{DC} = \frac{\alpha \cdot I_0}{2} \quad (4)$$

$$I_{AC}(\theta) = \alpha \cdot I_0 \cdot J_1(\delta_0) \cdot \sin \gamma \cdot \sin 2\theta \quad (5)$$

where $J_1(\delta_0)$ is the Bessel function. From Eq. (5), it is shown that the alternating component $I_{AC}(\theta)$ of the penetrating ray takes the maximum value at $\theta = n\pi + \pi/4$ (n indicates integers). On the other hand, the ratio of the alternating component $I_{AC}(\theta)$ to the direct component I_{DC} is independent of both intensity of incident laser ray and the transparency of the stressed material.

$$\frac{I_{AC}(\theta)}{I_{DC}} = A \cdot \sin \left(\frac{2\pi C (\sigma_1 - \sigma_2) \cdot d}{\lambda} \right) \cdot \sin 2\theta \quad (6)$$

where A is a constant which is also independent of the stress state and transparency of the specimen. The constant A was obtained with a quarter-wave plate as a sample and it is determined that $A = 0.199$ for this experimental equipment.

2.2 Photoelastic coefficient of glass

A glass plate of 1 mm thickness (super frost white; commercially available) was adopted for the stress measurement sample in the experiments, because of its small photoelastic constant and small initial optical anisotropy.

Figure 2 shows the experimental result of $I_{AC}(\theta)/I_{DC}$ obtained by applying the tensile load to a glass plate 46 mm in length, 5.0 mm in width and 1 mm in thickness. The measurement of $I_{AC}(\theta)$ and I_{DC} was carried out while rotating the stressed glass plate. The abscissa indicates the angle θ of the primary principal stress direction. The peak values of $I_{AC}(\theta)/I_{DC}$ (the value at $\theta = 45^\circ$) increase in proportion to the tensile load as shown in Fig. 2. Figure 3 shows the relation between the peak values of $I_{AC}(\theta)/I_{DC}$ and the load per unit width $\sigma \times d$. Since the photoelastic coefficient of the glass is sufficiently small compared

Fig. 2 The change of $I_{AC}(\theta)/I_{DC}$ of the glass plate subjected to uniaxial tensile load during rotation

Fig. 3 The relation between the maximum value of $I_{AC}(\theta)/I_{DC}$ and applied uniaxial stress for the glass plate

with polymers such as epoxy resins, the relation between $I_{AC}(45^\circ)/I_{DC}$ and $\sigma \times d$ is linear as shown in Fig. 3. The term of the stress in Eq. (6) can be expanded with a linear equation. The relation between the stress of the glass plate of 1 mm thickness and $I_{AC}(\theta)/I_{DC}$ can be approximated by,

$$\frac{I_{AC}(\theta)}{I_{DC}} = 9.52 \times 10^{-3} (\sigma_1 - \sigma_2) \cdot \sin 2\theta \quad (7)$$

where the unit of the stress is MPa. The resolution of stress measurement can be estimated from Eq. (7). In the experiment, I_{DC} was adjusted to be 2~3 V. On the other hand, the electrical noise of I_{AC} was less than ± 1 mV. Therefore the resolution of the principal stress difference, $(\sigma_1 - \sigma_2)$ is better than ± 0.05 MPa.

Fig. 4 The new synthesized method. Both principal stress difference $(\sigma_1 - \sigma_2)$ and principal stress direction θ at each point are calculated from two measurement data at $\phi = 0$ and 45°

3. Stress Distribution Measurement by New Synthesized Method

Figure 4 explains the new synthesized method. The (x, y) coordinate system is fixed on the stressed specimen and the ξ direction coincides with the PEM axis. The crossing point of ξ and η is the stress measuring point. The principal stress direction θ is defined in (x, y) coordinates as the angle from x -direction. The stressed specimen is rotated around the incident laser light. When the angle between the x -axis and the ξ -axis is ϕ , the ratio of I_{AC} and I_{DC} is expressed by,

$$Z_\phi = \frac{I_{AC}(\theta + \phi)}{I_{DC}} = B \cdot (\sigma_1 - \sigma_2) \cdot \sin 2(\theta + \phi) \quad (8)$$

where $B = 9.52 \times 10^{-3} \text{ MPa}^{-1}$.

The principal stress difference $(\sigma_1 - \sigma_2)$ and direction θ at each point are simply calculated from two measurements of Z_ϕ at $\phi = 0$ and $\phi = 45^\circ$,

$$\begin{aligned} Z_0 &= B \cdot (\sigma_1 - \sigma_2) \cdot \sin 2\theta \\ Z_1 &= B \cdot (\sigma_1 - \sigma_2) \cdot \cos 2\theta. \end{aligned} \quad (9)$$

The principal stress difference $(\sigma_1 - \sigma_2)$ and the principal stress direction θ can be separated from each other,

$$\theta = \frac{1}{2} \tan^{-1} \frac{Z_0}{Z_1} \quad (10)$$

$$\sigma_1 - \sigma_2 = \frac{\sqrt{Z_0^2 + Z_1^2}}{B \cdot d} \quad (11)$$

where the range of θ is determined by the signs of Z_0 and Z_1 . The sign of Z_ϕ corresponds to that of $I_{AC}(\theta + \phi)$ and the sign of the alternating component is obtained from the difference between phases of PEM

applied voltage and $I_{Ac}(\theta + \phi)$.

We had measured the stress state at a point by rotating the sample more than 180° in our previous method⁽¹⁾⁻⁽⁵⁾. On the other hand, the distributions of Z_0 and Z_1 can be obtained quickly by scanning the laser on the specimen, and thus high speed and high spatial-resolution measurement are realized by this new synthesized method.

Fig. 5 The size and shape of the specimen of glass plate with a center hole. The holes are made by machining. Two holes of 4.0 mm diameter are made for loading

4. Result of Stress Measurement

Using the synthesized method described above, stress measurement was carried out for a glass plate with a round hole in the center to which is applied a tensile load. Figure 5 shows the size and shape of specimen used in the experiment. The origin of coordinates was taken at the center of the hole and the loading direction was defined as the y -direction. The tensile load of 40.2 N was applied with the pins inserted into round holes at both ends of the specimen. The measurement was conducted at the positions of $y = 0, 1.25, 1.75$ and 2.25 mm in the y -direction, at 0.1 mm intervals in the vicinity of the hole in the x -direction, and at 0.2 mm intervals away from the hole. The experimental results are shown in Fig. 6. The length of the lines in Fig. 6 indicates the magnitude of the principal stress difference and the direction of the lines shows the primary principal stress direction. The stress direction on the line $y=0$ does not agree with the theoretical estimation, because the

Fig. 6 The experimental stress distributions of the glass plate with a center hole subjected to tensile load of 40.2 N

Fig. 7 The theoretical stress distributions of the infinite glass plate with a center hole subjected to uniaxial tensile load

(a)

(b)

Fig. 8 Comparison of the distributions of the experimental and theoretical principal stress differences in the x -direction

measuring position of the origin was shifted about 0.1 mm in the y -direction.

The theoretical solution of stress state of an infinite plate having a round hole is given by the following equations,

$$\begin{aligned}\sigma_r &= \frac{\sigma_0}{2} \left(1 - \frac{a^2}{r^2} \right) - \frac{\sigma_0}{2} \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2} \right) \cos 2\theta \\ \sigma_\varphi &= \frac{\sigma_0}{2} \left(1 + \frac{a^2}{r^2} \right) + \frac{\sigma_0}{2} \left(1 + \frac{3a^4}{r^4} \right) \cos 2\theta \\ \tau_{r\varphi} &= -\frac{\sigma_0}{2} \left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2} \right) \sin 2\theta\end{aligned}\quad (12)$$

where σ_r , σ_φ and $\tau_{r\varphi}$ are stress components in (r, φ) cylindrical coordinates, a is the diameter of the round hole and σ_0 is the applied tensile stress at the infinite point. Figure 7 shows the calculated principal stress differences and directions with lines. The complex changes of principal stress differences and directions near the hole are accurately measured and the experimental result agrees with the theoretical stress distribution.

The lines showing the experimental results in Fig. 6 are inclined about 2 deg to the right. This inclination was caused by the incongruity of the PEM axis with the coordinate axis of the specimen. Figure 8 shows the experimental and theoretical principal

(a)

(b)

Fig. 9 Comparison of the distributions of the experimental and theoretical principal stress directions

stress differences in the x direction at $y=0$ and 1.25 mm, and Fig. 9 shows the experimental and theoretical principal stress directions at the same points.

5. Conclusions

We developed a new synthesized stress measuring method for laser photoelasticity using a photo-elastic modulator and a frequency modulation technique. By this new synthesized method, the stress distribution can be obtained with two sets of scanning measurements. The stress distribution of a glass plate with a hole was measured and it was confirmed experimentally that the spatial resolution of stress measurement was the same as the diameter of the laser light. The stress at many points could be obtained quickly by the synthesized method and scanning stress distribution measurement was realized.

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