Wave Propagation in a Thick Cylindrical Bar Due to Longitudinal Impact*

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We present results of a solution to longitudinal impact of thick bars, obtained via an exact three-dimensional theory. Detailed investigation of the behavior of the solution has been carried out, including a number of subsidiary calculations. The results obtained made it possible to improve on the knowledge of the physical nature of the problem investigated. By comparing the exact theory with calculations performed in terms of an elementary bar theory, the successive transition from a three-dimensional (3D) description to a one-dimensional (1D) one could be demonstrated. The obtained dependences of displacements and stresses have been compared with Skalak's asymptotic solution and with results of experiments by Miklowitz.

Key Words: Elastic Waves, Wave Propagation, Impact of Bodies, Analytical Solution, Thick Cylindrical Bar

1. Introduction

Problems related to a transient load acting on a terminal face of a semi-infinite cylindrical bar (a special case is a longitudinal impact of bars) have been widely investigated for many years. Usually, a terminal face of a bar is assumed to be subjected to a step change in stress or velocity. The solution to this problem is well known in terms of elementary one-dimensional (1D) theory. A pulse, propagating at velocity c_0 throughout the bar, remains undistorted⁽¹⁾. The 1D theory is the basis of the most experimentally oriented papers, see, e.g., Ref. (2). It is also the basis of papers investigating nonprismatic bars⁽³⁾, and of those considering nonlinear material characteristics⁽⁴⁾. The elementary theory holds for long-wave components of the pulse, but it cannot be used to describe the

propagation of pulses with a steep wavefront. A certain improvement of the elementary 1D theory leads to the Love theory which considers inertial effects in the radial direction of a bar⁽⁵⁾. The Love theory extends the frequency region of applicability of the 1D theory, however, the hyperbolicity of the equation of motion is lost⁽⁶⁾. In spite of this imperfection, the "one-mode" theory is often successfully applied to long waves and far fields, instead of the more complex "two-mode" approximate Mindlin-Hermann theory or "three-mode" Mindlin-McNiven theory (both theories are hyperbolic; see Refs. (7) and (8)).

The first three-dimensional study, which is continued here, led to the Skalak solution for longitudinal impact of semi-infinite cylindrical bars⁽⁹⁾. Among other papers on this topic solved at that time within 3D theory we mention the paper by Folk et al.⁽¹⁰⁾, which describes a wave progression due to a suddenly applied stress at a terminal face of a semi-infinite bar. From papers appeared later we adduce the Motoyama solution for longitudinal impact of bars with an elliptical cross section⁽¹¹⁾ or the Tanaka solution for longitudinal impact of bars with a rectangular cross section⁽¹²⁾. The usual approach to studying

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3D problems is the use of integral transforms. This approach is used by Skalak and other authors^{(7),(10)-(12),(15)-(17)}. All the analytical approaches to 3D problems which are known to us differ from the present solution in the use of only an approximate method for evaluation of inverse transforms. In the majority of cases, the "saddle point method" has been used, which appears also in Skalak's paper⁽⁹⁾. The relations for displacements and stresses, obtained in this way, are valid only over a long time and at long distances from the place of impact.

In 1983 Valeš completed the exact analytical solution started by Skalak. He first carried out basic numerical calculations (see Refs. (13), (14)) which were later to become more exact.

The main objectives of the present paper are the

- i) determination of the near-field and short-time response, and
- ii) demonstration of the relationship between 3D theory and 1D theory for the far-field and long-time response.

Notation

- r, ϑ, z : cylindrical coordinate system
 a : radius of a bar
 t : time
 v_0 : initial velocity of colliding bars
 $r/a, z/a$: dimensionless geometrical coordinates
 $c_1 t/a, c_2 t/a$: dimensionless time parameters
 u_r^*, u_z^* : dimensionless radial and axial displacements
 $\sigma_r^*, \sigma_z^*, \sigma_\vartheta^*, \tau_{rz}^*$: dimensionless radial, axial, circumferential and shear stresses
 I_0, I_1 : modified Bessel functions of the first kind
 J_0, J_1 : Bessel functions of the first kind
 σ_0^* : dimensionless axial stress in a thin bar
 $k_1: k_1 = (c_2/c_1)^2 = (1-2\nu)/\{2(1-\nu)\}$
 ν : Poisson's ratio
 λ : Lamé's constant
 E : Young's modulus
 G : shear modulus
 γa : dimensionless integration variable (wave number)
 ζ_n : root of frequency equation
 n : addition subscript of series (sequence of dispersion curve)
 c_1 : velocity of dilatational wave in unbounded medium
 c_2 : velocity of shear (equivolumetric) wave in unbounded medium
 c_0 : velocity of longitudinal waves in a thin bar
 c_R : velocity of Rayleigh surface waves in half-space
 P, S, vS, R, P_0 : symbols denoting wavefront positions or arrival times of dilatational wave,

shear wave, von Schmidt wave, Rayleigh wave and longitudinal thin-bar wave

2. Analytical Solution

Longitudinal impact of thick bars is understood to mean the collision of two semi-infinite cylindrical bars which move coaxially against each other at identical velocities v_0 . All the points of the plane faces of the bars are assumed to meet at the same instant $t=0$ when they are suddenly stopped. The diameter of the bars is $2a$, the material of the bars is considered elastic, and the validity of the small deformation theory is assumed. Equations of motion of a three-dimensional elastic linear continuum are solved in a standard manner through integral transforms, separation of variables (see Refs. (15), (16) and (17)) and using Skalak's⁽¹⁾ decomposition⁽⁹⁾.

To illustrate the form of results of the solution presented in Ref. (14), we shall show here, as an example, the formula for evaluating a dimensionless displacement in the radial direction u_r^* . The other quantities have a similar form. For instance, in the region $1 \leq \zeta_n \leq 1/\sqrt{k_1}$ it holds that

$$u_r^* = \frac{c_2}{v_0} \cdot \frac{u_r}{a} = \frac{4\nu}{(1-\nu)\pi} \sum_{n=1}^{\infty} \int_0^{\infty} \frac{U_2\left(\frac{r}{a}, \gamma a, \zeta_n\right)}{\gamma a \zeta_n \sqrt{1-k_1 \zeta_n^2} M_2(\gamma a, \zeta_n)} \times \sin\left(\frac{c_2 t}{a} \cdot \gamma a \cdot \zeta_n\right) \cos\left(\frac{z}{a} \cdot \gamma a\right) d(\gamma a), \quad (1)$$

where

$$\begin{aligned} U_2\left(\frac{r}{a}, \gamma a, \zeta_n\right) &= (2 - \zeta_n^2) I_1\left(\frac{r}{a} \gamma a \sqrt{1 - k_1 \zeta_n^2}\right) \\ &\times J_1\left(\gamma a \sqrt{\zeta_n^2 - 1}\right) - 2 I_1(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \\ &\times J_1\left(\frac{r}{a} \gamma a \sqrt{\zeta_n^2 - 1}\right), \\ M_2(\gamma a, \zeta_n) &= -(\gamma a)^2 \cdot \left[\frac{(2 - \zeta_n^2)^2}{\sqrt{\zeta_n^2 - 1}} + 4 k_1 \sqrt{\zeta_n^2 - 1} \right] \\ &\times I_0(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \cdot J_0(\gamma a \sqrt{\zeta_n^2 - 1}) + \gamma a \\ &\times \left[(2 - \zeta_n^2) \left(4 + \frac{2 - \zeta_n^2}{\zeta_n^2 - 1} \right) + 2 k_1 \zeta_n^2 \right] \\ &\times I_0(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \cdot J_1(\gamma a \sqrt{\zeta_n^2 - 1}) \\ &+ 2 \gamma a (2 - \zeta_n^2) \frac{\sqrt{1 - k_1 \zeta_n^2}}{\sqrt{\zeta_n^2 - 1}} I_1(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \\ &\times J_0(\gamma a \sqrt{\zeta_n^2 - 1}) - \left\{ 2 \sqrt{1 - k_1 \zeta_n^2} \right. \\ &\times \left[2(\gamma a)^2 - \frac{2 - \zeta_n^2}{\zeta_n^2 - 1} \right] - k_1 (\gamma a)^2 \frac{(2 - \zeta_n^2)^2}{\sqrt{1 - k_1 \zeta_n^2}} \left. \right\} \\ &\times I_1(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \cdot J_1(\gamma a \sqrt{\zeta_n^2 - 1}). \end{aligned}$$

For values $\zeta_n < 1$ and $\zeta_n > 1/\sqrt{k_1}$ the expressions

⁽¹⁾ The problem of a longitudinal impact of bars can also be solved without using Skalak's decomposition - see Ref. (13).

for displacements and stresses have formally the same form.

Parameter ζ_n is related to integration variable γa through the frequency equation (dispersive relation), which for

$$1 \leq \zeta_n \leq \frac{1}{\sqrt{k_1}} \text{ is}$$

$$(2 - \zeta_n^2)^2 I_0(\gamma a \sqrt{1 - k_1 \zeta_n^2}) J_1(\gamma a \sqrt{\zeta_n^2 - 1}) - 4 \sqrt{1 - k_1 \zeta_n^2} \sqrt{\zeta_n^2 - 1} I_1(\gamma a \sqrt{1 - k_1 \zeta_n^2}) \times J_0(\gamma a \sqrt{\zeta_n^2 - 1}) + \frac{2 \zeta_n^2}{\gamma a} \sqrt{1 - k_1 \zeta_n^2} \times I_1(\gamma a \sqrt{1 - k_1 \zeta_n^2}) J_1(\gamma a \sqrt{\zeta_n^2 - 1}) = 0, \quad (2)$$

where γa = dimensionless wave number, λ = wavelength, $\zeta_n = c/c_2$, and c = phase velocity

Equation (2) is the well-known Pochhammer equation (see, e.g., Refs. (1), (18), (19)). Its graphical representation consists of dispersion curves for longitudinal modes, as shown in Fig. 1.

3. Numerical Evaluation of Relations for Displacements and Stress Components

The relations for displacements and stresses (see Eq. (1)) have the form of an infinite series of improper integrals. A detailed analysis of frequency equation (2) (see Ref. (20)) and, moreover, a detailed analysis of integrands and series in Eq. (1) (see Ref. (21)) have been carried out. For a numerical evaluation of integrals the Simpson method has been used, and for evaluating the roots of the frequency equation, the step-halving method and the regula falsi method have been applied (the roots are simple).

The basic problems in the numerical evaluation of relations (1) and (2) are as follows:

- determination of optimum magnitude of integration step $\delta(\gamma a)$,
- determination of integration limits,
- determination of optimum number of terms in series.

By analyzing the behavior of integrands, it is possible to estimate the maximum value of the integration step in the form (see Ref. (21))

$$\delta(\gamma a) \leq \frac{1}{3} \cdot \frac{\pi}{\frac{c_1 t}{a} + \frac{z}{a}}. \quad (3)$$

The nomogram for relation (3) is shown in Fig. 2. A zero lower limit of integration cannot be used in numerical calculation since for $\gamma a = 0$ the integrands become indeterminate expressions, however, with finite limit values (from analytical point of view). The value $(\gamma a)_{\min} = 0.005$ has been used. For long times and great distances from the place of impact of bars, the greatest increment to the values of integrals is obtained through integration at the beginning of the first dispersion curve. Here we used a linear interpola-

Fig. 1 Dispersion curves for longitudinal modes in a bar of radius a

Fig. 2 Integration step nomogram

tion to improve the exactness of results. The magnitude of the upper limit of integrals must be chosen expediently in order that those regions of integration variable which give the greatest increment to the value of the evaluated integral may be properly respected.

The values of the main parameters, used in the calculation, are as follows:

- minimum step of the Simpson method $\delta(\gamma a) = 0.005$,
- maximum number of terms summed in series $n_{\max} = 50$,
- lower integration limit $(\gamma a)_{\min} = 0.005$,
- upper limit of integrals $(\gamma a)_{\max} = 100.005$ (depends on the choice of $n_{\max}$).

The above values preserve reliable results, and the errors in the region $(c_1 t/a + z/a) \leq 200$ are only several percent.

4. Results of Calculation

The calculations were aimed at the determination of stress and deformation states in thick bars subjected to a longitudinal impact for wide intervals of time and space coordinates. The results show, among others, the mutual relation between solutions obtained via the 3D and 1D approaches. The results presented in this paper are obtained for Poisson's ratio $\nu=0.3$.

First let us show the shapes of wavefronts immediately after the impact, to delimitate the regions with a characteristic state of the stress. Figure 3(a) shows the wavefronts at a dimensionless time $c_1 t/a = 0.8$. From the plane, in which all the points of impacted ends of the bars were in contact at the instant $c_1 t/a = 0$, a plane pressure dilatational wave starts propagating at velocity c_1 . Behind this wavefront the same state of stress as caused by the impact of half-spaces (hatched region) occurs. However, the size of this region is successively decreased by the unloading waves generated at points lying on circular edges of colliding faces of the bars. The unloading dilatational waves are followed by shear waves propagating from the same points at velocity c_2 . The envelope of the unloading dilatational waves is represented by the surface of torus illustrated in the sectional view by a part of a circle with radius $c_1 t/a$. The envelope of the unloading shear waves is part of a circular torus with radius $c_2 t/a$ and part of a conic area forming the wavefront of the von Schmidt waves. Figure 3(b) shows the position of wavefronts for longer time intervals for $c_1 t/a > 10$. The hatched area in Fig. 3(b) (with the same state of stress as that caused by collision of half-spaces) is now diminished practically to zero.

To illustrate the situation depicted in Fig. 3(a), we can draw a comparison between the calculated values of displacements and stresses and the known values obtained by the impact of half spaces. The comparison is made at the "points" $z/a=0.5$, $r/a=0.2$, ($c_1 t/a=0.8$). The results are listed in Table 1.

Dimensionless displacements and stresses are defined by the following relations.

$$\begin{aligned} u_r^* &= \frac{c_2}{v_0} \frac{u_r}{a} & u_z^* &= \frac{c_2}{v_0} \frac{u_z}{a} & \sigma_r^* &= \frac{\sigma_r}{\left(\lambda \frac{v_0}{c_1}\right)} \\ \sigma_z^* &= \frac{\sigma_z}{\left(\lambda \frac{v_0}{c_1}\right)} & \sigma_\theta^* &= \frac{\sigma_\theta}{\left(\lambda \frac{v_0}{c_1}\right)} \\ \tau_{rz}^* &= \frac{\tau_{rz}}{\left(\lambda \frac{v_0}{c_1}\right)} & \sigma_0^* &= \frac{\sigma_0}{\left(\lambda \frac{v_0}{c_1}\right)} \end{aligned}$$

Table 1 Impact of bars and half-spaces
($z/a=0.5$, $r/a=0.2$ and $c_1 t/a=0.8$)

quantity	impact of half-spaces	impact of bars	
		results of calculation	abs. error
u_r^*	0	0.0000590	0.0000590
u_z^*	-0.267261	-0.267252	0.000009
σ_r^*	-1	-0.993580	0.006420
σ_z^*	-2.3	-2.330632	0.002701
σ_θ^*	-1	-0.996306	0.003694
τ_{rz}^*	0	-0.001418	0.001418

Fig. 3

Table 1 shows the exactness of calculations carried out for the region considered, using the parameters given at the end of the preceding paragraph above.

We shall now present basic results obtained for locations near the impacted end of the bar. Figure 4 (a) illustrates the selected cross sections a) to f) along with the positions of the wavefronts at the instant $c_1 t/a = 1.2$. Figure 4(b) shows the radial stress σ_r^* versus radial coordinate r/a in these cross sections. In sections e) and f) which are free of unloading dilatational waves there appears the same state of stress as that caused by the impact of half-spaces (in Fig. 4(a) from the axis of the bar to points A^e and A^f). Sudden changes in the slope in sections c), d), e) and f) correspond to the position of the wavefront of von Schmidt's wave (in Fig. 4(a), see points B^c, B^d, B^e and B^f). The boundary condition of zero radial stress is well satisfied for $r/a=1$ (surface of a bar).

(a) Wavefront positions for $c_1t/a=1.2$ and selected cross sections a) to f) of a bar

(b) Radial stress versus radial coordinate in cross sections depicted in Fig. 4(a) for time $c_1t/a=1.2$

Fig. 4

The z/a dependence of axial stress σ_z^* at a short distance from the axis and corresponding wavefront positions are shown in Figs. 5(a) and (b). In these diagrams, drawn for time $c_1t/a=2$ and $r/a=0.05$, we can see how the response is influenced by wavefronts of unloading waves (see points A and B). In the region between point A and C the value of axial stress is again identical to the value of stress appearing due to the impact of half-spaces. At point C the stress should undergo a step change from $-2.333\cdots$ to zero. From the figure we can see that the calculated dependence is strongly influenced by using a finite number of terms summed in series of the Eq. (1) type (because

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Fig. 5 Wavefront positions (a) and axial stress versus z/a coordinate (b) for $c_1t/a=2.0$ and $r/a=0.05$

of the finite number of modes considered). Figure 5(b) illustrates the summation of the first 40 terms of the series. By considering a greater number of terms the result will be considerably improved (see the detail in Fig. 5(b)).

All stress dependences are strongly influenced by von Schmidt waves. This is illustrated in Fig. 6 which shows the axial stress σ_z^* distribution along the radial coordinate. The great change in the slope of axial stress shown in Fig. 6(b) occurs in the neighborhood of point A (for $r/a=0.5$), i.e., in proximity to von Schmidt wavefront position (see Fig. 6(a)). In Fig. 7 we can see that the influence of von Schmidt waves upon the distribution of axial displacement u_z^* is considerably less pronounced. From the above figures we see that the distribution of the quantities considered over the cross section is not uniform. Cross-sectional warping effects are present. Accordingly, at this time $c_1t/a=5.0$, impact of bars cannot be described by the elementary theory, since this theory

Fig. 6 Von Schmidt wavefront position (a) and axial stress distribution along radial coordinate (b) for $c_1 t/a = 5.0$ and $z/a = 2.62$

Fig. 7 Von Schmidt wave influence on the distribution of axial displacement

assumes the quantities considered to be uniformly distributed over a cross section (details will be discussed below). In the elementary theory the radial distribution of axial stress is defined by a straight line σ_0^* (see Fig. 6(b)). For longer time (see also Figs. 11, 12) there appears a successive conversion to a one-dimensional state of stress. Figure 8 shows the axial distribution of axial stress σ_z^* on the surface of the bar ($r/a = 1$) at time instant $c_1 t/a = 20$. In terms of the one-dimensional theory a steady value of stress σ_0^* , as shown in the figure, is expected. At this time instant no distinct wavefront propagating at velocity c_0 has developed as yet. In the direction toward the place of collision the axial stress approaches the value σ_0^* .

It is to be noted that the near-field analysis reveals another important characteristic related to stress waves of the Rayleigh type. In Figs. 9 and 10 we can see the time course of axial stress σ_z^* on the surface of the bar at $z/a = 2$. While the stress response shown in Fig. 9 was determined by summing the first 50 terms of the series (50 modes), the diagram depicted in Fig. 10 is due only to the contribution of the first dispersion curve. From the coincidence of the two waveforms at the wavefront defined by velocity c_R we can conclude that the Rayleigh wave is comprised of the first-mode (see Ref. (24)).

Now we direct our attention to the distributions referring to large values of $c_1 t/a$. Keeping in mind Figs. 6 and 7, we examine Figs. 11 and 12, which show the situation at the instant when the wavefront of the primary dilatational wave covered the distance of 50 diameters from the place of collision of bars. The axial stress σ_z^* and displacement u_z^* are shown in the

Fig. 8 The axial distribution of axial stress on the surface of a bar for $c_1 t/a = 20.0$

Fig. 9 The time dependence of axial stress on the surface of a bar for $z/a=2.0$ (50 modes)

Fig. 10 The time dependence of axial stress on the surface of a bar for $z/a=2.0$ (first mode only)

cross section immediately behind the wavefront of a longitudinal wave in a thin bar ($z/a \doteq 83.6$; $c_0 t/a \doteq 86.2$).

Displacement and stress deviations from the uniform distribution (predicted by elementary bar theory) over the cross section are smaller than those depicted in Figs. 6 and 7. This means that the warping of the cross section diminishes for far fields and long time.

The mutual relation between the 3D and 1D theories is evident in Fig. 13. The figure shows the

Fig. 11 The axial stress versus radial coordinate; comparison with a thin bar theory

Fig. 12 The axial displacement versus radial coordinate for $c_1 t/a=100.0$ and $z/a=83.60$

distribution of axial stress σ_z^* on the surface of a bar which is dependent on z/a at $c_1 t/a=2, 10, 20, 40$ and 60 . The undistorted step pulse depicted by the dashed line is obtained in terms of the elementary 1D theory, while the solid line refers to the 3D solution. It is seen that for short times there occurs a great overshoot behind the wavefront propagating at velocity c_0 , but this overshoot rapidly decreases in time, and at $c_1 t/a=60$ it becomes small. Furthermore, we can see expansion of the region with zero axial stress behind the wavefront propagating at velocity c_1 .

Fig. 13 The distribution of the axial stress on the surface of a bar in the dependence on z/a for $c_1t/a=2.0$; 10.0; 20.0; 40.0; 60.0. The mutual relation between the 3D and 1D theory

Fig. 14 The radial distribution of radial displacement for $c_1t/a=20.0$ and $z/a=10.0$; comparison of 3D (solid line) and Love theories (dashed line)

Up to now the solution obtained in terms of the 3D theory has been compared with that obtained via the elementary 1D theory. A certain improvement of this elementary theory is represented by the Love theory. By the Love theory the radial displacement u_r^* for a given z is considered to be a linear function of r . Figure 14 shows that the above assumption is satisfied as early as in the relatively short time $c_1t/a=20$. The figure is drawn for $z/a=10$. The dashed and solid lines respectively correspond to the Love theory and 3D theory. Figure 15 shows the time dependence

Fig. 15 The time dependence of radial displacement on the surface of a bar for $z/a=10.0$ (the value 0.186 corresponds to the Love theory)

of radial displacement u_r^* on the surface of the bar for $z/a=10$. It is seen that this 3D solution oscillates about the value $u_r^*=0.186$ obtained via the Love theory. Point A defines the time corresponding to the preceding figure.

Skalak's solution⁽⁹⁾ gives a valid approximation for long time and great distances from the colliding faces of bars. The same results as from Skalak's approximation were obtained by Miklowitz⁽¹⁸⁾ using the Love equation. This procedure is employed also in Ref. (23). Figure 16 shows a comparison between the solution obtained through the one-dimensional theory (dashed line) and the asymptotic Skalak solution (solid line). Figure 17 illustrates the differences between the one-dimensional solution (dashed line) and the 3D solution (solid line). From the above figures it is seen that Skalak's solution closely approximates the 3D solution which, however, better complies with the elementary theory both at the wavefront of disturbance (i.e., for $z/a=c_0t/a$) and in the neighborhood of the place of impact of the bars (i.e., for $z/a=0$). In spite of the fact that the time interval being considered is very long ($c_1t/a=160$), the axial stress σ_z^* is not exactly in agreement with the results of the elementary theory. This is due to dispersion, the influence of which, unfortunately, increases as distance and time increase. The results for displacements are considerably better, as can be seen from Fig. 18 which shows the axial displacement u_z^* at $c_1t/a=100$ with reference to axial coordinate z/a . In terms of the elementary 1D theory the

Fig. 16 The axial stress versus z/a ; comparison between the 1D theory (dashed line) and asymptotic Skalak's solution (solid line)

Fig. 17 The axial stress versus z/a ; comparison between the 1D theory (dashed line) and 3D theory (solid line)

displacement is expected to be linearly distributed with reference to z/a . Moreover, at the time instant under consideration the value of the displacement at the wavefront propagating at velocity c_0 is $u_z^* = -53.452$. The same value must occur ahead of this wavefront. The 3D and 1D distributions are practically identical. Finally, Fig. 19 shows the comparison between time dependences of radial displacements u_r^* obtained through calculations via the 3D theory and those obtained through experiments by Miklowitz⁽²²⁾. It is noted that the boundary conditions in the calculation and in the experiment were not identical. In the

Fig. 18 The axial displacement as a function of z/a for $c_1t/a = 100.0$ and $r/a = 1.0$

Fig. 19 The time dependence of radial displacement for $r/a = 1.0$ and $z/a = 40.0$; comparison between 3D theory and experiment (after Miklowitz⁽²²⁾)

experiment the face of the bar was subject to a step change in axial load, whereas in the calculation a step change in velocity was assumed. The good agreement between the two dependences indicates that far from the place of collision (where the curves oscillate about or nearly about "Love's" value $u_r^* = 0.186$; see Fig. 15) the influence of boundary conditions decreases. This result supports previous indications that the behavior of colliding bars can be described (for the far field) by

the 1D model assuming a linear dependence between axial stress and axial velocity. In view of dispersion which always belongs to 3D model, the response at the wavefront shows no sharp discontinuity of pulse arrival unlike that predicted by the elementary theory.

5. Conclusion

We presented a comprehensive description of a longitudinal impact of thick cylindrical semi-infinite elastic bars. The development of the state of stress in the first phase of the impact in the proximity of colliding faces of bars was investigated, since the majority of theories using the one-dimensional approach fail to solve this problem. Between the place of impact of bars and the wavefront of the primary dilatational wave at the beginning of the process, there occurs the same state of stress as that caused by the impact of elastic half-spaces. At the same time, unloading dilatational and shear waves emitted from free circular edges of the colliding faces of the bars propagate through this region, and thus considerably influence the original simple state of stress. The magnitudes of displacements and stresses have been employed here as important check values for calculations.

Furthermore, the von Schmidt waves considerably influence deformation and the stress state in bars.

On the surface of the bars the existence of the Rayleigh-type waves has been proven. The waves are comprised of the first-mode.

Another part of the study is the investigation of a successive transition of the exact 3D solution to the solution of the elementary 1D theory with Rayleigh-Love correction, and the comparison with the asymptotic Skalak solution and the result of the experiment by Miklowitz. With increasing time it was proven that there appears a successive development of the wavefront propagating at the same velocity as of waves in a thin bar c_0 . This wavefront eventually becomes dominant, while the wavefront of dilatational waves propagating at velocity c_1 vanishes. The warping of the cross section of the bars continues to decrease. The axial stress appearing ahead of the wavefront propagating at velocity c_0 successively vanishes, while behind the wavefront the axial stress is almost uniformly distributed throughout the cross section of the bar, and its value is approximated as the value of σ^* . The distribution of axial displacement relative to coordinate z/a is linear, similarly to that caused by the impact of half-spaces. Radial displacements approximately correspond to those of the Love theory.

It was found that the choice of parameters in the

calculation is satisfactory from the viewpoint of exactness of the solution. Displacements and stresses can be evaluated over a sufficient extent of geometrical coordinates and time that the results may be compared with those obtained via the one-dimensional theory.

The comparison with the asymptotic Skalak solution was carried out for long values of time. The similarity of results obtained in this way to the present results give the evidence of only small deviations from the exact 3D solution. For the longest time considered the 1D solution better coincided with the 3D calculations than with the Skalak approximation.

Finally, the radial displacements evaluated via the 3D theory were compared with the experimental values obtained by Miklowitz, and good correspondence was found.

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