Assessment of J -Integral for Three-Dimensional Surface Crack at Notch Root*

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The contour J -integral values were calculated using the three-dimensional elastic-plastic finite element method for semielliptical surface cracks originated at notch roots under tensile loading. Then these were compared with the J -integral values calculated using the conventional equation we proposed in earlier papers, which we refer to as J hereafter. The difference between the two was quite small at the deepest point A in the semielliptical surface crack, but was fairly large at the edge of a crack on the specimen surface, C. The conventional equation was modified by considering stress/strain gradients developed in the cross section at the notch root. The J -integral value calculated using the proposed equation, J' was in quite good agreement with the contour J -integral value at both points, A and C in a semielliptical surface crack.

The difference between the ratio J/J' and 1.0 increased with increase in crack depth, depending upon the aspect ratio of the surface crack, magnitude of the applied stress and notch root radius. The parameter R_w was proposed, so that J/J' could be expressed by a unique curve as a function of R_w over quite wide ranges of notch root radius, surface-crack shape and magnitude of stress. The application limit of the simple J -integral estimation proposed earlier was assessed using this curve. The crack growth rates obtained from fatigue tests of notched specimen were successfully plotted against J -integral range calculated using the modified conventional equation.

Key Words: J -integral, Three-Dimensional Surface Crack, Notch Root, F. E. M. Analysis, Fatigue Crack Growth

1. Introduction

J -integral has been recognized as a useful fracture mechanics parameter for assessing elastic-plastic fatigue crack growth. Recently, the research of J -integral has been performed for not only the two-dimensional crack but also three-dimensional crack based on finite element analysis⁽¹⁾⁻⁽¹⁰⁾.

In general, most of machine components have stress raiser such as a notch where cracks initiate under cyclic loading. Since the complex stress/strain field develops around notch root, the assessment of growth of three-dimensional surface crack initiated there is accompanied with great difficulties. In the earlier papers, one of the authors proposed the conventional equation^{(11),(12)} for calculating the cyclic J -integral range ΔJ for semielliptical surface crack existing at dully notch root, and showed that this successfully expressed growth of the relatively short

surface fatigue crack in the circumferentially notched specimen. Since stress/strain gradient is not considered in this equation, however, it would not be valid for a long crack at mild notch root and for a crack at sharp notch root.

In the present study, the contour J -integral value was calculated for semielliptical surface crack originated at notch root under tensile loading, where three-dimensional elastic-plastic finite element method was employed. Then it was compared with the J -integral value calculated through the conventional equation proposed in the earlier papers. Furthermore, the conventional equation was modified to be applicable to large crack generated at mild notch root and to crack at sharp notch root, considering the stress/strain gradient developed around notch root.

2. Analysis

2.1 Models for analysis

Figures 1(a) and (b) show the shape and dimensions of notched specimen with the notch root radii $R = 3$ and 1 mm employed for this analysis. The elastic stress concentration factors of these specimens are

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Fig. 1 Shape and dimension of notched specimens

2.04 and 2.95, respectively. Depth and aspect ratio of the semielliptical surface crack were in the ranges of $a=0.5$ to 1.0 mm and $a/c=0.5$ to 1.0 in the present analysis, where the axial loading was applied.

2.2 Contour J -integral

The three-dimensional elastic-plastic finite element method was used for the calculation of contour J -integral. The cyclic stress-strain response obtained from 0.35%C medium carbon steel was employed as the basic material property in the F. E. M.; the strain hardening exponent and the cyclic yield stress were $n=0.225$ and $\sigma_{cy}=198$ MPa. In addition, Young's modulus and Poisson's ratio of this material were $E=206$ GPa and $\nu=0.303$.

The mesh division for the F. E. M. calculation of the J -integral values was conducted considering the symmetry of the specimen; the mesh division of the notched specimen with $R=3$ mm was shown in Fig. 2(a)-(c), where 20-nodal isoparametric elements are presented. The number of elements and nodal points were 294 and 1662 for both the notched specimens. The contour J -integral values at the bottom A

Fig. 2 Mesh division for F. E. M. analysis in contour J -integral calculation

and the edge on the specimen surface C of the semielliptical surface crack were calculated along the paths shown in Fig. 2(c), using the following equation⁽¹³⁾,

$$J = \int_{\Gamma} \left(W dy - \sigma_{ij} n_j \frac{\partial U_i}{\partial x} d\Gamma \right) - \int_A \frac{\partial(\sigma_{iz} \partial U_i / \partial x)}{\partial z} dA, \quad (1)$$

where Γ is contour surrounding the crack tip, n_j is the outward unit normal to Γ , σ_{ij} and U_i are the stress

tensor and displacement vector, and W is the strain energy density given by

$$W = \int^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}.$$

2.3 Conventional equation for calculating J -integral for three-dimensional surface crack

One of the authors proposed the conventional equation for estimating J -integral values at the points A and C in the semielliptical surface crack in his earlier papers^{(11),(12)}. The J -integral values at the points A and C, J_A and J_C are given by the equations,

$$J_A = 2\pi F_A^2 (W_{eA} + f_A W_{pA})_C \quad (2.a)$$

and

$$J_C = 2\pi F_C^2 (W_{eC} + f_C W_{pC})_C, \quad (2.b)$$

where F_A and F_C are the geometrical correction factors of the stress intensity factors at the points A and C. These values were shown in Table 1. The elastic and plastic strain energy densities at the points A and C, W_{eA} , W_{eC} , W_{pA} and W_{pC} were determined from the stress-strain response calculated at the corresponding points in the cross-section at the notch root containing no crack. The number of elements and nodal points were 207 and 1 174 for the notched specimen with $R=3$ mm, and were 213 and 1 193 for the notched specimen with $R=1$ mm. The f_A and f_C are expressed as function of depth and aspect ratio of the crack, and strain hardening exponent, which correspond to the points A and C, respectively.

3. Results and Discussions

3.1 Stress/Strain distribution in cross-section at notch root

Figure 3(a) shows the distributions of the axial stress σ_z and axial total strain ϵ_z^t in the cross-section at notch root of the notched specimen with $R=3$ mm in which no crack exists. According to this figure, the gradient of distribution curve of σ_z in the vicinity of the notch root decreases as the applied stress level increases. Such a phenomenon is induced by multi-axial stress developed greatly in the vicinity of the notch root at high stress level. Meanwhile, the gradient of the distribution curve of ϵ_z^t increases with increase in the applied stress. Such a tendency is much more enhanced in the notched specimen with $R=1$ mm than the notched specimen with $R=3$ mm. Figure 3(b) shows the distribution of the elastic and plastic strain energy densities, W_e and W_p in the cross-section at the notch root. From making comparison between Fig. 3(a) and 3(b), it is found that W_e and W_p show the similar configuration to σ_z and ϵ_z^t in the distribution and that the plastic strain energy density becomes greatly large beyond the elastic strain energy density at high applied stress. Accordingly, the J -integral value calculated by Eq. (2) are significantly governed

Table 1 F values in Eq. (2) for several surface cracks

Crack Depth a	Aspect Ratio a/c	F		
		0.5	0.75	1.0
0.5 (mm)	F_A	0.5861	0.6170	0.6174
	F_C	0.4731	0.6230	0.7163
0.75	F_A	0.5861	0.6176	0.6196
	F_C	0.4790	0.6213	0.7204
1.0	F_A	0.5863	0.6179	0.6261
	F_C	0.4860	0.6206	0.7148

(a) Axial stress and total strain

(b) Elastic and plastic strain energy densities

Fig. 3 Distribution of stress and strain, and strain energy density in cross section at notch root of $R=3$ mm

by the plastic strain energy density at high stress level.

3.2 Comparison between J -integral values

J -integral values at the points A and C in the semielliptical surface crack were calculated through the contour J -integral and the conventional equations (2.a) and (2.b). The J -integral values for the surface crack of the crack depth $a=1.0$ mm and the aspect ratio $a/c=1.0$ generated at the notch root of $R=3$ mm and 1 mm are plotted against nominal stress in Figs. 4(a) and (b), where the average of the J -integral values calculated along three paths shown in

(a) Notched specimen with $R=3$ mm

(b) Notched specimen with $R=1$ mm

Fig. 4 Comparison of J -integral values estimated from contour J -integral method and Eq. (2)

Fig. 2(c) is presented for contour J -integral. According to the figures, the J -integral values calculated by the conventional equation were in good agreement with the contour J -integral values at the point A in the notched specimen with $R=3$ mm, and the former is slightly smaller than the latter in the notched specimen with $R=1$ mm. Meanwhile, the difference between the two is fairly large at the point C in both the specimens.

The conventional equation (2) was originally proposed for semielliptical surface crack generated in smooth specimen under the tensile loading. Therefore, Eq. (2) is derived on the basis of the stress/strain distribution schematically shown by the dashed lines in Fig. 5(a) and 5(b), which are uniform all over the cross-section at the notch root. Meanwhile, the stress and strain actually vary as represented by the solid lines, depending on the location in the cross-

(a) Point A

(b) Point C

Fig. 5 Schema for expressing stress and strain distribution around notch root

section at the notch root. The stress/strain value described by the dashed line, which is introduced into Eq. 2(a), is smaller than the one described by the solid line in the region between the points E and F, and is larger in the region between the points O and E. In the calculation of the J -integral value based on the uniformly distributed stress/strain value at the point A, the overestimate of stress/strain in the region from the points O to E is counterbalanced by underestimate in the region from the points E to F. This is why the J -integral values J_A calculated through conventional equation differs little from the contour J -integral value. On the other hand, since the stress/strain distribution shown by the dashed line in Fig. 5(b) is larger than the actual stress in the whole region, the J -integral value J_C calculated on the basis of the uniformly distributed stress/strain through the conventional equation at the point C becomes larger than the contour J -integral value. Then the authors derived the new conventional equations from modifying the conventional equation (2) by taking into account the stress/strain gradient generated around the notch root as follows.

They are

$$\left. \begin{aligned} J_{TA} &= 2\pi F_{TA}^2 (W_{eA} + f_A W_{pA}) C \\ J_{BA} &= 2\pi F_{BA}^2 \{ (W_{eC} + f_A W_{pC}) - (W_{eA} + f_A W_{pA}) \} C \\ J'_A &= J_{TA} + J_{BA} \end{aligned} \right\} \quad (3.a)$$

and

$$\left. \begin{aligned} J_{TC} &= 2\pi F_{TC}^2 (W_{eA} + f_C W_{pA}) C \\ J_{BC} &= 2\pi F_{BC}^2 \{ (W_{eC} + f_C W_{pC}) - (W_{eA} + f_C W_{pA}) \} C \\ J'_C &= J_{TC} + J_{BC} \end{aligned} \right\} \quad (3.b)$$

Table 2 F_B values in Eq. (3) for several surface cracks

Crack Depth a	Aspect Ratio a/c	F_B	0.5	0.75	1.0
0.5 (mm)	F_{BA}		0.1737	0.2023	0.2109
	F_{BC}		0.3300	0.4530	0.5156
0.75	F_{BA}		0.1659	0.1892	0.1972
	F_{BC}		0.3432	0.4398	0.5141
1.0	F_{BA}		0.1664	0.1842	0.1912
	F_{BC}		0.3514	0.4321	0.4974

(a) Notched specimen with $R=3$ mm

(b) Notched specimen with $R=1$ mm

Fig. 6 Comparison of J -integral values estimated from contour J -integral method and Eq. (3)

at the points A and C in the semielliptical surface crack generated at notch root, respectively. In the above equations, F_{TA} and F_{TC} are equal to F_A and F_C in Eqs. (2.a) and (2.b), and F_{BA} and F_{BC} are expressed by the following equations,

$$F_{BA}^2 = \frac{J_A E - \pi c F_{TA}^2 \sigma_A^2}{\pi c (\sigma_C^2 - \sigma_A^2)} \quad (4.a)$$

and

$$F_{BC}^2 = \frac{J_C E - \pi c F_{TC}^2 \sigma_A^2}{\pi c (\sigma_C^2 - \sigma_A^2)}, \quad (4.b)$$

where σ_A and σ_C are the axial stresses at the points A and C in the cross-section at the notch root containing no crack, which are calculated by elastic F.E.M. Their values are shown in Table 2. The J -integral values estimated through Eq. (3) are plotted with the contour J -integral values against nominal stress in Figs. 6(a) and (b). These figures show that the former is in quite good agreement with the latter at both the points A and C in the notched specimens with $R=3$ and 1 mm.

3.3 Effect of stress/strain gradient on J -integral value

To express the effect of the stress/strain gradient generated around the notch root on J -integral value, we calculated the ratio of the J -integral value calculated through Eqs. (2.a) and (2.b), which we refer to as J hereafter, to the one calculated through Eqs. (3.a) and (3.b), which we refer to as J' . Figures 7(a) and (b) show the variation of J/J' at the points A and C with increase in crack depth in the notched specimen with $R=3$ mm. The separation in the J/J' values from 1.0 expands monotonously as crack depth increases. This tendency is more distinguished in the notched specimen with $R=1$ mm than in the one with $R=3$ mm. Furthermore, it is found that the value of J/J' depends on the aspect ratio of crack and applied stress level. Thus the effect of stress/strain gradient developed around notch root on J -integral value is fairly complicated.

We derived J_A/J'_A and J_C/J'_C from Eqs. (2) and (3) as follows.

$$\frac{J_A}{J'_A} = \frac{1}{1 + (F_{BA}/F_{TA})^2 \{1/(1-R_w) - 1\}} \quad (5.a)$$

and

$$\frac{J_C}{J'_C} = \frac{1}{1 + \{(F_{BC}/F_{TC})^2 - 1\} R_w}, \quad (5.b)$$

where

$$R_w = 1 - \frac{W_{eA} + f_{A,C} W_{pA}}{W_{eC} + f_{A,C} W_{pC}}.$$

Table 3 shows the calculated values of F_{BA}/F_{TA} and

F_{BC}/F_{TC} . This shows that these values are little affected by the shape of the surface crack. Then we derived following equations, assuming F_{BA}/F_{TA} and F_{BC}/F_{TC} as constants.

$$\frac{J_A}{J'_A} = \frac{1}{1 + C_1 \{1/(1 - R_W) - 1\}} \quad (6.a)$$

Table 3 F_B/F_T values in Eq. (5) for several surface cracks

Crack Depth a mm	Aspect Ratio a/c F_{BA}/F_{TA}	0.5	0.75	1.0
0.5	F_{BA}/F_{TA}	0.2964	0.3278	0.3415
	F_{BC}/F_{TC}	0.7013	0.7271	0.7199
0.75	F_{BA}/F_{TA}	0.2830	0.3063	0.3183
	F_{BC}/F_{TC}	0.7164	0.7078	0.7136
1.0	F_{BA}/F_{TA}	0.2839	0.3063	0.3080
	F_{BC}/F_{TC}	0.7230	0.6962	0.6959

Fig. 7 Variations of J/J' with increase in crack depth in notched specimen with $R=3$ mm

and

$$\frac{J_C}{J'_C} = \frac{1}{1 + C_2 R_W}, \quad (6.b)$$

where C_1 and C_2 are the constants. Thus J_A/J'_A and J_C/J'_C are expressed by a function of only one parameter R_W .

A parameter R_W is a term for assessing the effect of the stress/strain gradients on J -integral. According to these equations, J/J' values are 1.0 at $R_W=0$ and are separated from 1.0 with increase in R_W . The values of J_A/J'_A and J_C/J'_C are plotted against R_W in Figs. 8(a) and (b). The solid lines were determined from the least square's regression analysis of J_A/J'_A vs. R_W and J_C/J'_C vs. R_W plots in the elastic condition, where the plastic terms included in the related expressions were neglected. We obtained the constants of $C_1 = 0.0926$ and $C_2 = -0.496$ in Eqs. 6(a) and (b) from such a elastic calculations. The relationships between

Fig. 8 Relationships between J/J' and R_W

J_A/J'_A and R_w , and J_c/J'_c and R_w were calculated for the surface cracks in the ranges of depth from 0.5 to 1.0 and the aspect ratio from 0.5 to 1.0 in the nominal stress ranges from 0 to 600 MPa, and these were shown by open and solid circles for the notched specimen with $R=3$ and 1 mm in the figures. Almost all the circles fall on the solid lines in the figures, evidencing the effectiveness of the proposed parameter R_w for evaluating J -integral in the notched components. The effect of the stress/strain gradients developed around the notch root on the J -integral value and the applicability of the conventional equations (2.a) and (2.b) could be estimated using $J/J' - R_w$ relationships shown in Figs. 8(a) and (b).

3.4 Application of conventional equation to analysis of fatigue crack growth rate

The newly proposed conventional equation was applied to the analysis of three dimensional surface crack growth at notch root.

Medium carbon steel of 0.35 wt. percent carbon content was machined into the shape and dimensions shown in Fig. 9. Push-pull fatigue tests were performed under axial load-controlled condition, where the stress ratio, nominal stress range and cyclic frequency were $R_\sigma = -1$, $\Delta\sigma_n = 300, 450$ and 600 MPa, and $f = 0.1$ to 5 Hz, respectively. The crack opening loading point was determined from a load-strain hysteresis loop in which load and strain were led from load cell and strain gages affixed at the positions just above the crack surface. The respective strain energy densities calculated at the notch root were substituted for the corresponding terms of W_{eA} , W_{eC} , W_{pA} and W_{pC} in Eq. (3), and then ΔJ was estimated.

Crack growth rates at the points A and C, da/dN and dc/dN in the notched specimen with $R=3$ mm are plotted against ΔJ in Figs. 10(a) and (b), respectively, where crack growth rate was measured for the crack of the size ranged from $2c=500$ to 3000 μm . The da/dN vs. ΔJ curves for the long through-thickness crack⁽¹⁴⁾ are also given by the solid line in these figures, where ΔJ values were calculated through the

Fig. 9 Shape and dimension of fatigue test specimen

conventional equation proposed by Dowling⁽¹⁵⁾. At the point A, the difference between the ΔJ values calculated through Eqs. (2) and (3) for a given crack growth rate is quite small, as shown in Fig. 10(a). The da/dN vs. ΔJ plots estimated through Eq. (3) fall nearer to the solid line than those through Eq. (2) at all stress levels. Meanwhile, at the point C, ΔJ values calculated through Eq. (3) are quite small compared with those through Eq. (2), and the difference between the two becomes larger in the larger ΔJ region at each stress level. As the result, the gradient of $\log (dc/dN) - \log \Delta J$ curve is slightly steeper in the one evaluated through Eq. (3) than in the one through Eq. (2), and the former comes closer to the $da/dN - \Delta J$ curve for long through-thickness crack than the latter. Thus it is found that the modified conventional equation gives the better estimation of J -integral for the surface fatigue crack generated at notch root than the conventional equation presented in

(a) Point A

(b) Point C

Fig. 10 Crack growth rate plotted against J -integral range in notched specimen with $R=3$ mm

the earlier paper.

4. Conclusions

The conventional equations for estimating J -integral for relatively large semielliptical surface crack generated at notch root was proposed. The results were concluded as follows.

(1) The conventional equation proposed in the earlier paper gave a good estimation of J -integral at the deepest point A in the semielliptical surface crack, which is in good agreement with the contour J -integral. Meanwhile, at the edge of a crack on the specimen surface C, the difference between the two was fairly large. Then the conventional equation was modified by considering stress/strain gradients developed around the notch root. The J -integral estimated from the modified conventional equation was in quite good agreement with contour J -integral values at both the points A and C.

(2) A parameter $R_w = 1 - (W_{eA} + f_{A,C} W_{pA}) / (W_{eC} + f_{A,C} W_{pC})$ was proposed to express the effect of the stress/strain gradient developed around notch root on J -integral value which depends on the depth and the aspect ratio of crack, the applied stress level and the notch root radius. This effect and the applicability of the conventional equations (2.a) and (2.b) could be estimated using $J/J' - R_w$ relationships shown in Fig. 8(a) and (b).

(3) The fatigue crack growth rates of semielliptical surface fatigue crack generated at the notch root with $R=3$ mm were successfully plotted against cyclic J -integral range ΔJ calculated by the modified conventional equation. It was found that the newly proposed conventional equation was useful in calculating cyclic J -integral range for semielliptical surface fatigue crack of the widely ranged size existing at notch root ranged widely in its root radius.

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