Transient Dynamic Large-Deflection Analysis of Panel Structure under Blast Loading*

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The transient dynamic response of a panel structure subjected to blast loading was investigated, as simulated by the step function and the exponentially decaying function. Based on the Bauer methodology, and applying Galerkin's method, the von Karman nonlinear equations of plates could be transformed into Duffing-type nonlinear ordinary differential equations and then solved by adopting the Lindstedt-Poincare perturbation method. The complete analysis of square plates was carried out for all-clamped and all-simply supported edge conditions. The results of deflection history and stress resultants using one perturbation term were sufficiently correlated with results both from related literature and obtained by the finite-element method.

Key Words: Large-Deflection Analysis, Blast Loading, Galerkin's Method

1. Introduction

Panels are often used in ships, aircraft and various industrial structures. The damage inflicted by explosions can consist of direct shock-wave damage to those panel structures and mechanical shock damage to attached equipment. A fundamental and detailed understanding of shock loading and its associated structural response is required for designing effective panel structures to withstand explosions.

Large deformation occurs in structures subjected to shock loads. These structural phenomena are thus no longer adequately analyzed by linear theory since the deflection of the panel does not remain small in

comparison with the thickness. The effects of large deflections on the response of panels, based on von Karman's large-deflection plate theory, have been studied by several investigators using various techniques. Balachandra and Gopalacharyulu⁽¹⁾ considered the nonlinear problem of an isotropic clamped rectangular plate undergoing large deflections under uniform normal pressure using modified Fourier series. Approximate solutions for the nonlinear bending of thin rectangular plates with various boundary conditions have been studied by Iyengar and Naqvi⁽²⁾. A sine series solution previously used for the study of the flexural vibration of rectangular isotropic plates was presented by Dickinson⁽³⁾. Eisley⁽⁴⁾ considered a large-amplitude vibration of buckled beams and rectangular plates by applying the Galerkin method. Gajendar⁽⁵⁾ presented an elastic plate resting on an elastic foundation and undergoing large natural vibrations. Ventres and Dowell⁽⁶⁾ discussed the fluttering behavior of clamped plates exposed to either transverse pressure loading or buckled by uniform thermal expansion. Chia⁽⁷⁾ investigated the large-deflection behavior of a rectangular anisotropic plate under uniformly loaded pressure. Young⁽⁸⁾ discussed the

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frequencies and modes of vibration of thin elastic plates. A method was proposed by Little⁽⁹⁾ for analyzing the large-deflection behavior of thin rectangular orthotropic plates. In these analyses, the work stresses involved studying static or flexure free vibrations analysis. The transient dynamic response on plates has only been considered in a few papers. Bauer⁽¹⁰⁾ developed approximation solutions for a large-deflection plate under pulse excitation. Chong et al.⁽¹¹⁾ analyzed the response of panels subjected to in-plane compressive loading and transverse blast loading.

The nonlinear transient dynamic response of the rectangular plate subjected to blast loading is obtained in this work for all-clamped and all-simply supported edges. A simulation is performed using the step function and the exponentially decaying function. A set of Duffing-type nonlinear ordinary differential equations in time, which is based on Bauer's⁽¹⁰⁾ methodology, is yielded by employing Galerkin's method derived from the von Karman nonlinear equations of plates. Thus the differential equation is modified to a form for which Lighthill's extension⁽¹²⁾ of Poincaré's perturbation method can be employed as the solution of the problem. A simple analytical procedure is developed which enables the large deflection to be easily computed for any arbitrary blast shape. For effective panel design against shock threats, this procedure can offer knowledge of shock loading with the structure response.

Notation

- D : bending stiffness
- E : Young's modulus
- F : Airy's stress function
- L : differential operators
- $P(t)$: shock load
- d_{ij} : Fourier coefficients
- h : thickness of the plate
- k : exponential decay time constant
- w : deflection of the plate
- ρ : mass density
- ν : Poisson ratio
- ϕ : time function
- λ : the ratio of length and width
- ϵ : small parameter
- w_0 : circular natural frequency
- σ^m : membrane stresses
- σ^b : bending stresses
- $(\cdot)_{,i}$: partial differentiation with respect to variable i

2. Basic Theory

A thin rectangular plate of length a , width b , and thickness h is subjected to a shock load $p(t)$. The

equations of motion for an isotropic plate are expressed according to von Karman's large-deflection plate theory as

$$\nabla^4 F = E(w_{,xy}^2 - w_{,xx}w_{,yy}) \quad (1)$$

$$D\nabla^4 w + \rho h w_{,tt} - h(w_{,xx}F_{,yy} + w_{,yy}F_{,xx} - 2w_{,xy}F_{,xy}) = p(t), \quad (2)$$

where w is the deflection of the plate; ρ is mass density; $D = Eh^3/12(1-\nu^2)$ is the bending stiffness; E is Young's modulus of elasticity; ν denotes the Poisson ratio; F is Airy's stress function; and $(\cdot)_{,i}$ indicates a partial differentiation with respect to variable i .

The transverse deflection is assumed to be of the following forms for all edges simply supported and for all edges clamped-in,

$$w(x, y, t) = h\phi(t) \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \quad (3)$$

$$w(x, y, t) = h\phi(t) \cos^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b}, \quad (4)$$

respectively, where $\phi(t)$ is an unknown time function that must be determined. This approximated deflection satisfies the geometrical boundary conditions.

Substituting expressions (3) and (4) into the compatibility Eq. (1) yields

$$\nabla^4 F = \frac{\pi^4 E h^2 \phi^2}{a^2 b^2} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} a_{pq} \cos \frac{2p\pi x}{a} \cos \frac{2q\pi y}{b}. \quad (5)$$

The general solution of Eq. (5), Airy's stress function F , can then be obtained as

$$F = E h^2 \phi^2 \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} d_{pq} \cos \frac{2p\pi x}{a} \cos \frac{2q\pi y}{b}, \quad (6)$$

where coefficients a_{pq} and d_{pq} are listed in Table 1 for various boundary conditions.

A system of differential equations is next considered. The equilibrium Eq. (2) can be rewritten as

$$L(w, F) = D\nabla^4 w + \rho h w_{,tt} - h(w_{,xx}F_{,yy} + w_{,yy}F_{,xx} - 2w_{,xy}F_{,xy}) - p(t), \quad (7)$$

in which L is a differential operator.

With Eqs. (3) and (4) for w and Eq. (6) for F , however, the equilibrium Eq. (2) generally can not be exactly satisfied. Instead, the Galerkin method is applied here toward minimizing the error function, in the form of

$$\int_0^{a/2} \int_0^{b/2} L(w, F) w dx dy = 0. \quad (8)$$

Equations (3), (4) and (6) are inserted into Eq. (8). A nonlinear ordinary differential equation for the time function ϕ is consequently obtained. For the simply supported case,

$$\rho h^2 \phi_{,tt} + \frac{\pi^4 E h^4}{a^4} \left[\frac{\phi}{3(1-\nu^2)} + 0.06492 \phi^3 \right] = \frac{16}{\pi^2} p(t), \quad (9)$$

Table 1 a_{pq} and d_{pq} for various bound conditions

parameter boundary condition	a_{pq}	d_{pq}
simply supported	$a_{01}=1/2$	$d_{01}=-.01623$
	$a_{10}=1/2$	$d_{10}=-.01623$
clamped-in	$a_{01}=1/2$	$d_{01}=-.01623$
	$a_{10}=1/2$	$d_{10}=-.01623$
	$a_{11}=1$	$d_{11}=-.022686$
	$a_{02}=1/2$	$d_{02}=-.006456$
	$a_{20}=1/2$	$d_{20}=-.006456$
	$a_{12}=1/2$	$d_{12}=-.006456$
	$a_{21}=1/2$	$d_{21}=-.006456$

All other a_{pq} and d_{pq} are zero

and for the clamped-in case,

$$\rho h^2 \phi_{,tt} + \frac{16\pi^4 E h^4}{9a^4} \left[\frac{2\phi}{3(1-\nu^2)} + 0.14903\phi^3 \right] = \frac{16}{9} p(t), \quad (10)$$

Equations (9) and (10) are divided by ρh^2 . These equations can then be rewritten into the form

$$\phi_{,tt} + w_0^2 \phi + \varepsilon w_0^2 \phi^3 = P \quad (11)$$

in which w_0 is the circular natural frequency of the linearized system and ε is a small value. Equation (11) is the well-known Duffing's equation. At time $t=0$, the plate is assumed to be at rest. The time function is then

$$\phi(0)=0 \quad \phi_{,t}(0)=0. \quad (12)$$

Let

$$\phi(t) = q(t) + g(t). \quad (13)$$

Inserting Eq. (13) into Duffing's Eq. (11) yields

$$q_{,tt} + w_0^2 q + g_{,tt} + w_0^2 g + \varepsilon w_0^2 (q+g)^3 = P. \quad (14)$$

Selecting $g(t)$ satisfied the equation

$$g_{,tt} + w_0^2 g = P. \quad (15)$$

$g(t)$ is the particular response of the linearized system, as yielded by $g(t) = g_p(t)$ as the particular solution of Eq. (15) under the new initial conditions

$$q(0) = -g_p(0), \quad q_{,t}(0) = -g_{p,t}(0). \quad (16)$$

Equation (14) then reduces to

$$q_{,tt} + w_0^2 q + \varepsilon w_0^2 (q + g_p)^3 = 0. \quad (17)$$

An approximate solution of Eq. (17) can be obtained by the perturbation method, which includes

the renormalization method, Lighthill's⁽¹²⁾ method and multiple time scales. Lighthill's extension of the perturbation method is used in this work. This method is achieved by introducing a new variable τ and expanding the time function $q(t)$ into a series of powers in the small parameter ε .

The approximation $q(\tau)$ is

$$q(\tau) = -(g_p(0)\cos w_0\tau + \frac{1}{w_0}g_{p,\tau}(0)\sin w_0\tau) + \varepsilon q_1, \quad (18)$$

where

$$q_1(\tau) = \frac{X(0)}{w_0} \cos w_0\tau - \frac{Y(0)}{w_0} \sin w_0\tau + \frac{Y(\tau)}{w_0} \sin w_0\tau - \frac{X(\tau)}{w_0} \cos w_0\tau \quad (19)$$

and

$$X(\tau) = \int N(\tau) \sin w_0\tau d\tau$$

$$Y(\tau) = \int N(\tau) \cos w_0\tau d\tau. \quad (20)$$

With $N(\tau)$ being the leftover right hand side of the differential equation of q_1

The solution to Eq. (11) consequently becomes

$$\phi(t) = g(t) - (g_p(0)\cos w_0\tau + \frac{1}{w_0}g_{p,\tau}(0)\sin w_0\tau) + \varepsilon q_1. \quad (21)$$

With these results, the deflection $w(x, y, t)$ and the Airy stress function $F(x, y, t)$ can be obtained from Eqs. (3), (4) and (6). The median membrane stresses are then defined by

$$F_{,yy} = \sigma_x^m, \quad F_{,xx} = \sigma_y^m, \quad -F_{,xy} = \tau_{xy}^m. \quad (22)$$

Additionally the bending stresses are

$$\sigma_x^b = -\frac{Ez}{1-\nu^2}(w_{,xx} + \nu w_{,yy})$$

$$\sigma_y^b = -\frac{Ez}{1-\nu^2}(w_{,yy} + \nu w_{,xx})$$

$$\tau_{xy}^b = -\frac{Ez}{1-\nu^2}w_{,xy}. \quad (23)$$

3. Time Function for Specified Blast Loading

The blasting effects of an explosion are in the form of a shock wave composed of a high-pressure shock front which expands outward from the center of detonation with a pressure intensity which decays with distance. As the wave front impinges on a structure, a portion of or the whole structure would be engulfed by the shock pressure. The magnitude and distribution of the blast load on the structure are a function of the type of explosive material, weight of explosive, the location of the explosion relative to the structure, and the interaction of the shock front with the structure itself. Far long distance from the explosion source, the spherical shock fronts are planes nearly parallel to the surface of the structure. Consequently, the shock wave can be represented by a

planar wave. For the shock wave history, the pressure is characterized by an almost instantaneous rise to a peak pressure p_0 which then rapidly decays. The step function and exponentially decaying function are adopted in this work for simulating the blast load.

According to the deflection function $w(x, y, t)$, Eqs. (3) and (4) and the stress function $F(x, y, t)$ (Eq. (6)), the time function $\phi(t)$ is unknown. However, the time function $\phi(t)$ is relative to the values of w_0 , ε and P . The time function $\phi(t)$ for a specified step function and exponentially decaying function are considered as follows.

3.1 Step function

A step function for which the load was a prescribed constant value during a given time interval and zero for other times is expressed as

$$p(t) = \begin{cases} p_0 & 0 \leq t \leq t_0 \\ 0 & t > t_0 \end{cases} \quad (24)$$

It is found that

$$g_p(t) = \frac{P}{w_0^2}, g_{p,t}(t) = 0. \quad (25)$$

Substituting Eq. (25) into Eq. (18) yields

$$q(\tau) = -\frac{P}{w_0^2} \cos w_0 \tau + \frac{\varepsilon P^3}{32 w_0^6} (-80 + 65 \cos w_0 \tau + 16 \cos 2w_0 \tau - \cos 3w_0 \tau). \quad (26)$$

The time function is

$$\phi(t) = \frac{P}{w_0^2} - \frac{P}{w_0^2} \cos w_0 \tau + \frac{\varepsilon P^3}{32 w_0^6} (-80 + 65 \cos w_0 \tau + 16 \cos 2w_0 \tau - \cos 3w_0 \tau), \quad (27)$$

where

$$\tau = t \left(1 + \frac{3\varepsilon P^2}{8w_0^4} \right). \quad (28)$$

3.2 Exponentially decaying function

From the shock wave history, the shock wave at a fixed location $p(t)$ is initiated with instantaneous pressures increasing to a peak pressure p_0 in less than 10^{-7} sec, which is followed by a decay in which its initial portion is typically approximated by an exponential function,

$$p(t) = p_0 e^{-kt}, \quad (29)$$

where k is the decay time constant.

It is found that

$$g_p(t) = \frac{P_0 e^{-kt}}{k^2 + w_0^2}, g_{p,t} = \frac{-k P_0 e^{-kt}}{k^2 + w_0^2}. \quad (30)$$

Substituting Eq. (30) into Eq. (18) yields

$$q(\tau) = \frac{-P_0}{k^2 + w_0^2} \cos w_0 \tau + \frac{P_0 k}{k^2 + w_0^2} \sin w_0 \tau + \frac{\varepsilon}{H} (A + B \cos w_0 \tau + C \cos 2w_0 \tau + D \cos 3w_0 \tau + E \sin w_0 \tau + F \sin 2w_0 \tau + G \sin 3w_0 \tau), \quad (31)$$

where A to H are functions of w_0 , ε and P .

The time function $\phi(t)$ is

$$\phi(t) = \frac{P_0 e^{-kt}}{k^2 + w_0^2} - \frac{P_0}{k^2 + w_0^2} \cos w_0 \tau + \frac{P_0 k}{k^2 + w_0^2} \sin w_0 \tau + \frac{\varepsilon}{H} (A + B \cos w_0 \tau + C \cos 2w_0 \tau + D \cos 3w_0 \tau + E \sin w_0 \tau + F \sin 2w_0 \tau + G \sin 3w_0 \tau). \quad (32)$$

The deflection function $w(x, y, t)$ and the stress function $F(x, y, t)$ can then be solved.

4. Example and Verification

The transient dynamic response of a square plate, with all-simply supported or all-clamped edges, subjected to a step function or exponentially decaying function loading is investigated by the presented method and confirmed by data in related literature.

4.1 All-simply supported square plate subjected to step blast load

A simply supported square plate which is subjected to step function loading has been described in detail by Akay⁽¹³⁾. The step function is shown in Fig. 1. The material properties of square plate are as follows:

length: $a = 243.8$ cm;
thickness: $h = 0.635$ cm;
Poisson ratio: $\nu = 0.25$;
Young's modulus: $E = 70.3$ GPa;
mass density: $\rho = 2547$ kg/m³;
uniform blast load: $p(t) = 48.82$ kPa.

The values w_0^2 , ε and P are obtained as

$$P = 770.62, w_0^2 = 1092, \varepsilon = 0.1$$

The time function $\phi(t)$ is given by Eq. (27) under the simply supported edge condition of

$$\phi(t) = 0.5754(0.9477 - \cos 34.21t + 0.0558 \cos 68.42t - 0.0035 \cos 102.64t).$$

With these results, the deflection function $w(x, y, t)$ and the Airy stress function $F(x, y, t)$ can be obtained from Eqs. (3), (4) and (6).

A similar problem of all simply-supported plates has been analyzed by Akay⁽¹³⁾ using a four-mode isoparametric quadrilateral element. Additionally, the COSMOS finite-element package is applied for large-deflection dynamic analysis for verification.

Fig. 1 All-simply supported square plate under a step function load

The central deflection time history $w(t)$ is displayed in Fig. 2 and the maximum deflection is 0.73 cm which is 1.15 times the plate thickness. The results of the Akay solution and the COSMOS solution are also compared. There is good agreement in the results. The central deflections obtained using the present method, static solution and linear solution are indicated in Fig. 3. The nonlinear effect is clear in this figure. Therefore this effect can no longer be analyzed using the linear theory for large deflection. The central bending stress and membrane stress time history in

Fig. 2 Time history of central deflection under a step function load

Fig. 3 Time history of central deflection for static, linear and present methods under a step function load

Fig. 4 Time history of central bending and membrane stress under a step function load

the x direction are displayed in Fig. 4. The bending stress in this figure is of greater practical significance than the membrane stress. The maximum central bending stress is 1.45 MPa and the maximum central membrane stress is 64 kPa at $t=0.95$ sec.

4.2 All-clamped square plate subjected to exponentially decaying blast load

An all-clamped square plate is next considered where load is due to DRES⁽¹⁴⁾ experimental results. Its exponentially decaying function is shown in Fig. 5. The material properties of the square plate are as follows:

- length: $a=508$ mm;
- thickness: $h=3.4$ mm;
- Poisson ratio: $\nu=0.3$;
- Young's modulus: $E=207$ GPa;
- mass density: $\rho=7770$ kg/m³;
- uniform blast load: $p(t)=202e^{-1640t}$ kPa.

The values w_0^2 , ε and P can be obtained as

$$P=3998066.3, w_0^2=599857.4, \varepsilon=0.203.$$

The time function $\phi(t)$ is provided by Eq. (32) under the clamped-in edge condition

$$\begin{aligned} \phi(t) = & 0.507e^{-4920t}(-0.0442 + 0.222e^{3280t} \\ & - 0.099e^{1640t}\cos 1082.92t - e^{4920t}\cos 1082.92t \\ & + 0.63e^{3280t}\cos 2165.84t + 0.361e^{4920t}\cos 3248.76t \\ & - 0.4255e^{1640t}\sin 1082.92t \\ & + 6.237e^{4920t}\sin 1082.92t \\ & - 0.9345e^{3280t}\sin 2165.84t \\ & - 0.1982e^{4920t}\sin 3248.76t). \end{aligned}$$

With these results, the deflection function $w(x, y, t)$ and Airy's stress function $F(x, y, t)$ can be obtained from Eqs. (3), (4) and (6).

Experimental results are next presented for the same problem with occurs DRES⁽¹⁴⁾ and a four-node isoparametric quadrilateral mixed element with a 12×12 mesh and $\Delta t=0.05$ msec for the COSMOS finite-element package.

The central deflection time history $w(t)$ is displayed in Fig. 6, and the maximum deflection is 9.4 mm, which is 2.77 times the thickness of the plate. The results of the DRES solution and the COSMOS

Fig. 5 All-clamped square plate under an exponentially decaying function load

Fig. 6 Time history of central deflection under an exponentially decaying function load

Fig. 7 Time history of central deflection for static, linear and present method under an exponentially decaying function load

Fig. 8 Time history of central bending and membrane stress under an exponentially decaying function load

solution are shown in the same figure for comparison. The results for the present method are observed to be close to those of the COSMOS solution since the solution of the present method is taken on the first approximation. The results can thus be approached as closely as desired by taking more approximation. The solution obtained by the present method is observed to deviate from the DRES experimental results. This difference might be attributable to three reasons.

First, the mounting test plate in the DRES test was not truly fixed, while the square plate was fully fixed. Second, the shock wave adopted here is an exponential function, which was inadequate in perfectly simulating the real shock load. Third, the analysis needs to include a damping term. The central deflection is obtained using the present method. The static solution and linear solution are shown in Fig. 7. The difference between these solutions are observed in this figure, with the nonlinear effect being obvious. The central bending stress and membrane stress time history in the x direction are shown in Fig. 8. The bending stress in this figure is found to be of greater practical significance than the membrane stress. The maximum central bending stress is 369 MPa.

5. Conclusion

A large-deflection transient dynamic analytical method based on the Galerkin method and the Lindstedt-Poincaré perturbation method was derived in this work for analyzing a square plate subjected to shock load. The step function and exponential decay function were selected here as loads for analysis.

The related parameter values w_0^2 , ϵ and P were provided according to various loads and boundary conditions. From these results, the deflection $w(x, y, t)$ and the Airy stress function $F(x, y, t)$ could be obtained. Consequently, the deflection time history and stress results of the plate were determined. This method of using only one perturbation term for describing the out-of-plane deflection would be observed to be close to the finite-element results.

The boundary condition could also possibly influence the analytical results if the mathematical boundary condition is not the same as the experimental one. Similar logic could be applied for the loading condition if the blast load cannot be perfectly simulated by the exponential function. Consequently, a certain degree of influence could be exerted on the accuracy of the predicted result.

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