Plastic Properties of Metal-Metal Composites : A Numerical Investigation*

Manabu GOTOH** and A. B. M. IDRIS***

A two-dimensional finite element model of an elastic-plastic solid (aluminum) is used to predict the plastic properties including stress-strain behaviour of aluminum composites containing up to 40 volume percent particulate reinforcements under combined loading up to 0.2 in equivalent logarithmic strain. The effects of reinforcement size, shape, contents, orientation, elastic properties and loading conditions on the overall behavior of the composite are investigated. The elastic modulus of the composites is isotropic, almost independent of the type of reinforcement, and controlled solely by the volume percentage of reinforcement present. The work hardening exponent of the composites (one of the plastic properties) is surprisingly influenced by the ratio (γ) of the elastic constants of the reinforcement and the matrix in an inverse manner. It is also affected by the volume fraction, size, shape, orientation and distribution of the reinforcement. The variation in flow stress is controlled primarily by volume fraction, type, distribution and γ . For various loading conditions, the parameters, namely, the work hardening exponent, elastic modulus and flow stress of the composites for all kinds of reinforcements, remain almost constant for a particular value of γ and volume fraction with a slight change in the values for plane strain tension. For porous solids, these parameters are affected slightly by the loading conditions. Furthermore, the degree of constitutive softening of porous solids is strongly dependent on the volume fraction and shape of voids. A comparison of properties with conventional aluminum shows that an improvement in the plastic property of a metal by combination with other metals could become an interesting subject, especially in the field of metal forming processes. For such research, the FEM model used here is a powerful tool.

Key Words : Plasticity, Composite Material, Finite Element Method, Work Hardening, Strain Softening

1. Introduction

In the area of composite elasticity, considerable work has been done, but few studies treat the elastoplastic behaviour of composite due to material nonlinearity. Among these few studies, ones on fiber-reinforced composites have drawn the primary attention.

In this paper, we focus on another class of materials — particle-reinforced plastic composites — an area which is also of vital technological importance. Improvement in the plastic properties of a metal by adding other metal(s) may become an attractive

subject, especially in the field of metal forming processes, and the optimum design of plastic composites relating to metal forming processes. The research in this area, i.e., the prediction of completely plastic properties of composites where the matrix and the inclusions both are work-hardened and highly strained (20%), has been completely dropped from the interest of investigators up to the present.

Eshelby^{(1),(2)} studied particle-reinforced elasticity by using a method for determining the stress in an elastic solid when an ellipsoidal region within it undergoes a change of form. Tanaka et al.⁽³⁾ calculated the stress and energy of a specimen with a spherical inclusion when plastic deformation occurred only in the matrix. Weng⁽⁴⁾ and Tandon & Weng⁽⁵⁾ studied particle-reinforced elasticity using Eshelby's^{(1),(2)} solution for an ellipsoidal inclusion and Mori-Tanaka's⁽⁶⁾ concept of average stress in the matrix. Other authors, such as, Chow⁽⁷⁾, Taya & Chou⁽⁸⁾ and Taya &

* Received 17th September, 1996.

** Department of Mechanical Engineering, Gifu University, 1-1 Yanagido, Gifu 501-11, Japan

*** Doctoral Graduate School, Department of Mechanical Engineering, Gifu University, 1-1 Yanagido, Gifu 501-11, Japan

Mura⁽⁹⁾ studied the inhomogeneity problem within the context of elasticity.

In the case of plasticity, the problem is easier if the matrix remains elastic and the particles deform plastically. In that case the problem can be solved directly by modifying Mori-Tanaka's original formulation to an equivalent transformation problem. Several years ago a theory of particle-reinforced plasticity was proposed by Tandon & Weng⁽¹⁰⁾ in which elastic, spherical particles are uniformly dispersed in the ductile, work hardening matrix. Using this theory, Zhao & Weng⁽¹¹⁾ have further investigated a class of inclusions. Their idea was to combine the Mori-Tanaka⁽⁶⁾ concept of mean-stress in elasticity and Hill's⁽¹²⁾ discovery of a decreasing constraint power of the matrix in polycrystal plasticity. However, these models do not take the effects of size, shape, volume fraction and distribution pattern of inclusions into consideration.

This paper deals with the numerical prediction of the overall mechanical properties of particle-reinforced two-dimensional composites in the plastic range where the matrix and the inclusion (the second metal phase) are both work-hardened. With respect to work-hardened inclusions, this model differs from others^{(10),(11)}. The average equivalent strain is high as it is raised to 0.2 logarithmically (more than 22% in nominal strain). The effects of inclusion size, shape, contents, orientation, elastic properties and loading conditions on the overall behavior of the composite have also been determined. This work initiates a new subject, in comparison to the preceding work, where only a few plastic aspects of a bi-metal composite are numerically investigated.

In addition to composites, porous materials are important from an engineering perspective. An important mechanism of ductile failure involves the nucleation, growth and coalescence of small internal voids. Such voids alter the mechanical properties of a material that can be regarded as a dilating continuum. For void nucleation, the hard second phase inclusions are always the common sites. The voids grow due to plasticity and eventually coalesce to form micro cracks or cause the propagation of existing cracks.

Previous analytical models of void growth (McClintock⁽¹³⁾; Rice & Tracey⁽¹⁴⁾; Budiansky et al.⁽¹⁵⁾) have been limited to the case of an isolated void embedded within an infinite block of material subjected to some remote deformation or stress field. In contrast, real mechanical components are non-infinite and often contain numerous irregularly spaced void-nucleating inclusions. Other studies of void interactions and finite porosity have been restricted to two-dimensional periodic arrays of holes in plates

(Needleman⁽¹⁶⁾, Tvergaard⁽¹⁷⁾) and spherical cavities within spheres (Gurson⁽¹⁸⁾) or axisymmetric rods (Andersson⁽¹⁹⁾; Bourcier et al.⁽²⁰⁾; Duva⁽²¹⁾; Koplik & Needleman⁽²²⁾). Some analytical studies (Hellan⁽²³⁾; Budiansky et al.⁽¹⁵⁾) have demonstrated a decrease in void growth rate with increased material hardening rate; however, these studies have been limited to considering rate hardening or viscous materials. In a more recent model (Worswick & Pick⁽²⁴⁾), a study of void growth and constitutive softening within a three-dimensional periodically voided material was presented. The influence of variations in void spacing, size and shape, however, are neglected.

In this work, a thorough investigation has been carried out for porous materials considering variations in void size, shape, contents and loading conditions. The distribution of voids was also changed from regular to random (irregular), making the model more real. Although the analysis is restricted to a two-dimensional case at present, in future work the three-dimensional case will be considered, which will make the model more real in terms of mechanical component. However, the simplicity of the two-dimensional model enables us to investigate systematically a wide variety of composites, generating much attractive and useful data. This is the most advantageous aspect of this model.

2. Model Description

2.1 Boundary conditions

Figure 1 depicts one of the models under investigation. It shows a small element of a composite sheet material in the plane stress state. Therefore, this model is assumed to retain rectangular shape during deformation.

Fig. 1 Initial shape of composite element with boundary condition. (A small square in an element shows that the element is the inclusion. This is also the case in Fig. 3)

2.2 Materials

The materials considered in this study are elastic-plastic, allowing incorporation of material work-hardening into the model. The properties of aluminum are used as the properties of the matrix material throughout the work for numerical illustration; its elastic and plastic constants are

$E=72$ GPa, $\sigma_0=50$ MPa, $\nu=0.3$, $n=0.2$, and for inclusion material $\sigma_0=500$ MPa, $\nu=0.3$, $n=0.01$

E is given by γ , where $\gamma=E_{\text{inclusion}}/E_{\text{matrix}}$.

The interface between the matrix and the second-phase metal is assumed not to debond during deformation.

3. Problem Formulation

For both the matrix and inclusion, the constitutive equation is assumed to follow the strain hardening in accordance with the power law given by $\sigma=c\varepsilon^n$, where, σ , ε , n , c are the stress, plastic strain, work-hardening exponent and plastic coefficient, respectively. The model is loaded bi-axially in various ways from equi-biaxial tension to bi-axial tension-compression affected by $\alpha=d\varepsilon_x/d\varepsilon_y$ when the y -axis is the major tensile axis, and $\alpha=d\varepsilon_y/d\varepsilon_x$ when the x -axis is the major tensile axis where $-1.0 \leq \alpha \leq 1.0$. Incremental calculation is applied, where, in each step of calculation, the increment of the major in-plane strain is as small as 0.001. All models are subjected to a high-level total strain of 0.2 logarithmically (more than 22% in engineering strain). For the two types of loading conditions, the variation of α is $\alpha=-1.0, -0.5, 0.0, 0.5, 1.0$. For clarity, $\alpha=d\varepsilon_x/d\varepsilon_y$ is ICASE=1, and $\alpha=d\varepsilon_y/d\varepsilon_x$ is ICASE=2. Four types of inclusions, namely, small triangular, large triangular, square and rectangular, are considered with variation in volume fraction from 1% to 40%. For determination of the effect of the elastic modulus of the inclusions, the value of γ is changed to 1.2, 1.5, 2.0, 5.0, 10.0, 0.001, where the value $\gamma=0.001$ is for the void case and the model is a porous material. The orientation of inclusions is also changed for each type of loading condition from regular to random. The analytical solution of the current problem is considered intractable; therefore, a finite element technique using the finite element code 'COMPOSIT', newly developed by one of the authors (Gotoh), is employed. A linear triangular finite element is adopted. For computation, an EWS (HP 9000-712/80) is used.

4. Evaluation of Overall Properties

4.1 Overall Young's modulus $\bar{E}$

For an arbitrary strain ratio α , the overall Young's modulus is determined at the first step of the calculation where deformation is elastic. For elastic-

ity in the plane stress state:

$$\varepsilon_x = (1/E)(\sigma_x - \nu\sigma_y), \quad \varepsilon_y = (1/E)(\sigma_y - \nu\sigma_x), \\ \varepsilon_z = (-\nu/E)(\sigma_x + \sigma_y); \quad \nu=0.3.$$

The equivalent strain $\bar{\varepsilon}$ is

$$(9/2) \bar{\varepsilon}^2 = (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 \\ = 2(1+\nu)^2(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2)/E^2 \\ = 2(1+\nu^2) \bar{\sigma}^2/E^2, \quad (1)$$

where $\bar{\sigma}$ is the equivalent stress and τ_{xy} is neglected, because the overall elasticity is almost isotropic. Thus

$$\bar{E} = 2/3 \times 1.3 \times (\bar{\sigma}/\bar{\varepsilon}) = 0.867 \times (\bar{\sigma}/\bar{\varepsilon}). \quad (2)$$

Using the overall values of $\bar{\varepsilon}$ and $\bar{\sigma}$ at the first step in the calculation, from the above equation we can calculate the overall value of $\bar{E}$ for any multiaxial loading.

4.2 Evaluation of overall quantities

As indicated previously, the model expresses an infinitesimal element subjected to biaxial straining and thus keeps its rectangular shape during deformation. The increment of deformation is prescribed by the incremental displacement of nodes on the side edges, where the step size of deformation is controlled at 0.001 in the in-plane major strain. Therefore, the overall in-plane strains $\bar{\varepsilon}_x$ and $\bar{\varepsilon}_y$ are readily determined as the summation of their increments at each calculation step. On the other hand, the overall thickness is determined as the average value of the thicknesses of all finite elements, and the current overall thickness strain $\bar{\varepsilon}_z$ is calculated as the logarithm of the ratio of the current thickness to the initial thickness. The corresponding overall equivalent strain is estimated using Eq.(1). The in-plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\tau}_{xy}$ are calculated as follows:

$$\bar{\sigma}_x = F_x/A_x, \quad \bar{\sigma}_y = F_y/A_y, \quad \bar{\tau}_{xy} = F_{xy}/A_y, \quad (3)$$

where F_x and F_y are the total nodal force in the X - and Y -directions along the sides parallel to the Y - and X -axes, respectively. The values A_x and A_y are the sectional edge areas subjected to F_x and F_y , respectively, and F_{xy} is the total nodal force in the X -direction acting along the A_y -edge. The corresponding overall equivalent stress $\bar{\sigma}$ is estimated using the following formula:

$$\bar{\sigma}^2 = \bar{\sigma}_x^2 - \bar{\sigma}_x \bar{\sigma}_y + \bar{\sigma}_y^2 + 3 \bar{\tau}_{xy}^2. \quad (4)$$

The stress-strain curve ($\bar{\sigma}$ vs. $\bar{\varepsilon}$) thus obtained is approximated by the n -th power law

$$\bar{\sigma} = \bar{c} \bar{\varepsilon}^{\bar{n}}, \quad (5)$$

where $\bar{n}$ is the overall n -value of the model. The values $\bar{n}$ and $\bar{c}$ are determined using the method of least-mean-squares, applied to the $\bar{\sigma}$ - $\bar{\varepsilon}$ curve for the range $\bar{\varepsilon}=0.001$ -0.2, using the values calculated at each increment of deformation (i.e., at the interval of $\bar{\varepsilon}=0.001$).

From the stress-strain curve (Fig. 2) of the composite, we can see that the point A (corresponding to

the yield strength) is the common point for both the plastic and the elastic region. Therefore, at point A, the plastic stress-strain relationship yields

$$\bar{\sigma}_0 = \bar{c} \bar{\epsilon}_0^{\bar{n}}, \quad (6)$$

where $\bar{\sigma}_0$ is the overall yield strength of the composite and $\bar{\epsilon}_0$ is the corresponding strain.

For the same point A, the elastic stress-strain relationship yields

$$\bar{\sigma}_0 = \bar{E} \bar{\epsilon}_0. \quad (7)$$

The combination of Eqs. (6) & (7), yields

$$\bar{\sigma}_0 = \bar{c} (\bar{\sigma}_0 / \bar{E})^{\bar{n}}. \quad (8)$$

By use of the above equation the overall yield strength $\bar{\sigma}_0$ can be easily evaluated.

5. Results and Discussion

Figure 1 shows one of the models under investigation with inclusions marked by squares where the inclusions are dispersed randomly. Figure 3 shows additional models, although for calculation many other models are considered with respect to reinforcement content, shape, size and distribution. The distribution is changed from regular to random for a particular volume fraction. For regular distribution, reinforcements are arranged with a constant interparticle

Fig. 2 A typical overall stress-strain curve for a composite showing overall yield strength ($\bar{\sigma}_0$)

spacing for each row or for every 2 or 3 rows of reinforcements, constrained by the volume fractions and shape of the reinforcements. At the same time, uniform distribution of reinforcements throughout the matrix model is maintained as far as the volume fractions and the shape of the inclusions permit. For the random case, uniform distribution of the reinforcements is done more easily than in the regular case for a particular volume fraction because the constraint due to maintaining constant interparticle spacing for the whole matrix does not play any role. In the computer programming, a random table is used to generate random numbers for the reinforcements in a random distribution. Sometimes, to have a more uniform randomness in the distribution, random numbers are generated manually.

For both ICASE=1 and ICASE=2 with various loading conditions characterized by α , the elastic and plastic properties of the composite, namely, the modulus of elasticity ($\bar{E}$), the yield strength ($\bar{\sigma}_0$), the work-hardening exponent ($\bar{n}$) and the work-hardening coefficient ($\bar{c}$), are almost invariable with α . This is one of the most interesting behaviors of composites discovered in this study. For better understanding of this fact, some typical numerical values for a particular volume fraction of reinforcements are shown in Table 1 as a reference. Only for $\alpha=0$, i.e., for the plane strain tension condition, all the parameters except modulus of elasticity have slightly different values.

Comparison among all inclusions shows that, for a particular volume fraction of reinforcements, square inclusions are most effective in strengthening the matrix. Table 2 lists some numerical values of yield strengths ($\bar{\sigma}_0$) of composites with different reinforcements. However, comparison between only triangular types shows that small ones are more effective than large ones; for a particular volume fraction of reinforcing particles of a given shape, the smaller their size, the more effective they are in hardening, because

Fig. 3 Examples of a composite model

Table 1 Typical numerical values of $\bar{E}$, $\bar{\sigma}_0$, $\bar{n}$, $\bar{C}$ for various loading conditions

		Overall Values											
		$\gamma=1.2$				$\gamma=5.0$				$\gamma=10.0$			
		$\bar{E} \times 10^2$ (GPa)	$\bar{\sigma}_0$ (MPa)	$\bar{n}$	$\bar{C} \times 10^3$ (MPa)	$\bar{E} \times 10^2$ (GPa)	$\bar{\sigma}_0$ (MPa)	$\bar{n}$	$\bar{C} \times 10^3$ (MPa)	$\bar{E} \times 10^2$ (GPa)	$\bar{\sigma}_0$ (MPa)	$\bar{n}$	$\bar{C} \times 10^3$ (MPa)
$\alpha=d\epsilon_x/d\epsilon_y$ (ICASE=1)	-1.0	0.75349	77.49	0.1651	0.2413	1.06431	87.12	0.1446	0.2435	1.25496	94.27	0.1359	0.2506
	-0.5	0.75361	75.26	0.1648	0.2350	1.06632	83.63	0.1469	0.2391	1.25601	90.97	0.1373	0.2455
	0.0	0.75354	69.97	0.1743	0.2363	1.05796	86.28	0.1339	0.2236	1.23140	96.73	0.1167	0.2228
	0.5	0.75336	77.35	0.1574	0.2285	1.04440	83.58	0.1442	0.2337	1.19789	88.60	0.1360	0.2362
	1.0	0.75326	80.48	0.1589	0.2387	1.03750	87.45	0.1422	0.2397	1.18304	91.81	0.1370	0.2449
$\alpha=d\epsilon_y/d\epsilon_x$ (ICASE=2)	-1.0	0.75349	77.12	0.1664	0.2425	1.06431	85.99	0.1480	0.2467	1.25496	92.60	0.1404	0.2549
	-0.5	0.75348	74.77	0.1670	0.2373	1.05648	81.68	0.1529	0.2443	1.23822	89.22	0.1430	0.2511
	0.0	0.75335	72.38	0.1677	0.2321	1.04231	71.48	0.1766	0.2588	1.20273	83.08	0.1542	0.2552
	0.5	0.75325	77.51	0.1583	0.2303	1.03542	82.06	0.1503	0.2400	1.18122	86.85	0.1453	0.2478

$\bar{E}$ = Young's modulus $\bar{\sigma}_0$ = initial yield strength

$\bar{n}$, $\bar{C}$: in n -th power law of work-hardening $\bar{\sigma} = C\bar{E}^n$.

Table 2 A few numerical values showing yield strengths($\bar{\sigma}_0$) of composites with various inclusions

Volume Fraction	Small triangular		Large triangular		Square		Rectangular	
	$\gamma=1.2$	$\gamma=10.0$	$\gamma=1.2$	$\gamma=10.0$	$\gamma=1.2$	$\gamma=10.0$	$\gamma=1.2$	$\gamma=10.0$
5%	57.95 (15.9%)*	60.34 (20.68%)	54.34 (8.68%)	56.78 (13.78%)	62.93 (25.86%)	66.86 (33.72%)	52.61 (5.22%)	53.13 (6.26%)
25%	76.42 (52.84%)	91.14 (82.28%)	70.13 (40.26%)	82.94 (65.88%)	90.91 (81.82%)	120.64 (141.28%)	64.93 (29.86%)	68.59 (37.18%)

* value inside the parenthesis indicates increment with respect to matrix ($\sigma_0 = 50$ MPa)

γ = Young's modulus of inclusion divided by that of matrix.

the smaller the particle size, the smaller the regions available for plastic flow bypassing the particles, since the mean distance between particles is smaller for smaller inclusions. Particle size enters in another way into the simplest continuum picture in which, corresponding to the dislocation loop picture, the particles act as barriers to prevent the matrix from slipping between weak places. The smaller the particle, the greater the increase in yield strength, until such small sizes are reached that particles are sufficiently coherent to deform plastically with the matrix.

For each inclusion and for each loading and distribution, three characteristic curves are plotted: (i) the work-hardening exponent($\bar{n}$) vs. the ratio of elastic constants(γ) (ii) the work-hardening exponent($\bar{n}$) vs. volume fractions(C) and (iii) the overall elastic modulus($\bar{E}$) vs. the ratio of elastic constants(γ). Using the principal stresses of the composite, yield curves are also drawn for equi-strains($\bar{\epsilon}$) 0.05, 0.10, 0.15, 0.20 for each type of inclusion.

In the following data, estimations on the basis of the 'Linear Law of Mixture' are also included if necessary. This law is simply expressed as

$$\bar{q} = C_{\text{mat}}q_{\text{mat}} + C_{\text{inc}}q_{\text{inc}}, \quad (9)$$

where q is an arbitrary physical quantity and $\bar{q}$ is its overall value; C denotes volume fraction. The suffixes 'mat' and 'inc' designate 'matrix' and 'inclusion', respectively.

5.1 Overall Young's modulus $\bar{E}$

The increase in average modulus of elasticity($\bar{E}$) with γ for particular volume fractions is shown in Figs. 4(a), 4(b), 4(c), and 4(d). Evaluations of these graphs show that the reinforcement content is the dominant factor in the improvement of modulus of elasticity in these composites, although the increase is not linear with reinforcement content. The effects of distribution and loading type on elastic modulus are not as prominent as in other properties of the composite. For lower values of γ for a particular distribution, the modulus tends to be isotropic with nearly equal values obtained for both ICASE=1 and ICASE=2. The modulus of the composite is also almost independent of the type of reinforcements.

Prediction of $\bar{E}$ by the linear law of mixture always gives higher values than that by FEM, irrespective of the C -value, the γ -value and the type of inclusions, although a linear relationship exists between $\bar{E}$ and γ , which is tangential at $\gamma=1$ (no inclusion) to the curve predicted by FEM.

5.2 The relation between overall $\bar{n}$ and γ

The surprising phenomenon that the elastic property of the composite affects its plastic properties is observed. Figure 5(a) shows the relationship between the overall work-hardening exponent($\bar{n}$) of the composite and the ratio of the elastic constants(γ), for small triangular inclusions of regular

(a) Small triangular inclusions(regular distribution)

(b) Large triangular inclusions(random distribution)

(c) Square inclusions(random distribution)

(d) Rectangular inclusions(random distribution)

Fig. 4 Effect of γ on modulus of elasticity of composites

(a) Small triangular inclusions(regular distribution)

(b) Large triangular inclusions(regular distribution)

(c) Square inclusions(random distribution)

(d) Rectangular inclusions(regular distribution)

Fig. 5 Effect of γ on work hardening exponent ($\bar{n}$) of composites

distribution with $\alpha = d\varepsilon_y/d\varepsilon_x$, i.e., ICASE=2. The relationship is inversely proportional to γ for larger volume fractions, although this effect is less evident for smaller volume fractions.

Figure 5(b) shows the $\bar{n}$ - γ relationship for large triangular inclusions. For regular distributions, ICASE=2 predicts a higher value of $\bar{n}$ than does ICASE=1, and for the random case, ICASE=1 predicts higher values than does ICASE=2 for volume fractions of 5% & 10%. For fractions of 25% & 40%, ICASE=2 predicts higher values.

Square inclusions (Fig. 5(c)) show the same relationship between $\bar{n}$ and γ . The effect of the distribution and loading on the $\bar{n}$ value is evident; for the same loading, the random distribution predicts higher $\bar{n}$ values than does the regular distribution. When the distributions are the same, ICASE=2 predicts higher values of $\bar{n}$. On the other hand, for the random case, ICASE=1 predicts higher values.

For rectangular inclusions the volume fractions are changed over a smaller range because of the convenience of element subdivision along the x , y axes of the specimen under investigation. These inclusions reveal the same relationship (Fig. 5(d)) between $\bar{n}$ and γ , but the effect of γ for higher volume fractions is less prominent than with other inclusions. For regular distribution, ICASE=2 predicts higher $\bar{n}$ values than does ICASE=1, but for the random case the opposite applies. For two groups of loadings, the regular distribution predicts higher $\bar{n}$ values in ICASE=1 except for volume fraction 10%.

For the void case, i.e., $\gamma=0.001$, the $\bar{n}$ value of the composite for some volume fractions is slightly lower than the matrix n value. In Fig. 5, $\bar{n}$ -values calculated using the linear law of mixture (Eq. (6)) are also plotted; these are constants with γ for a particular C -value. From the graphs in Fig. 4, for larger γ -value for other than (d) (i.e., the case of rectangular inclusions with regular distribution and void cases), the law of mixture overestimates the $\bar{n}$ -value. The discrepancy is larger for larger C -values. For smaller γ -values, the relation is not clear.

5.3 The relation between overall $\bar{n}$ and C

The variations in the work-hardening exponent ($\bar{n}$) with volume fraction (C) for particular values of γ are shown in Figs. 6(a), 6(b), 6(c), and 6(d) for all inclusions. Among all the inclusions for a particular value of γ , the $\bar{n}$ values are highest for rectangular inclusions and lowest for square ones for all volume fractions. With respect to distribution, the decrease in $\bar{n}$ values with the increase in volume fraction of reinforcements is greater for regular distributions in the case of square and small triangular inclusions, but for rectangular and large triangular ones, random

(a) Small triangular inclusions (random distribution)

(b) Large triangular inclusions (regular distribution)

(c) Square inclusions (random distribution)

(d) Rectangular inclusions (regular distribution)

Fig. 6 Effect of reinforcement content on work-hardening exponent ($\bar{n}$) of composites

distributions cause a greater decrease. For the same distribution, $\bar{n}$ values decrease more in ICASE=2 than in ICASE=1 with the exception of large triangular reinforcements in the random case.

5.4 Overall yield strength and subsequent equi-strain yield curve

The yield strength of the composite is affected most by the volume fraction C , type, and size of the reinforcements. For square inclusions the increment is highest and it is lowest for the rectangular type for a particular volume fraction and γ . Between triangular types, small ones cause a greater increase. Reinforcement content (C) is an important factor controlling the strength. The increment in the yield strength between two volume fractions increases with increasing γ . Therefore, a composite with a higher value of flow stress for a particular volume fraction is obtained by selecting an inclusion of much higher elastic modulus compared to that of the matrix.

Part of the explanation for the strength increase observed can be seen from a typical stress-strain curve for the composite (Fig. 7). The curve shows that the proportional limit stress, where the composite enters plastic flow, increases as the reinforcement content is increased for each type of inclusion. As the reinforcement content is increased, the elastic modulus increases and causes the stress-strain curve to enter plastic flow at a higher flow stress. The slope of the stress-strain curve also increases as the composite enters plastic flow, and a higher flow stress is required to reach a given plastic strain. This indicates that the strength increase is probably caused by closer packing of the reinforcement and smaller interparticle spacing in the matrix. This would cause increased interaction of dislocations with the reinforcement, resulting in increased strain-hardening.

Equi-strain yield curves of the composite are

Fig. 7 An example of the strengthening effect of reinforcement on the stress-strain behavior of a composite with 25% square inclusions

shown in Fig. 8(a), and 8(b) for various volume fractions and γ values along with yield curves of the matrix for comparison. At lower percentage of reinforcements and for random distributions, the composite as a whole tends to be isotropic, but for higher percentage and regular distributions the composite tends to be anisotropic as evident from the irregularities in the curves for two types of loading. This effect is more prominent for the yield surfaces at higher average equi-strains ($\bar{\epsilon}$). For the composite to be anisotropic, the loading conditions also play an important role. Except for $\alpha=1$ (equi-biaxial tension), other loadings cause the composite to elongate more along the major tensile axis, thereby making the composite change its initial shape with the course of plastic deformation. Up to 25% reinforcement

(a) Small triangular inclusions (regular distribution)
 $\gamma=1.2, 1.5, 2.0, 5.0, 10.0, C=1\%$

(b) Large triangular inclusions (random distribution)
 $\gamma=5.0, C=25\%$

Fig. 8 Equi-strain yield curves

(a) Small triangular inclusions(regular distribution), $\gamma=10.0$

(b) Square inclusions(random distribution), $\gamma=5.0$

(c) Large triangular inclusions(regular distribution), $\gamma=10.0$

Fig. 9 Equi-strain yield curves of composites with 40% reinforcement content
(Dashed curves are for the matrix only)

content, this phenomena are observed in all composites with various reinforcement contents. But for 40% reinforcement content and $\gamma=5.0$, and/or 10.0, it is very interesting to note from the yield curves for different composites(Figs. 9(a), 9(b), and 9(c)) that the equi-strain curves vary with strain levels from Mises' ellipse(approx.) to some other shapes with further strain. Most noticeable is the yield curve for an equivalent strain of 20% which shows strain-hardening for $\alpha=0$, i.e., plane strain tension for both loadings. This is one of the most interesting results in this study.

5.5 Materials with voids

Finally, we must consider the case of voids. The modulus of elasticity of porous materials is isotropic with nearly equal values obtained for the two types of loadings. The decrease in $\bar{E}$ value with the increase in the volume fraction of voids is not the same for

(a) Large triangular inclusions(random distribution) $\gamma=0.001$, $C=5\%$

(b) Small triangular inclusions(regular distribution) $\gamma=0.001$, $C=40\%$

Fig. 10 Constitutive softening effect on composites

voids of different shape and size. This observation leads to the conclusion that the modulus of elasticity of porous solids depends on the type and size of voids.

Constitutive softening and void growth within various materials have been studied under various loading conditions with variation in the size, shape, contents and distribution of voids. Figure 10 illustrates two examples of the equi-strain yield curves for this case. The degree of constitutive softening occurring in porous solids is function of void volume fraction and shape. Most constitutive softening occurs in the case of rectangular voids. For square voids, this effect is minimum for a particular volume fraction. With respect to the distribution of voids, the random case experiences a greater softening effect than regular distribution.

Different loading conditions play an important role in the behaviour of porous solids. For $\alpha = -1.0$, -0.5 , tension and compression occur on the specimen. During compression the porosity decreases, constituting a strengthening factor while during extension the material becomes less dense and its porosity grows, causing strain-softening. For other values of $\alpha (>0)$, only extension occurs, thereby, promoting strain-softening alone. This affects the $\bar{n}$ values and yield strength of the porous solids, as the $\bar{n}$ values and yield strengths for $\alpha = -1.0$, -0.5 are always higher than those of other values for α .

Decreasing equi-strain yield curves with increasing void volume fractions are shown in Figs. 10(a), and 10(b). Although for lower volume fractions the curves are essentially isotropic, for a higher percentage of voids, anisotropy evolves due to void growth (and/or contraction), change in void shape, and the interaction of voids with the course of plastic deformation.

6. Conclusions

1. The simple two-dimensional model used in this paper works well for systematic and wide-range investigations on plastic properties of metal-metal composites up to a high strain level of at least 0.2 in logarithmic equivalent strain.

2. Comparison among all kinds of reinforcements reveals that, for a particular volume percentage of reinforcements, square inclusions are most effective in strengthening the matrix. However, comparison between triangular types shows that small ones are more effective than large ones; i.e., for a given volume percentage of reinforcing particles of a given shape, the smaller their size, the more effective they are in hardening.

3. The overall elastic modulus is controlled primarily by the volume percentage but is almost in-

dependent of reinforcement type. Furthermore, for lower values of γ and for a particular distribution, the elastic modulus tends to be isotropic, with values nearly invariant with α (in-plane strain-ratio), where $\alpha = d\epsilon_x/d\epsilon_y$ (ICASE=1) or $\alpha = d\epsilon_y/d\epsilon_x$ (ICASE=2) with major strain axis Y or X , respectively.

4. The ratio of the elastic constants of reinforcement and matrix (γ) influences the elastic modulus ($\bar{E}$), the work-hardening exponent ($\bar{n}$), and the yield strength ($\bar{\sigma}_0$) of the composite. It is surprising that the relative elastic properties of the composite (γ) influence the plastic properties such as $\bar{n}$ and $\bar{\sigma}_0$. The value of $\bar{n}$ becomes smaller for larger $\gamma (>1)$. This effect is larger for a larger content of inclusions. A composite with a higher value of elastic modulus and yield strength for a particular volume fraction can be obtained by selecting an inclusion of much higher elastic modulus compared to that of the matrix.

The above phenomena are commonly found for all types of reinforcements, where small triangular inclusions experience the greatest of such effects, whereas the lowest are for the rectangular type.

5. The prediction of the elastic modulus and work-hardening exponent in the composites by the linear law of mixture is not reliable, because it predicts $\bar{n}$ values either higher (for triangular and square inclusions) or lower (for rectangular ones) and always predicts higher $\bar{E}$ values than those of the finite element solution. The discrepancy is larger for larger γ and larger content of inclusions.

6. For various types of loading for both ICASE=1 and 2 characterized by α , the parameters of the composites for all types of inclusions, namely, the elastic modulus ($\bar{E}$), the work-hardening exponent ($\bar{n}$), and the yield strength ($\bar{\sigma}_0$), remain almost constant for a particular value of γ and volume fraction, with slight variation in the values for $\alpha=0$, i.e., plane strain tension. For porous solids these parameters are affected slightly by the loading conditions. These are among the most interesting results obtained here.

7. For ICASE=1 and ICASE=2, the values of $\bar{n}$ and $\bar{\sigma}_0$ of the composite for a particular distribution (regular or random) are not the same, because the distributions of reinforcements are not symmetric with respect to the Cartesian axes. This criterion is common to all kinds of reinforcements but to a lesser degree to square inclusions, because square reinforcements are, by virtue of their shape, symmetric with respect to both axes.

8. Yield strength of the composites for all kinds of reinforcements is controlled by the type, content and distributions of reinforcements, because the mechanical properties of the reinforcements, except the elastic modulus, are kept constant.

9. The degree of constitutive softening occurring in porous solids is strongly dependent on the volume concentration and shape of voids. Rectangular voids cause the greatest softening effect, while this effect is least for square voids, with the same void content.

10. Investigation of yield surfaces shows that for a lower percentage and with a more random orientation of inclusions, the composites as a whole tend to be isotropic; i.e., the yield curves tend to the Mises' ellipse which is used for both the matrix and metal inclusion. For higher percentage and with a higher degree of preferred orientation of inclusions, the composites tend to be anisotropic. This effect is very evident for the yield surfaces at higher average equi-strains ($\bar{\epsilon}$). For the composites with large inclusion content (arbitrarily 40%), the yield curve is apt to elongate around $\alpha=0$ (plane strain) and to be suppressed around $\alpha=1$ (equi-biaxial tension), though the reasons for this are not clear at present. Furthermore, the shape of the yield surface for the composite is almost independent of the type of reinforcements at a given volume percentage.

References

- (1) Eshelby, J.D., The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems, *Proc. R. Soc. London, A* 241 (1957), p. 376.
- (2) Eshelby, J.D., The Elastic Field Outside an Ellipsoidal Inclusion, *Proc. R. Soc. London, A* 252 (1959), p. 561.
- (3) Tanaka, K., Mori, T. and Nakashima, T., Cavity Formation at the Interface of a Spherical Inclusion in a Plastically Deformed Matrix, *Phil. Mag.*, Vol. 21, No. 170 (1970), p. 267.
- (4) Weng, G.J., Some Elastic Properties of Reinforced Solids, With Special Reference to Isotropic Ones Containing Spherical Inclusions, *International J. of Engg. Science*, Vol. 22 (1984), p. 845.
- (5) Tandon, G.P. and Weng, G.J., Stress Distribution in and Around Spheroidal Inclusions and Voids at Finite Concentration, *ASME Journal of App. Mech.*, Vol. 53, No. 3 (1986), p. 511.
- (6) Tanaka, K., Wakashima, K. and Mori, T., Plastic Deformation Anisotropy and Work-Hardening of Composite Materials, *Journal of the Mechanics and Physics of Solids*, Vol. 21 (1973), p. 207.
- (7) Chow, T.S., Effect of Particles Shape at Finite Concentration on the Elastic Moduli of Filled Polymers, *Journal of Polymer Science: Polymer Physics Edition*, Vol. 16 (1978), p. 959.
- (8) Taya, M. and Chou, T.W., On Two Kinds of Ellipsoidal Inhomogeneities in an Infinite Elastic Body: An Application to a Hybrid Composite, *International Journal of Solids and Structures*, Vol. 17, No. 6 ((1981), p. 553.
- (9) Taya, M. and Mura, T., On Stiffness and Strength of an Aligned Short-Fiber Reinforced Composite Containing Fiber-End Cracks Under Uniaxial Applied Stress, *ASME Journal of App. Mech.*, Vol. 43, No. 2 (1981), p. 361.
- (10) Tandon, J.P. and Weng, G.J., A Theory of Particle-Reinforced Plasticity, *ASME Journal of App. Mech.*, Vol. 55, No. 1 (1988), p. 126.
- (11) Zhao, Y.H. and Weng, G.J., Theory of Plasticity for a Class of Inclusions and Fiber-Reinforced Composites, *Micromechanics and Inhomogeneity, The Toshio Mura 65th Anniversary Volume* (edited by Weng, G.J., Taya, M. and Abe, H.), (1990), p. 599, Springer-Verlag, New York.
- (12) Hill, R., Continuum Micro-mechanics of Elastoplastic Polycrystals, *J. Mech. Phys. Solids*, Vol. 13 (1965), p. 89.
- (13) McClintock, F.A., A Criterion for Ductile Fracture by the Growth of Holes, *ASME Journal of App. Mech.*, Vol. 35, No. 2 (1968), p. 363.
- (14) Rice, J.R. and Tracey, D.M., On the Ductile Enlargement of Voids in Triaxial Stress Fields, *J. Mech. Phys. Solids*, Vol. 17, No. 3 (1969), p. 201.
- (15) Budiansky, B., Hutchinson, J.W., and Slutsky, S., *Mechanics of Solids, The Rodney Hill 60th Anniversary Volume* (edited by Hopkins, H.G. and Sewell, M.J.), (1982), p. 13, Pergamon Press, Oxford.
- (16) Needleman, A., Void Growth in an Elastic Plastic Medium, *J. App. Mech.*, Vol. 39, No. 4 (1972), p. 964.
- (17) Tvergaard, V., Influence of Voids on Shear Band Instabilities Under Plane Strain Conditions, *Int. J. Fracture*, Vol. 17, No. 4 (1981), p. 389.
- (18) Gurson, A.L., Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I-Yield Criteria and Flow Rules for Porous Ductile Media., *Engng. Mater. Tech.*, Vol. 99, No. 1 (1977), p. 2.
- (19) Andersson, H., Analysis of a Model for Void Growth and Coalescence Ahead of a Moving Crack Tip, *J. Mech. Phys. Solids*, Vol. 25, No. 3 (1977), p. 217.
- (20) Bourcier, R.J., Koss, D.A., Smelser, R.E., and Richmond, O., The Influence of Porosity on the Deformation and Fracture of Alloys, *Acta Metall.*, Vol. 3, No. 12 (1986), p. 2443.
- (21) Duva, J.M., A Constitutive Description of Non-linear Materials Containing Voids, *Mech. Mater.*, Vol. 5, No. 2 (1986), p. 137.
- (22) Koplik, J. and Needleman, A., Void Growth and Coalescence in Porous Plastic Solids, *Int. J. Solids. Struct.*, Vol. 11, No. 8 (1988), p. 835.
- (23) Hellan, K., An Approximate Study of Void Expansion by Ductility or Creep, *Int. J. Mech. Sci.*, Vol. 17, No. 5 (1975), p. 369.
- (24) Worswick, M.J. and Pick, R.J., Void Growth and Constitutive Softening in a periodically Voided Solid, *J. Mech. Phys. Solids*, Vol. 38, No. 5 (1990), p. 601.