Computer Simulation to Evaluate Effective Elastic Moduli of Sintered Spherical Particles* (Examination of Simulation Technique by 2D Model)

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A simulation method has been proposed to estimate the effective elastic moduli of a porous elastic body made of sintered spherical particles by analyzing the response of a random network of springs with 6 degrees of freedom to uniform deformations; the spring network is constructed by computer simulation of a three-dimensional random packing of spherical particles, whereas the spring constants of a pair of sintered particles are estimated by the FEM. In order to verify the proposed method, its two-dimensional version has been applied to an estimation of the effective Young modulus, and the specific procedures of the proposed method have been examined in detail.

Key Words: Simulation, Effective Elastic Modulus, Sintered Spherical Particles, Random Packing, Spring Network, Spring Constant, FEM, 2 D Model

1. Introduction

Effective elastic moduli of the porous elastic solids prepared by sintering an aggregate of spherical particles have been estimated^{(1),(2)} using either the Hill-Budiansky-Wu self-consistent composite model⁽³⁾⁻⁽⁵⁾ or the Berryman self-consistent model based on elastic wave scattering⁽⁶⁾.

However, the estimation using these self-consistent models and all or almost all other composite models have some difficulties as follows. (1) In the above estimation^{(1),(2)} based on the self-consistent models, it was assumed that pores (that is, fillers which have no rigidity) have the shape of needles of infinite length, taking into account the fact that pore fluid flows through the sintered spherical-particle aggregates; this shape is completely different from

that of the aggregates. (2) The estimates of the effective elastic moduli by the composite models are largely different from each other, especially for the case of large porosity, since the sintered particle aggregates are such an extreme case of 'composite materials' that one phase of the composites has vanishing rigidity. Furthermore, there is no theoretical ground for judging which is most or more reasonable among the composite models that yield largely different estimates. (3) Plona⁽⁷⁾ showed good agreement between the phase velocities observed for water-saturated sintered glass beads and the theoretical estimates based on Berryman's self-consistent model and Biot's dynamic elasticity theory of fluid-saturated porous materials. In spite of this good agreement, it can be theoretically concluded that the phase velocities of fast compressional and shear waves should approach zero when the porosity approaches that for the nonsintered random packing of rigid spherical particles, since the effective elastic moduli vanish at this limit; this is not the case for Berryman's model. In this regard, it may not be appropriate to employ Berryman's model to estimate the effective elastic moduli of sintered spherical particles.

* Received 23rd January, 1995. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 59, No. 563, A (1993), pp. 1748-1755. (Received 2nd November, 1992)

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In the present paper we propose a simulation method to estimate the effective elastic moduli of a sintered spherical particle aggregate by analyzing the response of a random network of springs with 6 degrees of freedom to uniform deformations; the network is constructed by computer simulation of a three-dimensional random packing of spherical particles, whereas the spring constants of a pair of sintered particles are estimated by the FEM. In order to verify the proposed method, its two-dimensional version has been applied to an estimation of the effective Young modulus, and its specific procedures have been examined in detail.

It should be emphasized that the proposed method can overcome all the difficulties mentioned above.

2. Proposed Simulation Method

The simulation method proposed is that for estimating the effective elastic moduli of a fully three-dimensional sintered spherical particle aggregate. Since the procedures of simulation will be examined in detail using two-dimensional circular discs in two-dimensional space in the following sections, the terms “three-dimensional,” “spherical particle,” and “6 degrees of freedom” in this section should be understood to correspond to “two-dimensional,” “circular disc”, and “3 degrees of freedom” there, respectively.

The flow of simulation is shown in Fig. 1. In Part (A), spring constants are estimated by the FEM for extension (contraction), bending, torsion, and shearing of a pair of neighboring spherical particles taken out of the aggregate. It may not cause any serious

Fig. 1 Flow chart of the simulation to evaluate the effective elastic moduli of a porous elastic body made of sintered spherical particles

errors to select the pair of spherical particles and regard it as an elastic spring, since deformations of the pair concentrate around the connecting neck. The pair has 6 degrees of freedom in the three-dimensional case, since bending and shearing have two degrees of freedom around and in the two perpendicular directions normal to the axis of the pair.

Part (B) concerns the computer construction of a three-dimensional random packing of spherical particles by dropping particles one after another from a point determined by random numbers, to fill a virtual rectangular box; these procedures are executed by solving motion equations numerically. Particles falling one after another are not expected to result in a big difference in packing structure from reality, although countless particles are thrown into the box all at once in the experiment.

Regarding the random packing structure constructed in Part (B) as a random network of springs with the spring constants obtained in Part (A), the static response of the network to macroscopically uniform deformations is analyzed in a similar manner as in the FEM, resulting in the desired effective elastic moduli.

In order to check the validity, as well as to examine the details of the method proposed in this section, we will apply it to the two-dimensional case in what follows.

3. FEM Estimation of Spring Constants of a Pair of Circular Discs with 3 Degrees of Freedom

3.1 FEM analysis and elements

Consider the pair of two-dimensional particles (circular half-discs) shown in Fig. 2. The two discs with radius R contact each other and are connected by added portions which are surrounded by the two discs and an arc with radius r tangent to them. This connection is introduced to describe the connection

Fig. 2 A pair of connected semicircular discs as an element spring of the random spring network

formed due to sintering and the degree of sintering is given by the angle θ between the two center lines (this angle will be called a "contact angle" hereafter). The static response of this pair to contraction, shearing and bending at its ends is analyzed by the FEM; the ratios of (generalized) reaction forces to the forced deformations yield the spring constants. The FEM analysis is employed for the hatched quarter-region in Fig. 2. Figure 3 depicts an example of the division of the region into triangular elements for the case of contact angle $\theta = 30$ degrees. In the figure, the circled number is the element number, and the non-circled number is the node number. The numbers of elements and nodes are 441 and 800, respectively, for all cases.

3.2 Boundary conditions

In the Cartesian coordinate system (x, y) having the origin at point O_1 in Fig. 2, the x and y axes fall in the horizontal and vertical directions. Let u and v denote the displacement components in the respective directions. Stress components are denoted as σ_x , σ_y and τ . The boundary conditions for contraction (extension), shearing and bending are shown in the following; the curved boundaries are free from stress in all cases.

(1) Contraction: Consider the hatched region in Fig. 2 (quarter-disc) sandwiched by two rigid plates; there is no friction between the quarter-disc and the plates. The lower plate is forced to move upward by v_0 . Taking into account the symmetry of the deformation, the boundary conditions are given by

$$\begin{cases} x=0 : u=0, \tau=0, \\ y=0 : v=v_0, \tau=0, \\ y=R : v=0, \tau=0. \end{cases} \quad (1)$$

(2) Shearing: Similarly, the disc is sandwiched by the rigid plates. In this case, however, the two plates are completely bonded to the quarter-disc; the lower plate slides in the horizontal direction by u_0 , the distance between the plates remaining constant. Taking into consideration the symmetric character of the shearing problem, the boundary conditions are

$$\begin{cases} x=0 : v=0, \sigma_x=0, \\ y=0 : u=u_0, v=0, \\ y=R : u=0, \sigma_y=0. \end{cases} \quad (2)$$

(3) Bending: The lower rigid plate is forced to rotate around the fixed point O_1 by angle γ_0 , while the upper plate stays at the original position. No friction acts between the disc and the plates. The boundary conditions and fixed condition are as follows:

$$\begin{cases} x=0 : v=0, \sigma_x=0, \\ y=0 : v=\gamma_0 x, \tau=0, \\ y=R : v=0, \tau=0, \\ x=0, y=0 : u=0. \end{cases} \quad (3)$$

3.3 Estimation of spring constants and its results

The spring constants denoted by K_F , K_S , and K_M for contraction, shearing and bending are defined by

$$K_F = \frac{F_y}{v_0}, \quad K_S = \frac{F_x}{u_0}, \quad K_M = \frac{M}{\gamma_0}, \quad (4)$$

where F_x , F_y , and M are the sums of nodal forces in the x and y directions and their moments around point O_1 along the lower boundary (from node No. 1 to No. 21). These total forces and moment are given by

$$F_y = \sum_{i=1}^{21} F_{yi}, \quad F_x = \sum_{i=1}^{21} F_{xi}, \quad M = \sum_{i=1}^{21} F_{yi} X_i. \quad (5)$$

The results of the spring constants calculated by the FEM are shown in Figs. 4 and 5. The former is for contraction and shearing, while the latter is for

Fig. 3 Finite element idealization of the two-dimensional region shown in Fig. 2

Fig. 4 Spring constants for compression and shear

Fig. 5 Spring constant for bending

bending. In both figures, the abscissa is the contact angle, which reflects the degree of sintering. Particle material is assumed to be Crown glass, whose bulk and shear moduli are 41.2 and 29.2 GPa.

It is natural that all the spring constants are larger for the larger contact angle (the larger degree of sintering). The curves for contraction and shearing are upward convex, whereas the curve for bending is downward convex. This may be understood from the fact that the constants for the former are approximately proportional to the width of the neck of the pair, whereas the constant for the latter may roughly vary with the cubic width of the neck, as inferred from the beam theory where the bending stiffness is the cubic of beam height.

4. Simulation of Random Packing of Circular Discs

To construct a two-dimensional random packing of circular discs in a computer, simulation of the discs of equal diameter falling into a virtual rectangular box filled with linear viscous fluid has been executed by solving motion equations for each disc. In the motion equations, the acceleration term is omitted (that is, quasi-static motion is assumed) in order to avoid bouncing of the discs off the bottom of the rectangular box or other setting discs. The discs fall one after another from points which are determined by pseudorandom numbers generated by the computer. To simplify the algorithm, the bottom of the rectangular box and the setting discs, which never move even if they collide with a new disc, are assumed to have many linear infinitesimal springs on their surfaces; this assumption makes the simulation simple because we need not take into account kinematical constraints for disc motion. Furthermore, friction between the discs and their rotation are neglected to save computing time. In conclusion, each disc rests at the position where gravity force is in equilibrium with

the spring forces from the setting discs or the bottom surface under contact.

Below, more detail will be given on the simulation procedures.

4.1 Quasi-static motion equations

Coordinates (X, Y) of the position occupied by the center of the I th disc at time T can be determined by solving the quasi-static motion equations while taking into account disc collision against the bottom surface or some of the $(I-1)$ resting discs. If M, C , and G denote the mass of the disc, the viscosity coefficient and the gravity acceleration, the motion equations are given by

$$\begin{cases} C \frac{dX}{dT} = F_X(X, Y), \\ C \frac{dY}{dT} = F_Y(X, Y) - MG, \end{cases} \quad (6)$$

where $F_X(X, Y)$ and $F_Y(X, Y)$ denote the forces in the X and Y directions acting on the disc in motion. These forces are given by

$$\begin{cases} F_X(X, Y) = \sum_{j=1}^{I-1} k_1 d(J) \frac{dX(J)}{\sqrt{dX(J)^2 + dY(J)^2}}, \\ F_Y(X, Y) = \frac{1}{N} \left\{ k_2 Y_1 + \sum_{j=1}^{I-1} k_2 d(J) \frac{dY(J)}{\sqrt{dX(J)^2 + dY(J)^2}} \right\}, \end{cases} \quad (7.a) \quad (7.b)$$

where k_1 and k_2 are the spring constants in the X and Y directions for contact between the discs. The latter is also the spring constant against the bottom. The number of contacts of the disc in motion against neighboring discs and the bottom surface is given by N .

The reason why the factor $(1/N)$ is multiplied to the right-hand side of Eq.(7.b) is as follows. If the motion equations (6) are solved analytically, the disc will stop at the equilibrium position of gravity and the forces from the springs. However, since we solve them numerically or discretely, the disc in motion may in many cases go through the equilibrium position because of finite incremental falling distance ΔY ; in this case, the disc is pushed back up by the spring and goes upward through the equilibrium position at the next step, taking a long time until rest due to this type of oscillation. Our trial simulations have shown that such a case seems to occur more frequently when the disc contacts many neighboring discs; therefore we introduced $1/N$ to bring the disc to rest at the equilibrium position more quickly.

In Eq.(7), Y_1 is the penetration of the disc into the floor (bottom surface); $dX(J)$ and $dY(J)$ are the differences in the X and Y coordinates between the centers of the I th disc in motion and the J th disc in contact with it among the $(I-1)$ discs at rest; $d(J)$ is the mutual penetration of discs in contact. If the disc

is not in contact with the other resting discs or the floor, $d(J)$ and Y_1 are taken to be zero. Judgement of whether the disc is in contact or not is based on the sign of the penetrations. For discs with diameter D (radius R), these penetrations are defined by

$$Y_1 = R - Y(I) > 0, \\ d(J) = D - \sqrt{dX(J)^2 + dY(J)^2} > 0, \quad (8)$$

with

$$dX(J) = X(I) - X(J), \quad dY(J) = Y(I) - Y(J). \quad (9)$$

From the above, it can be understood that solution of the nonlinear simultaneous differential equations of first order given by Eqs. (6) yields the coordinates of the I th disc at time T . To solve them, we have employed the Runge-Kutta-Gill method; if the absolute values of disc center displacements for each time step ΔT is smaller than some given small value (i.e., ε_1 for the X direction and ε_2 in the Y direction), the disc is assumed to come to rest. This procedure is repeated from $I=1$ to $I=K$, with K being the total number of discs thrown into the box. Thus, we obtain a random packing of equal-radius circular discs.

To accelerate the computation in the above procedure, the following measures are adopted: (1) The height from which each disc starts to fall is set immediately over the highest position of all the resting discs in order to shorten the time of free fall. (2) Until the falling disc collides with other discs, its mass is set to be larger than its original mass in order to accelerate its free fall.

The spring constants k_1 and k_2 and the small parameters ε_1 and ε_2 are determined by the trial and error method: $k_1=30.8$, $k_2=56.5$, $\varepsilon_1=10^{-5}$, $\varepsilon_2=10^{-3}$ for $C=1$, $M=1$, $G=9.8 \times 10^{-2}$.

4.2 Elimination of the wall effect by cyclic boundaries

When the discs are packed into a box of finite dimensions, the structure of the random packing near the box walls must be different from that far from the walls; the packing may be more sparse and regular immediately near the walls than far from them. This adverse effect may be greater for a smaller box. In order to eliminate this adverse effect, the cyclic boundary method has been introduced; when a disc goes out through, for example, the left wall, another disc comes in through the right wall exactly in a symmetric manner and vice versa, as shown in Fig. 6. Each disc starts to fall from some point within the width B shown in the figure. This width is called "effective width".

4.3 Adjustment of porosity by adhesional probability

According to powder engineering, the structure of

Fig. 6 Cyclic boundary conditions

particle packing is strongly affected by adhesion between the particles. The porosity has been adjusted in the simulation by introducing "adhesion" between the disc under motion and the resting disc upon their collision; that is, a specific probability as to whether a disc adheres to another resting disc upon their first collision is set in the trial of simulation before it starts and, if a pseudorandom number uniform in the interval from 0 to 1 assigned to each disc is smaller than the adhesion probability, the disc is made to stop and rests at the position of its first collision. If the pseudo number is larger than the probability, the disc continues to move under the influence of gravity and spring forces until it reaches its equilibrium position.

4.4 Results of simulation

First, neglecting adhesion (that is, zero adhesion probability), a series of simulations were carried out for various values of B/D ; it has been confirmed that the porosity approaches some constant value for large B/D and never varies for $B/D > 10$. In any case, the porosity ranges from 17% to 19%. The porosities for the regular tetragonal (the sparsest) and hexagonal (the densest) systems are 9.3% and 21.5%, respectively; the present values obtained fall between them and the packings may be said to be rather sparse.

Since the present simulation imitates the free fall of particles, there is a possibility that the results obtained by this method reflect the effect of the gravity; that is, the structure in the horizontal direction is different from that in the gravity direction. Thus, in order to examine the structural dependence on the direction, a line segment is introduced within the disc-packed structure and a so-called "line porosity" is defined as the ratio of the segment length crossing pores to the entire segment length. The examination of the direction dependence of the line porosities shows that there is no marked gravity effect on the structures constructed by the free fall of discs for the case of no adhesion.

Figures 7 to 9 show examples of packing structures for the adhesion probabilities 0%, 10% and 20%, respectively. The respective porosities are 17.78%, 19.94%, and 23.29%. It can be seen from the figures

Fig. 7 Random packing (Adhesion probability=0%)

Fig. 8 Random packing (Adhesion probability=10%)

that more and larger pores appear with larger adhesion probability.

5. Response Analysis of Spring Network with Three Degrees of Freedom

Consider the network of springs whose ends are at the centers of the accumulated discs; each spring has three degrees of freedom for contraction, shearing and bending, and its spring constants have already been obtained in section 3. Analyzing the static response of the spring network, that is, calculating the forces associated with appropriate displacements given at the network periphery, yields the Young

Fig. 9 Random packing (Adhesion probability=20%)

Fig. 10 A spring element and local coordinates

modulus as a ratio of the corresponding average tensile stress to the average strain. If the case is a three-dimensional one, Poisson's ratio will be obtained from the associated transverse strain; since we are dealing here with the two-dimensional case, we obtain only Young's modulus.

The static response of the network has been analyzed in a similar manner as that of the FEM: first, obtain stiffness matrices of each spring, superpose them on each other and construct the global stiffness matrix, and then solve the linear algebraic equations with respect to nodal displacements, where the equations result from the equilibrium of nodal (generalized) forces and the boundary conditions.

5.1 Element stiffness matrix and stiffness matrix for whole network

The ends of a typical spring in the network are referred to as nodes i and j ; the displacement components and rotation (u_i, v_i, γ_i) , (u_j, v_j, γ_j) at these ends are defined as in Fig. 10. The corresponding forces and moment are denoted by (X_i, Y_i, M_i) , (X_j, Y_j, M_j) . If generalized nodal displacements and generalized nodal forces F are defined by

$$\delta = [u_i \ v_i \ \gamma_i \ u_j \ v_j \ \gamma_j]^T, \quad (10)$$

$$F=[X_i \ Y_i \ M_i \ X_j \ Y_j \ M_j]^T, \quad (11)$$

with superscript T denoting the transpose of the matrix, there is the following relation between them:

$$F=K\delta, \quad (12)$$

where K is the element stiffness matrix. This matrix is obtained from the equilibrium of the spring under the generalized forces and expressed by

$$K=\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad (13)$$

with

$$A=\begin{bmatrix} K_F \cos^2 \alpha + K_S \sin^2 \alpha & (K_F - K_S) \sin \alpha \cos \alpha \\ (K_F - K_S) \sin \alpha \cos \alpha & K_F \sin^2 \alpha + K_S \cos^2 \alpha \\ -K_S L \sin \alpha & K_S L \cos \alpha \\ -K_S/2 \cdot L \sin \alpha & K_S/2 \cdot L \cos \alpha \\ K_M + K_S/2 \cdot L^2 \end{bmatrix}, \quad (14)$$

$$B=\begin{bmatrix} -(K_F \cos^2 \alpha + K_S \sin^2 \alpha) \\ -(K_F - K_S) \sin \alpha \cos \alpha \\ K_S L \sin \alpha \\ -(K_F - K_S) \sin \alpha \cos \alpha & -K_S/2 \cdot L \sin \alpha \\ -(K_F \sin^2 \alpha + K_S \cos^2 \alpha) & K_S/2 \cdot L \cos \alpha \\ -K_S L \cos \alpha & -K_M + K_S/2 \cdot L^2 \end{bmatrix}, \quad (15)$$

$$C=\begin{bmatrix} -(K_F \cos^2 \alpha + K_S \sin^2 \alpha) \\ -(K_F - K_S) \sin \alpha \cos \alpha \\ -K_S L \sin \alpha \\ -(K_F - K_S) \sin \alpha \cos \alpha & K_S/2 \cdot L \sin \alpha \\ -(K_F \sin^2 \alpha + K_S \cos^2 \alpha) & -K_S/2 \cdot L \cos \alpha \\ K_S L \cos \alpha & -K_M + K_S/2 \cdot L^2 \end{bmatrix}, \quad (16)$$

$$D=\begin{bmatrix} K_F \cos^2 \alpha + K_S \sin^2 \alpha & (K_F - K_S) \sin \alpha \cos \alpha \\ (K_F - K_S) \sin \alpha \cos \alpha & K_F \sin^2 \alpha + K_S \cos^2 \alpha \\ K_S L \sin \alpha & -K_S L \cos \alpha \\ K_S/2 \cdot L \sin \alpha & -K_S/2 \cdot L \cos \alpha \\ K_M + K_S/2 \cdot L^2 \end{bmatrix}, \quad (17)$$

where K_F , K_S , and K_M are the spring constants for contraction, shearing and bending, respectively. The length of the spring and the angle between its direction (i, j) and the x -axis are denoted by L and α . The spring length is the same as the distance between the associated nodal points, that is, $L=D$.

Taking into account that resultant forces must vanish at all nodal points, except those where the displacement boundary conditions are specified, and superposing the element stiffness matrices given by Eq.(12) yields the overall equilibrium condition of the random network. If N is the number of all nodal points, the equilibrium condition is a linear system of $(3 \times N)$ equations.

5.2 Displacement boundary conditions and effective Young modulus

For the analysis of the static response of the network, we have dealt with all discs existing fully

and even partially within the rectangle between lines $B1$ and $B2$ and between $H1$ and $H2$ with the distance being H , as shown in Fig. 11. The displacement in the vertical direction is specified for all the discs which cross lines $H1$ and $H2$ by the following quantity for the disc with the y -coordinate being y_i :

$$v=v_0(2y_i-H)/H, \quad (18)$$

where v_0 denotes a specified small displacement. These are the boundary conditions for the random network analysis. These conditions have been introduced into the linear system in the similar manner as in the FEM analysis. Once the displacement of the center (node) of each disc has been determined, it is easy to obtain an internal force of each spring (element).

Denoting as Y_1 and Y_2 the sums of the y -components of the internal force of all springs (disc pair) crossing lines $H1'$ and $H2'$ distant from $H1$ and $H2$ by D , we defined the average stress by

$$\sigma=\frac{1}{2B}(Y_1+Y_2), \quad (19)$$

whereas we defined the average strain by

$$\varepsilon=\Delta v/\Delta y, \quad (20)$$

where Δv and Δy are the differences in the average displacement and the average y -coordinate of all the associated disc centers, respectively. Thus, the effective Young modulus can be estimated by

$$E=\sigma/\varepsilon. \quad (21)$$

6. Results and Discussion

Trial numerical calculations of the effective Young modulus and the porosity have been made for the adhesion probabilities $AP=0\%$, 10% , and 20% in the simulation of random packing, for the contact angles $\theta=10^\circ, 20^\circ$, and 30° as a parameter of the

Fig. 11 Region for analysis of random packing (spring network)

Table 1 Numerical results of the porosities and effective Young modulus ($B/D=16$)

	AP [%]	θ [°]	Samp. A	Samp. B	Samp. C	Samp. D	Samp. E	Ave.	Dev.
Porosity [%]	0	10	17.09	17.06	15.96	16.96	17.10	16.83	0.441
		20	16.63	16.59	15.47	16.50	16.65	16.37	0.450
		30	15.59	15.54	14.39	15.48	15.64	15.33	0.471
	10	10	21.05	20.31	21.41	20.22	20.89	20.78	0.450
		20	20.64	19.90	21.01	19.81	20.49	20.37	0.455
		30	19.73	18.98	20.12	18.88	19.59	19.46	0.467
	20	10	23.72	22.40	24.04	24.54	23.49	23.64	0.711
		20	23.35	22.02	23.66	24.18	23.12	23.27	0.720
		30	22.52	21.16	22.83	23.39	22.30	22.44	0.738
Effective Young's Modulus [GPa]	0	10	21.10	22.93	22.90	21.23	21.31	21.89	0.836
		20	26.12	27.99	28.54	26.46	26.50	27.12	0.958
		30	29.93	31.77	32.86	30.46	30.42	31.09	1.076
	10	10	16.33	16.61	15.53	16.33	15.76	16.11	0.401
		20	21.19	20.96	19.57	20.97	20.31	20.60	0.593
		30	25.06	24.32	22.71	24.61	23.90	24.12	0.800
	20	10	11.57	13.04	13.21	12.16	11.26	12.25	0.774
		20	15.81	17.11	17.24	16.17	14.74	16.21	0.916
		30	19.12	20.27	20.42	19.34	17.52	19.33	1.039

Fig. 12 Effective Young modulus versus porosity ($B/D=16$)

degree of sintering and for the ratios $B/D=16, 24$, and 30 as a scale parameter of the packings. The trial calculations have shown that there is no essential difference in the results between the cases of $D/B=16, 24$, and 30 , so only the results for $B/D=16$ are shown in Table 1 and Fig. 12. The porosities and effective Young moduli are tabulated for five different series

(samples $A \sim E$) of initial positions of the discs generated by five different series of the pseudorandom numbers, together with their averages and standard deviations. It is natural that the porosity is larger and the effective modulus is smaller with larger adhesion probability. The standard deviations of both the porosity and effective modulus for the five samples do not seem to be small enough in comparison with their averages, so more samples seem to be needed; we cannot, however, be certain of obtaining a smaller standard deviation even if we use more samples. This remains for our further research.

7. Conclusion

(1) A new method of simulation has been proposed to estimate the effective elastic moduli of a porous elastic body prepared by sintering an aggregate of spherical particles with equal diameter; the method involves constructing an aggregate of spherical particles of equal diameter in a computer by means of solution of the motion equations of each particle under gravity and spring forces acting between the particle and other setting particles and/or a floor, by evaluating the spring constants of a pair of connected neighboring particles by the FEM, and by analyzing, in a similar manner as the FEM, the static response of a random network of springs, whose constants are already obtained and which have 6 degrees of freedom, to some macroscopically uniform deformation.

(2) By executing the two-dimensional simulation of the proposed method to the estimate Young modulus of a porous elastic two-dimensional body, the method and its detailed procedures have been verified to be effective.

Acknowledgements

The authors are grateful to Mr. Yoshiyuki Goto for his assistance in preparing this manuscript. The present study was partially supported by a Grant-in-Aid for Scientific Research (C) No. 0450076 from the Ministry of Education, Science and Culture.

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