Application of Damage Mechanics to the Analysis of Spall Damage*

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A systematic method of spall damage analysis was developed on the basis of damage mechanics. By use of a scalar damage variable, the damage evolution equation of Lemaitre et al. and the viscoplastic constitutive equation of Perzyna modified for damaged materials were incorporated into the commercial finite difference program MANJUSRI-3 D for nonlinear dynamic analysis. The spall damage process, and stress wave propagation and temperature histories were analyzed for plate impact of OFHC copper disc targets. The computational results for the stress history and the particle velocity at the rear surface of the target plates were compared with the corresponding experimental results.

Key Words: Damage Mechanics, Stress Wave, Finite Difference Method, Spall Damage, Plate Impact, OFHC Copper, Elastic-Viscoplastic-Damage Coupling, Axisymmetric Analysis

1. Introduction

When a material is subjected to strong impact, the compressive waves generated by the impact induce a localized region of high tensile stress as a result of their reflection and interference. If the amount of the induced tensile stress is high enough, localized damage will develop and will result in fracture or separation of the material. This damage is usually called spall damage⁽¹⁾. Particularly, in the case of metals, ductile spall is induced by relatively low tensile stress and becomes an important problem of design and analysis of structural components subject to shock loading.

Some spall damage models have been proposed to describe the damage behavior and the resulting change in the mechanical properties of materials. Seaman and his coworkers⁽²⁾, for example, performed a series of plate-impact tests, and proposed spall damage evolution equations by taking account of the nucleation and growth mechanisms of voids. Recently, Eftis et al.⁽³⁾ extended Perzyna's viscoplastic constitutive model of dynamic fracture⁽⁴⁾ to allow for the nonlinear isotropic hardening law and discussed a systematic method for spall damage analysis. However, these models include a considerable number of material parameters, and necessitate complicated procedures to determine these parameters.

Recently, *continuum damage mechanics* (CDM) has been applied successfully to a large variety of static problems, such as creep damage, plastic damage and elastic-brittle damage^{(5),(6)}, and provides a useful method for modeling the process of nucleation and growth of microcavities in materials. CDM describes systematically the mechanical effects of distributed microvoids in terms of proper internal state variable together with their evolution in the framework of continuum mechanics. However, CDM has been

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applied to only a few dynamic problems⁽⁷⁾ and its applicability has not been investigated sufficiently.

In the present paper, we will discuss the applicability of damage mechanics to spall damage analysis in metals as a prospective systematic approach. The mechanical effects of microcavities are first represented by a scalar damage variable, and the evolution equation of Lemaitre et al.⁽⁸⁾ for elastic-plastic damage is used. The inelastic constitutive equation of Perzyna⁽⁹⁾ is extended to take into account the damaged material. Then, these equations are incorporated into the commercial finite difference program MANJUSRI 3D (CRC Research Institute)⁽¹⁰⁾ for inelastic dynamic analysis that allows for coincident deformation, damage and thermal effects. The resulting computer program is applied to the analyses of the one-dimensional spall damage process in the plate impact of OFHC copper. Stress wave propagation, the development and spatial distribution of damage, the heat generation brought on by deformation and the degradation of mechanical properties due to damage are elucidated. Finally, the results obtained from this analysis are compared with the corresponding experimental results reported to date, and the validity of the present analysis is examined.

2. Damage Variable and Its Evolution Equation

2.1 Damage variable

In *continuum damage mechanics* (CDM), the mechanical effects of cavities due to damage are represented in terms of a proper macroscopic damage variable^{(5),(6)}. In particular, when the development of cavities does not show salient anisotropy as in the case of elastoplastic damage in ductile materials, the damage can be described by a scalar damage variable D ($0 \leq D \leq 1$) such that

$$\left. \begin{aligned} D=0 & \quad (t=0; \text{ initial undamaged state}) \\ D=1 & \quad (t=t_R; \text{ final ruptured state}) \end{aligned} \right\}. \quad (1)$$

The damage variable D is often interpreted as the reduction of load-carrying area (effective area), and thus, the effect of Cauchy stress is magnified by the damage as follows:

$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{1-D}. \quad (2)$$

2.2 Damage evolution equation

If we postulate that the development of microcavities and the degradation of the mechanical properties are approximately isotropic, we can employ the evolution equation of isotropic damage developed by Lemaitre et al.⁽⁸⁾ Namely, if the material is ductile and the development of damage is mainly governed by hydrostatic pressure, the evolution equation of D is given as follows:

$$dD = \left\langle \frac{P - P_D}{(1-D)S_D} \right\rangle^{q_D} \frac{\langle dP \rangle}{S_0}, \quad (3)$$

where $\langle \rangle$ are Macauley brackets, $P = 1/3 \sigma_{kk}$, P_D are the hydrostatic stress and its threshold value of damage development, respectively, and S_D , q_D are material parameters.

3. Constitutive Equation

3.1 Elastic constitutive equation

The stiffness of a material is decreased by the development of microcavities due to spall damage. We postulate that the decrease of material stiffness is isotropic and the thermoelastic constitutive equation is given by applying the principle of strain equivalence^{(5),(6)} to the Duhamel-Neuman equation, i.e., by replacing the Cauchy stress σ_{ij} in the Duhamel-Neuman equation with the corresponding effective stress $\bar{\sigma}_{ij} = \sigma_{ij}/(1-D)$ as follows:

$$\epsilon_{ij}^e = \frac{1+\nu}{E} \frac{\sigma_{ij}}{1-D} - \frac{\nu}{E} \frac{\sigma_{kk}}{1-D} \delta_{ij} + \alpha \Delta \theta \delta_{ij}, \quad (4)$$

where E , ν are the elastic modulus and Poisson's ratio, respectively, and α , θ are the thermal expansion coefficient and the temperature, respectively.

3.2 Viscoplastic constitutive equation

Ductile materials subject to high stress at high rate of strain usually show irreversible, rate dependent and viscoplastic deformation, besides elastic deformation as described in section 3.1. In the present paper, the Perzyna viscoplastic constitutive equation⁽⁹⁾ is used as a constitutive relation of dynamic response of undamaged materials for a wide range of strain rates:

$$\dot{\epsilon}_{ij}^p = \gamma \Phi(F) \frac{\partial f}{\partial \sigma_{ij}}, \quad (5)$$

$$\Phi(F) = \left\langle \frac{f}{\kappa} - 1 \right\rangle^m, \quad (6)$$

where $\dot{\epsilon}_{ij}^p$, σ_{ij} , f are the viscoplastic strain rate tensor, the stress tensor and the yield function, respectively, and γ , κ , m are the viscosity constant, the quasi-static yield stress and a material constant, respectively.

For damaged materials, in particular, we represent the yield function f of equations (5) and (6) by combining the Mises type yield function and the principle of strain equivalence as follows:

$$f = \left(\frac{1}{2} \frac{s_{ij}}{1-D} \frac{s_{ij}}{1-D} \right)^{1/2} = \frac{\sqrt{J_2}}{1-D}, \quad (7)$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{s_{ij}}{2\sqrt{J_2}(1-D)}, \quad (8)$$

where $s_{ij} = \sigma_{ij} - 1/3 \sigma_{kk} \delta_{ij}$, J_2 are the stress deviator and its second invariant. By assuming nonlinear isotropic strain hardening of the material, the yield stress κ is represented in the following form⁽³⁾:

$$\kappa = q + (\kappa_0 - q) e^{-\beta \epsilon_{eq}^p}, \quad (9)$$

$$\dot{\epsilon}_{eq}^p = \int_0^t \frac{2}{\sqrt{3}} (\dot{\epsilon}_{pq}^p \dot{\epsilon}_{pq}^p)^{1/2} dt, \quad (10)$$

where ϵ_{eq}^p is the equivalent viscoplastic strain, and q , β are material hardening parameters.

Thus, the inelastic constitutive equation for the damaged material is represented by the following relation:

$$\dot{\epsilon}_{ij}^p = \frac{\gamma}{2} \left\langle \frac{\sqrt{J_2}}{(1-D)(q + (\alpha_0 - q)e^{-\beta \epsilon_{eq}^p})} - 1 \right\rangle^m \frac{s_{ij}}{\sqrt{J_2}(1-D)}, \quad (11)$$

3.2 Constitutive equation for volumetric strain

In addition to the isochoric viscoplastic strain of Eq. (11), the dynamic response of material at high rate of strain is affected by the hydrostatic pressure, and thus an equation describing the constitutive relationship between the hydrostatic pressure and the volumetric strain is also needed.

For this purpose, we will employ the shock-particle velocity equation of Grüneisen⁽¹¹⁾. Namely, the hydrostatic pressure p ($= -P$) for the undamaged material is given as a function of the specific internal energy e and the current density ρ as follows:

$$p = p_H + \Gamma \rho_0 (e - e_H), \quad (12)$$

where, Γ is Grüneisen ratio. p_H , e_H are Hugoniot pressure and Hugoniot energy, respectively, which are given by

$$p_H = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2}, \quad e_H = \frac{p_H \eta}{2\rho_0}, \quad \eta = 1 - \frac{\rho_0}{\rho}. \quad (13)$$

The symbols ρ_0 and s in this equation are the initial density and a material constant specifying Hugoniot pressure. The symbol c_0 is a parameter given by use of the initial volumetric elastic modulus K_0 as follows:

$$c_0 = \sqrt{\frac{K_0}{\rho_0}}. \quad (14)$$

In the case of damaged material the reduction of the volumetric elastic modulus K is given as follows if we assume the postulate of strain equivalence:

$$K = K_0(1 - D). \quad (15)$$

The constitutive equation of volumetric strain for the damaged material, therefore, is expressed as follows:

$$\dot{p} = (1 - D)(\dot{p}_H - \Gamma \rho_0 \dot{e}_H) + \Gamma \rho_0 \dot{e}. \quad (16)$$

4. Field Equations

The analyses in this paper are performed by incorporating Eqs. (3), (4), (11) and (16) into the dynamic finite difference program MANJUSRI-3D. In the program, the conservation laws of mass, momentum and energy in the dynamic process are described by the following forms:

$$\dot{\rho} + (\rho u_i)_{,i} = 0, \quad (17)$$

$$\rho \dot{u}_i - \sigma_{ij,j} = 0, \quad (18)$$

$$\rho \dot{e} = \sigma_{ij} \dot{\epsilon}_{ij}, \quad (19)$$

where ρ , u_i , e are density, displacement and specific internal energy of the damaged material, respectively. The symbols, $(\)_{,i}$ and $(\)_{,\dot{t}}$ denote the differentiation with respect to coordinates and the material time derivative, respectively. The effect of body force in the conservation law (18) has been neglected, and we further assume that the duration of spall phenomenon is so short that the heat convection in the energy conservation law (19) can be ignored and all the external work changes into heat.

In the spall damage process, local temperature rise is expected as a result of significant inelastic deformation. In order to analyze this process, the temperature and the internal energy are related by the following relation:

$$\dot{e} = C_v \dot{\theta}, \quad (20)$$

where C_v is the isovolumetric specific heat.

In the present analysis, in order to prevent the numerical instability due to abrupt change of stress, an artificial viscosity⁽¹²⁾

$$q = \max[0, -\rho L \dot{\epsilon}_v (C_q L |\dot{\epsilon}_v| + C_L a)], \quad (21)$$

will be employed and will be added to the constitutive equation of the hydrostatic pressure p . The symbols C_L , C_q in Eq. (21) are the viscosity parameters related to the linear and the quadratic term of $\dot{\epsilon}_v$, respectively, and are taken as $C_L = 0.1$, $C_q = 4.0$. The symbol a , furthermore, denotes the velocity of stress wave propagation, and the characteristic length L is given by $L = \sqrt[3]{V}$ in terms of the volume of the element V .

5. Analysis of Elastic-Viscoplastic Spall Damage

5.1 Analysis of the plate impact experiment of Seaman et al.⁽²⁾ for OFHC copper

5.1.1 Method of analysis We first analyze the plate impact experiment of Seaman et al.⁽²⁾, in which a target disc 1.6 mm thick is impacted by a flyer disc 0.6 mm thick at 160 m/sec as shown in Fig. 1. The radii of the flyer and the target are large in comparison with their thickness, thus, the effect of the reflection of waves from the side of disc can be disregarded. Therefore, the radial component of the

Fig. 1 Model of plate impact

displacement u_r is taken to be 0, and one-dimensional analysis allowing only for the axial component u_x is performed. The region $x=0-1.6$ mm of the target disc of Fig. 1 is divided into 100 finite difference elements. In the analysis of two-body impact, apparent oscillation of the stress wave is often caused by the numerical procedure. To avoid this instability and also to save calculation time, the impact loading by the flyer disc is replaced by the constant loading $\sigma_x = -2900$ MPa acting at the impact surface of the target disc for the time interval $t=0.30$ μ s. The magnitude of the stress and duration were determined to simulate the incident stress wave by performing numerical analysis of the two-body impact in advance. The initial temperature of the material was specified to be 300 K. The following values of the material constants for OFHC copper are employed with reference to the literature⁽³⁾:

$$\left. \begin{aligned} C_V &= 3.924 \times 10^2 \text{ J/kg} \cdot \text{K}, \gamma = 337 \text{ sec}^{-1}, m = 2 \\ \kappa_0 &= 93.1 \text{ MPa}, q = 125 \text{ MPa}, \beta = 6.14 \\ p_D &= 1800 \text{ MPa}, q_D = 0.7, s_D = 445 \text{ MPa} \\ c_0 &= 3901 \text{ m/sec}, \Gamma = 1.51, s = 1.99 \end{aligned} \right\} \quad (22)$$

Below, the time of impact will be taken $t=0$, and the compressive stress will be taken positive.

5.1.2 Results of analysis Numerical results for the distribution of the axial stress σ_x in the axial direction of the target disc for the time interval $t=0-0.62$ μ s are shown in Fig. 2-4. The propagation speed of the elastic stress wave is 0.48 cm/ μ s, and thus the stress wave travels and is reflected through the material in 0.67 μ s.

As shown in Fig. 2, the elastic wave of approximately 300 MPa precedes the plastic wave of 2900 MPa, and the range of the elastic wave extends gradually, as the stress wave propagates. This is simply due to the fact that the speed of the elastic wave is higher than that of the plastic wave, and the distance between the fronts of these waves increases with time. The stress wave profile at $t=0.4$ μ s shows a stress of 2600 MPa at the range between $x=1.3$ mm and 1.5 mm. At this time, the front of the elastic wave

Fig. 2 Stress distribution in target disc ($t=0.1\sim0.4$ μ s)

has already been reflected from the rear surface, i.e., a free surface, and the reflected tensile stress wave has negated the compressive wave in this region. The stress wave oscillation observed at this time in the range between $x=0$ mm and 0.35 mm is attributable to the instability caused by the abrupt unloading at the impact surface.

Figure 3 shows the stress wave profiles at the times from $t=0.45$ μ s to 0.55 μ s. The compressive wave which had arrived at the rear surface of the target is reflected to the tensile wave, and cancels the arriving compressive wave, leading to a diminished magnitude of stress. The range of the compressive wave becomes narrow with time as shown in this figure. In the case of the ideal elastic wave, the value of the stress would become zero throughout the material at a certain time. In the actual material, however, both elastic and plastic waves exist and their propagation speeds differ. Thus, we have the history of stress waves as shown in Fig. 3.

The stress wave profiles for $t=0.57$ μ s through 0.62 μ s are shown in Fig. 4. Though the value of stress is nearly zero over the disc at $t=0.55$ μ s, at $t=0.57$ μ s, a large localized tensile stress is induced around $x=1.0$ mm and the region of the tensile stress spreads thereafter.

The damage distribution in the disc for times corresponding to those in Fig. 4 ($t=0.57$ μ s through $t=0.62$ μ s) is shown in Fig. 5. As observed in this figure, the damage process proceeds rapidly after the induction of tensile stress, and tends to the final

Fig. 3 Stress distribution in target disc ($t=0.45\sim0.55$ μ s)

Fig. 4 Stress distribution in target disc ($t=0.57\sim0.62$ μ s)

Fig. 5 Distribution of damage in target disc ($t = 0.57 \sim 0.62 \mu\text{s}$)

Fig. 6 Distribution of temperature in target disc ($t = 0.57 \sim 0.62 \mu\text{s}$)

ruptured state $D=1.0$ around $x=1.0$ mm at $t=0.62 \mu\text{s}$. This location corresponds to the spall surface. This result almost coincides with the experimental result of Seaman et al. The stress in this region reduced to zero at $t=0.62 \mu\text{s}$ as shown in Fig. 4.

Finally, Fig. 6 shows the changes of the temperature distribution at $t=0.57 \mu\text{s}$ to $t=0.62 \mu\text{s}$. The temperature is elevated with the damage development. At $t=0.62 \mu\text{s}$, in particular, the temperature is elevated about 45°C locally. This may be accounted for by the significant localized deformation in this region due to concentrated material damage.

5.1.3 Comparison with the experimentally measured stress history Now, we will discuss the validity of the present analysis by comparing the numerical result of the stress wave history with the corresponding available experimental result. In the impact test of copper discs, Seaman et al.⁽²⁾ placed a polymethylmethacrylate (PMMA) plate embedded with a manganin wire gauge behind the target disc as a buffer material, and measured the stress wave propagating from the target disc. Thus, we also performed an impact analysis by incorporating the PMMA buffer material as shown in Fig. 7, and calculated the stress wave history in PMMA. The thickness of PMMA is taken to be 1.6 mm, and we assumed that the PMMA is unaffected by the reflected wave at the free surface for a duration of $1.2 \mu\text{s}$. The material parameters of PMMA are taken as follows:

Fig. 7 Model of target disc and buffer material

Fig. 8 Stress history in PMMA

$$\left. \begin{aligned} \rho &= 1185 \text{ kg/m}^3, \quad G = 1.56 \text{ GPa} \\ c_0 &= 2430 \text{ m/sec}, \quad \Gamma = 2.5, \quad s = 1.0 \end{aligned} \right\} \quad (23)$$

Figure 8 shows the calculated and the measured stress-time profiles in PMMA 0.5 mm from the target back surface. Though the predictions of the arrival time of the first stress wave and that of the wave reflected at the spall surface are in good agreement with the experimental results, the numerical result for the largest stress is about 10% larger than that obtained experimentally. This trend is observed also in the analysis of Johnson⁽¹³⁾, and is probably attributable to the accuracy of the experiment or that of the constitutive modeling of PMMA. The difference between the largest stress and the succeeding minimum stress is related to the spall stress, and the minimum stress agrees with the experimental result. Regarding the value of the second stress peak, the numerical value is larger than that obtained experimentally. This may be accounted for by the fact that the energy dissipation predicted in the analysis may be smaller than that of the experiment because of the more rigid constraint condition in the analysis. The trend of stress variation in the analysis agrees with the experimental result as a whole, and thus this analysis represents the actual phenomenon qualitatively, and also quantitatively in part.

5.2 Comparison with the plate impact test of Rajendran et al.⁽¹⁴⁾ for OFHC copper

5.2.1 Method of analysis We will further analyze the experiment conducted by Rajendran et

Fig. 9 Particle velocity at back surface of target disc

al.⁽¹⁴⁾, where a flyer disc 2 mm thick impacts a target disc 9 mm thick at 185 m/sec. The material is OFHC copper similar to that described in section 5.1. The history of the back surface velocity of the target disc was measured by *velocity interferometer system for any reflector* (VISAR). The analysis was performed by a method similar to that described in subsection 5.1.1. The target disc was divided into 500 finite difference elements, and the shock loading is replaced by a constant load 300 MPa continuing for 1.0 μ s.

5.2.2 Results of analysis Though the distributions and the histories of stress, damage and temperature similar to those shown in Figs. 2–6, were calculated in the analysis, they are omitted here.

Figure 9 shows the comparison between the results of analysis and those of experiment for the particle velocity history at the back surface of the target disc. The time of the stress wave arrival and that of the stress wave rising up agree well with each other. The largest value of the particle velocity and the spall pullback velocity also show similar values for the analysis and the experiment. Regarding the history of the velocity following the secondary peak, the numerical results show larger values than those of the experiment. This may be accounted for similarly to the discrepancy shown in Fig. 8. Nevertheless, the numerical and the experimental results for the time dependent change of the phenomenon agree with each other, and the analysis predicts the actual phenomenon quantitatively.

6. Conclusions

The applicability of continuum damage mechanics to the systematic analysis of spall damage was discussed. The mechanical effect of microvoids was represented by a scalar damage variable, and its development was represented by the evolution equation of Lemaitre et al. for elastic-plastic damage. The viscoplastic constitutive equation of Perzyna was extended to the damaged material. The resulting equations were incorporated into a commercial inelastic dynamic finite difference program MANJUSRI-

3 D, and one dimensional analysis for the plate impact damage of OFHC copper was performed. The results obtained from these analyses are summarized as follows.

(1) The proposed method can predict the process of stress wave propagation, the initiation and growth of spall damage and inelastic deformation and the resulting temperature elevation.

(2) The predicted location of the spall surface coincides with that obtained experimentally.

(3) Stress history detected in the PMMA placed behind the back surface of the target disc can be predicted reasonably well by the analysis. In particular, the time dependent change of the phenomenon and the first minimum value of stress caused by the initiation of spall agree well with those obtained experimentally.

(4) Regarding the history of the particle velocity at the back surface of the target disc, numerical predictions of time dependent change of the velocity, the value of the first peak velocity and the spall pullback velocity agree with those obtained experimentally.

(5) The theoretical model developed in the present paper is considered to be a systematic method of spall damage analysis for elastic-viscoplastic materials.

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