Evaluation of Creep Compliance of Carbon-Fiber-Reinforced Composites by Homogenization Theory*

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Effective creep compliance of carbon-fiber-reinforced composites is evaluated for applications of composites at elevated temperatures. A homogenization theory with two-scale asymptotic expansion in the Laplace domain is used to solve viscoelastic problems of composites. Effective constitutive equations and microscopic disturbed displacements are derived from the homogenization theory. A hexagonal array of fibers is employed for the microstructure of the composite and a hexagonal unit cell is placed in the microscopic field to represent a boundary value problem. In numerical calculations, a carbon-fiber-reinforced composite with thermoplastic matrix is considered at the glass transition temperature of the matrix. The matrix is viscoelastic and is represented using the generalized Maxwell model at the glass transition temperature, and fibers are considered as a transversely isotropic elastic medium. The effective creep compliance of the composite is determined by numerically solving a set of equations for macro- and microscopic fields.

Key Words: Composite Material, Viscoelasticity, Numerical Analysis, Boundary Element Method, Creep Compliance, Homogenization Theory

1. Introduction

Carbon-fiber-reinforced composites with thermoplastic matrix have received attention as composites with high toughness. Since the thermoplastic matrix has been improving in heat resistance, the composites are possible to use for applications at elevated temperatures. It is important to study time-dependent behavior of the composites in high-temperature environment.

Studies on linear and nonlinear viscoelastic properties of composites have been made with cylinder model⁽¹⁾, self-consistent model⁽²⁾ and finite element model⁽³⁾. Theoretical study applying a homogenization theory is also presented⁽⁴⁾. The homogenization theory has been developed to evaluate properties of composites in detail including microscopic behavior, and effects of interfacial properties and nonlinearity of matrix on the mechanical behavior of composites

have been evaluated by the method⁽⁵⁾⁻⁽⁸⁾. As the method has high compatibility for use with a finite element method on microscopic analysis, it is applied to the preprocessing and postprocessing for the finite element method⁽⁹⁾.

In this study, viscoelastic analysis of a carbon-fiber-reinforced composite is made by using the homogenization theory in Laplace domain to evaluate creep compliance of the composite. Response for macroscopic stress to macroscopic strain is generally derived from the homogenization theory⁽¹⁰⁾ with asymptotic expansion of displacement. However, it was difficult to obtain the response for the macroscopic strain to constant macroscopic stress for determination of the creep compliance. In view of integral equations for disturbed displacement, it is presented in the study that the creep compliance of the composite is obtained by solving the macro- and microscopic fields at the same time.

A carbon-fiber composite with thermoplastic matrix is treated in numerical calculation, where master curve of the matrix at the glass transition temperature is identified by generalized Maxwell model. The creep compliance of the composite is

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determined.

2. Homogenization Theory for Viscoelasticity

Consider a unidirectional carbon-fiber composite with periodic array of fibers to apply a homogenization theory. As the composite is usually supposed to be transversely isotropic, hexagonal array of fibers is chosen. Regular hexagonal unit cells are made to represent microstructure of the composite. The model of the composite is shown in Fig. 1 with two coordinate systems. Large-scale coordinate $\mathbf{x}=(x_1, x_2, x_3)$ with x_3 axis for fiber direction and small-scale coordinate $\mathbf{y}=(y_1, y_2, y_3)$ whose origin is located at the center of the fiber are employed in the analysis for homogenization. The size of the composite is supposed to be sufficiently large in comparison with the size of the unit cell.

When the composite is regarded as an inhomogeneous viscoelastic medium where fibers are also extended to be viscoelastic, the stress-strain relation of the composite may be expressed in $\mathbf{x}$ and $\mathbf{y}$ coordinate systems as follows:

$$\sigma_{ij}(\mathbf{x}, \mathbf{y}, t) = \int_0^t C_{ijkl}(\mathbf{y}, t-\tau) \frac{\partial \varepsilon_{kl}(\mathbf{x}, \mathbf{y}, \tau)}{\partial \tau} d\tau \quad (1)$$

where σ_{ij} are the stress components, ε_{ij} are the strain components, C_{ijkl} are the components of relaxation modulus, and t is the time. Inversely, the strain-stress relation is

$$\varepsilon_{ij}(\mathbf{x}, \mathbf{y}, t) = \int_0^t J_{ijkl}(\mathbf{y}, t-\tau) \frac{\partial \sigma_{kl}(\mathbf{x}, \mathbf{y}, \tau)}{\partial \tau} d\tau \quad (2)$$

where J_{ijkl} are the components of creep compliance. As properties of the fiber and matrix are different, C_{ijkl} and J_{ijkl} are expressed as functions in $\mathbf{y}$.

In order to apply the correspondence principle to viscoelastic analysis, Laplace transform is made to the governing equation of the composite. Laplace transform of the stress-strain relation (1) is written as

$$\bar{\sigma}_{ij}(\mathbf{x}, \mathbf{y}, p) = p \bar{C}_{ijkl}(\mathbf{y}, p) \bar{\varepsilon}_{kl}(\mathbf{x}, \mathbf{y}, p) \quad (3)$$

where $\bar{\cdot}$ indicates the Laplace transform of a function, and p is a variable of the Laplace transform. Similarly, the equilibrium equation and strain-displacement relation are given as

$$\left. \begin{aligned} \frac{\partial}{\partial x_j} \bar{\sigma}_{ij}(\mathbf{x}, \mathbf{y}, p) + \bar{F}_i(\mathbf{x}, p) &= 0 \\ \bar{\varepsilon}_{ij}(\mathbf{x}, \mathbf{y}, p) &= \frac{1}{2} \left[\frac{\partial \bar{u}_i(\mathbf{x}, \mathbf{y}, p)}{\partial x_j} + \frac{\partial \bar{u}_j(\mathbf{x}, \mathbf{y}, p)}{\partial x_i} \right] \end{aligned} \right\} \quad (4)$$

where F_i are the body force components and u_i are the displacement components.

To derive the homogenized equations of the composite with inhomogeneity, the displacement with

Fig. 1 Periodic array of fibers and two coordinate systems

large-scale coordinate $\mathbf{x}$ and small-scale coordinate $\mathbf{y}$ can be expressed by asymptotic expansion with parameter λ as⁽¹⁰⁾

$$\bar{u}_i(\mathbf{x}, \mathbf{y}, p) = \bar{u}_i^{(0)}(\mathbf{x}, \mathbf{y}, p) + \lambda \bar{u}_i^{(1)}(\mathbf{x}, \mathbf{y}, p) + \lambda^2 \bar{u}_i^{(2)}(\mathbf{x}, \mathbf{y}, p) + \dots \quad (5)$$

where λ characterizes smallness of the microstructure in the composite. Relationships between large- and small-scale coordinate systems, i.e., $\mathbf{x}$ and $\mathbf{y}$, and the differential operators are given as

$$\mathbf{y} \rightarrow \frac{\mathbf{x}}{\lambda}, \quad \frac{\partial}{\partial x_i} \rightarrow \frac{\partial}{\partial x_i} + \frac{1}{\lambda} \frac{\partial}{\partial y_i} \quad (6)$$

First, by substituting Eqs. (5) and (6) into Eq. (4), λ^{-1} order term of the equilibrium equation is obtained from extension of Ref. (6) as

$$\frac{1}{\partial y_j} \{ p \bar{C}_{ijkl}(\mathbf{y}, p) [\bar{\varepsilon}_{kl}^{(1)}(\mathbf{y}, p) + \bar{\varepsilon}_{kl}^*(p)] \} = 0 \quad (7)$$

where

$$\left. \begin{aligned} \bar{\varepsilon}_{ij}^{(1)}(\mathbf{y}, p) &= \frac{1}{2} \left[\frac{\partial \bar{u}_i^{(1)}(\mathbf{y}, p)}{\partial y_j} + \frac{\partial \bar{u}_j^{(1)}(\mathbf{y}, p)}{\partial y_i} \right] \\ \bar{\varepsilon}_{ij}^*(p) &= \frac{1}{2} \left[\frac{\partial \bar{u}_i^{(0)}(\mathbf{x}, p)}{\partial x_j} + \frac{\partial \bar{u}_j^{(0)}(\mathbf{x}, p)}{\partial x_i} \right] \end{aligned} \right\} \quad (8)$$

Here the other higher terms of the displacement can be ignored. The macroscopic strain $\bar{\varepsilon}_{ij}^*(p)$ derived from the macroscopic displacement $\bar{u}_i^{(0)}$ is uniform in the unit cell. Moreover, the disturbed displacement $\bar{u}_i^{(1)}$ is expressed as a function of $\mathbf{y}$ because of the periodicity in the microstructure of the composite.

Next, by integrating the λ^0 order term for whole region of the unit cell, the macroscopic equilibrium equation is obtained as

$$\frac{\partial}{\partial x_j} \left\{ \frac{1}{V} \int_V p \bar{C}_{ijkl}(\mathbf{y}, p) [\bar{\varepsilon}_{kl}^{(1)}(\mathbf{y}, p) + \bar{\varepsilon}_{kl}^*(p)] dV(\mathbf{y}) \right\} + \bar{F}_i(p) = 0 \quad (9)$$

where V is the volume of the unit cell per unit thickness for fiber direction. As the term in the braces $\{ \}$ of above equation corresponds to macroscopic stress, the homogenized stress-strain relation of the composite is written as

$$\begin{aligned}\bar{\sigma}_{ij}^*(p) &= p \bar{C}_{ijkl}^*(p) \bar{\varepsilon}_{kl}^*(p) \\ &= \frac{1}{V} \int_V p \bar{C}_{ijkl}(\mathbf{y}, p) [\bar{\varepsilon}_{kl}^{(1)}(\mathbf{y}, p) + \bar{\varepsilon}_{kl}^*(p)] dV(\mathbf{y})\end{aligned}\quad (10)$$

where C_{ijkl}^* are the components of the effective relaxation modulus of the composite.

3. Creep Compliance of Composite

The homogenized strain-stress relation of the composite is also written as

$$\bar{\varepsilon}_{ij}^*(p) = p J_{ijkl}^*(p) \bar{\sigma}_{kl}^*(p) \quad (11)$$

where J_{ijkl}^* are the components of the effective creep compliance of the composite. The creep compliance of the composite is determined by applying constant macroscopic stress as $\bar{\sigma}_{kl}^*(p) = \sigma_{kl}^0/p$ into Eq.(11). Thus

$$\bar{J}_{ijkl}^*(p) = \frac{1}{\sigma_{kl}^0} \bar{\varepsilon}_{ij}^*(p) \quad (12)$$

However, as the macroscopic strain $\bar{\varepsilon}_{ij}^*(p)$ is not presented in Eqs.(7) and (10) explicitly, it is difficult to obtain the creep compliance directly like Eq.(12). Following method is considered in this study.

In Eq.(7) for the inhomogeneous medium, the equation is reduced by separating the unit cell region into fiber and matrix phases as follows:

$$\frac{\partial}{\partial y_j} \bar{\sigma}_{ij}^\beta(\mathbf{y}, p) = 0 \quad (13)$$

$$\bar{\sigma}_{ij}^\beta(\mathbf{y}, p) = p \bar{C}_{ijkl}^\beta(p) [\bar{\varepsilon}_{kl}^{(1)}(\mathbf{y}, p) + \bar{\varepsilon}_{kl}^*(p)] \quad (14)$$

where symbol β takes m or f , and m and f indicate the matrix and fiber phases, respectively. Equations (13) and (14) can be regarded as governing equations of a viscoelastic medium with eigenstrain terms. The solution of disturbed displacement for the boundary value problem is expressed as

$$\begin{aligned}\bar{u}_i^{\beta(1)}(\mathbf{y}, p) &= \int_{\Gamma_\beta} [\bar{U}_{ij}^\beta(\mathbf{y}, \mathbf{y}', p) \bar{t}_j^{\beta(1)}(\mathbf{y}', p) \\ &\quad - \bar{T}_{ij}^\beta(\mathbf{y}, \mathbf{y}', p) \bar{u}_i^{\beta(1)}(\mathbf{y}', p)] d\Gamma(\mathbf{y}')\end{aligned}$$

Fig. 2 Local coordinate systems and contours

$$- \int_{\Omega_\beta} \bar{S}_{ijkl}^\beta(\mathbf{y}, \mathbf{y}', p) \bar{\varepsilon}_{kl}^*(p) d\Omega(\mathbf{y}') \quad (15)$$

where Γ_β and Ω_β are integral contours on the boundary and internal regions, respectively, as shown in Fig. 2. t_i^β is the surface traction on the boundary Γ_β , and U_{ij}^β , T_{ij}^β and S_{ijkl}^β are fundamental solutions with respect to displacement, traction and stress, respectively, where an instantaneous point load is subjected to an infinite viscoelastic medium. These fundamental solutions are determined from linear elastic solution by correspondence principle.

As the strain of the composite do not change for fiber direction in case of a unidirectional fiber composite, the disturbed displacement can be solved as the generalized plane strain problem. When the matrix is isotropic viscoelastic medium, the fundamental solution $\bar{U}_{ij}^m$ for displacement in Laplace domain can be written from elastic solution by correspondence principle⁽¹¹⁾ as follows:

$$\begin{aligned}\bar{U}_{ij}^m(\mathbf{y}, \mathbf{y}', p) &= \frac{[1 + p \bar{\nu}_m(p)]}{4\pi[1 - p \bar{\nu}_m(p)] p \bar{E}_m(p)} \\ &\quad \times \left\{ \left[3 - 4p \bar{\nu}_m(p) \right] \log\left(\frac{1}{s}\right) \delta_{ij} + \frac{\partial s}{\partial y_i} \frac{\partial s}{\partial y_j} \right\}, \\ (i, j &= 1, 2)\end{aligned}\quad (16)$$

$$\bar{U}_{33}^m(\mathbf{y}, \mathbf{y}', p) = \frac{[1 + p \bar{\nu}_m(p)]}{\pi p \bar{E}_m(p)} \log\left(\frac{1}{s}\right) \quad (17)$$

where

$$s = \sqrt{(y'_1 - y_1)^2 + (y'_2 - y_2)^2}$$

and δ_{ij} is Kronecker's delta. E_m and ν_m are the time-dependent moduli corresponding to Young's modulus and Poisson's ratio, respectively. When carbon-fiber is transversely isotropic, the fundamental solutions for displacement are

$$\begin{aligned}U_{ij}^f(\mathbf{y}, \mathbf{y}') &= \frac{E_T(1 + \nu_T)^2}{4\pi E_T(E_L - \nu_{LT}^2 E_T)} \\ &\quad \times \left\{ \left[3 - \frac{4(\nu_T E_L + \nu_{LT}^2 E_T)}{E_L(1 + \nu_T)} \right] \log\left(\frac{1}{s}\right) \delta_{ij} \right. \\ &\quad \left. + \frac{\partial s}{\partial y_i} \frac{\partial s}{\partial y_j} \right\}, \quad (i, j = 1, 2)\end{aligned}\quad (18)$$

$$U_{33}^f(\mathbf{y}, \mathbf{y}') = \frac{1}{2\pi G_{LT}} \log\left(\frac{1}{s}\right) \quad (19)$$

where E_L , ν_{LT} and G_{LT} are the Young's modulus, Poisson's ratio and shear modulus for longitudinal direction of the fiber, respectively. E_T and ν_T are the Young's modulus and Poisson's ratio for transverse direction of the fiber, respectively.

Creep compliance J_{ijkl}^* is determined by solving simultaneous equations which consist of the disturbed displacement (15) and macroscopic stress (10). Since the disturbed displacement is expressed as an integral equation, the macroscopic stress is also reduced to surface integral on the boundary with principle of Gauss as follows:

$$\bar{\sigma}_{ij}^*(p) = \frac{1}{V} \left\{ \int_{\Gamma_m} \frac{1}{2} p \bar{C}_{ijkl}^m(p) [\bar{u}_k^{m(1)}(\mathbf{y}, p) n_l \right.$$

$$\begin{aligned}
& + \hat{u}_i^{m(1)}(\mathbf{y}, p) n_k] d\Gamma(\mathbf{y}) + p \hat{C}_{ijkl}^m(p) \hat{\varepsilon}_{kl}^*(p) V_m \\
& + \int_{\Gamma_f} \frac{1}{2} C_{ijkl}^f [\hat{u}_k^{f(1)}(\mathbf{y}, p) n_l + \hat{u}_l^{f(1)}(\mathbf{y}, p) n_k] \\
& \times d\Gamma(\mathbf{y}) + C_{ijkl}^f \hat{\varepsilon}_{kl}^*(p) V_f \} \quad (20)
\end{aligned}$$

where n_i are the components of normal vector on the boundary, and V_m and V_f are the volumes of matrix and fiber in the unit cell, respectively. The unknown disturbed displacement can be treated on the boundary Γ_m and Γ_f . Creep compliance of the composite is determined as response for strain ε_{ij}^* to given constant stress σ_{kl}^0 in macroscopic field, where the simultaneous equations in Laplace inverse transform are solved numerically.

4. Numerical Calculation of Solution

Creep compliance of the composite is determined by solving the boundary integral equations of fiber and matrix and macroscopic stress in Laplace inverse transform. The boundary integral equations and macroscopic stress (20) are made discrete equations including time domain in Laplace inverse transform.

The time domain is divided into M parts, and the contours Γ_m and Γ_f in the integral equations are divided into N_m and N_f parts, respectively. The discrete boundary integral equations of matrix and fiber are written as

$$\begin{aligned}
& \frac{u_j^{m(1)}(\mathbf{y}, \tau_M) - u_j^{m(1)}(\mathbf{y}, \tau_{M-1})}{2(\tau_M - \tau_{M-1})} \\
& = \sum_{r=1}^M \sum_{s=1}^{N_m} \frac{t_i^m(\mathbf{y}_s, \tau_r) - t_i^m(\mathbf{y}_s, \tau_{r-1})}{\tau_r - \tau_{r-1}} \\
& \times \int_{\tau_{r-1}}^{\tau_r} \int_{\Gamma_m^s} U_{ij}^m(\mathbf{y}, \mathbf{y}', \tau_M - \tau) d\Gamma(\mathbf{y}') d\tau \\
& - \sum_{r=1}^M \sum_{s=1}^{N_m} \frac{u_i^{m(1)}(\mathbf{y}_s, \tau_r) - u_i^{m(1)}(\mathbf{y}_s, \tau_{r-1})}{\tau_r - \tau_{r-1}} \\
& \times \int_{\tau_{r-1}}^{\tau_r} \int_{\Gamma_m^s} T_{ij}^m(\mathbf{y}, \mathbf{y}', \tau_M - \tau) d\Gamma(\mathbf{y}') d\tau \\
& - \sum_{r=1}^M \frac{\varepsilon_{kl}^*(\tau_r) - \varepsilon_{kl}^*(\tau_{r-1})}{\tau_r - \tau_{r-1}} \\
& \times \int_{\tau_{r-1}}^{\tau_r} \int_{\Omega_m} S_{jkl}^m(\mathbf{y}, \mathbf{y}', \tau_M - \tau) d\Omega(\mathbf{y}') d\tau \quad (21)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} u_j^{f(1)}(\mathbf{y}, \tau_M) = \sum_{s=1}^{N_f} t_j^f(\mathbf{y}_s, \tau_M) \int_{\Gamma_f^s} U_{ij}^f(\mathbf{y}, \mathbf{y}') d\Gamma(\mathbf{y}') \\
& - \sum_{s=1}^{N_f} u_i^{f(1)}(\mathbf{y}_s, \tau_M) \int_{\Gamma_f^s} T_{ij}^f(\mathbf{y}, \mathbf{y}') d\Gamma(\mathbf{y}') \\
& - \varepsilon_{kl}^*(\tau_M) \int_{\Omega_f} S_{jkl}^f(\mathbf{y}, \mathbf{y}') d\Omega(\mathbf{y}') \quad (22)
\end{aligned}$$

where τ_r is the r -th time in the divided times, and Γ_m^s and Γ_f^s are the s -th integral contours of matrix and fiber, respectively. Similarly, the discrete equation for Eq. (20) can be made as

$$\begin{aligned}
& \sigma_{ij}^*(\tau_M) \\
& = \frac{1}{V} \left\{ \sum_{r=1}^M \sum_{s=1}^{N_m} \frac{1}{2} \left[\frac{u_k^{m(1)}(\mathbf{y}_s, \tau_r) - u_k^{m(1)}(\mathbf{y}_s, \tau_{r-1})}{\tau_r - \tau_{r-1}} n_l \right. \right. \\
& \left. \left. + \frac{u_l^{m(1)}(\mathbf{y}_s, \tau_r) - u_l^{m(1)}(\mathbf{y}_s, \tau_{r-1})}{\tau_r - \tau_{r-1}} n_k \right] D_{ij}^m \right.
\end{aligned}$$

$$\begin{aligned}
& \times \int_{\tau_{r-1}}^{\tau_r} C_{ijkl}^m(\tau_M - \tau) d\tau \\
& + V_m \sum_{r=1}^M \frac{\varepsilon_{kl}^*(\tau_r) - \varepsilon_{kl}^*(\tau_{r-1})}{\tau_r - \tau_{r-1}} \int_{\tau_{r-1}}^{\tau_r} C_{ijkl}^m(\tau_M - \tau) d\tau \\
& + \sum_{s=1}^{N_f} \frac{1}{2} C_{ijkl}^f [u_k^{f(1)}(\mathbf{y}_s, \tau_M) n_l + u_l^{f(1)}(\mathbf{y}_s, \tau_M) n_k] D_{ij}^f \\
& \left. + V_f C_{ijkl}^f \varepsilon_{kl}^*(\tau_M) \right\} \quad (23)
\end{aligned}$$

where D_m^s and D_f^s are the lengths of integral contours Γ_m^s and Γ_f^s , respectively.

Linear simultaneous equations from Eqs. (21)–(23) are solved with initial conditions and boundary conditions under suitable loading condition. The initial conditions at $t=0^-$ are given as

$$\begin{aligned}
& \left. \begin{aligned} u_i^{m(1)}(\mathbf{y}, 0^-) &= 0, & t_i^m(\mathbf{y}, 0^-) &= 0 \\ u_i^{f(1)}(\mathbf{y}, 0^-) &= 0, & t_i^f(\mathbf{y}, 0^-) &= 0 \\ \varepsilon_{ij}^*(0^-) &= 0 \end{aligned} \right\} \quad (24)
\end{aligned}$$

Cartesian coordinate system (y_1, y_2, y_3) and cylinder coordinate system (r, θ, z) with origins at the center of the fiber are shown in Fig. 2. And local coordinate systems ξ_j ($j=1, 2, 3, \dots, 6$) with origins at the center of each side of the hexagon are also placed to indicate the boundary conditions on the outside of the unit cell, see Fig. 2. For the local coordinate system, the boundary conditions on the outside of the unit cell are given as

$$\begin{aligned}
& u_i^{m(1)}(\xi_j) = -u_i^{m(1)}(-\xi_j), \quad t_i^m(\xi_j) = t_i^m(-\xi_j), \\
& (i=1, 2, 3; j=1, 2, 3, \dots, 6) \quad (25)
\end{aligned}$$

When it is assumed that the interface between fiber and matrix is perfect bond, the boundary conditions at the interface ($r=a$) are written as

$$\begin{aligned}
& \left. \begin{aligned} u_r^m &= -u_r^f, & t_\theta^m &= -t_\theta^f, & t_z^m &= -t_z^f \\ u_r^{m(1)} &= u_r^{f(1)}, & u_\theta^{m(1)} &= u_\theta^{f(1)}, & u_z^{m(1)} &= u_z^{f(1)} \end{aligned} \right\} \quad (26)
\end{aligned}$$

where subscripts r, θ and z indicate components of radius, circumferential and longitudinal directions of the fiber, respectively.

The simultaneous Eqs. (21)–(23) are solved with the initial conditions, boundary conditions and macroscopic stress, and the variation of macroscopic strain is obtained with respect to time. When the symmetry of the unit cell for the origin is considered in the actual numerical calculation, number of unknown variables on the boundary can be reduced.

5. Numerical Results and Discussion

On the basis of aforementioned method, numerical calculation is carried out for carbon-fiber composite with thermoplastic matrix AS 4/PACM-12⁽¹²⁾. Time-dependent creep compliance of the composite is evaluated. The carbon fiber is AS 4 (Hercules) and is transversely isotropic. The following properties are used: $E_L=235$ GPa, $E_T=14$ GPa, $\nu_{LT}=0.2$, $\nu_T=0.25$, $G_{LT}=28$ GPa from Ref. (13). Thermoplastic matrix is PACM-12 (DuPont) and the viscoelastic property is

identified by using master curve at the glass transition temperature $T=145^\circ\text{C}$ from Ref.(14). The properties of the matrix is presented by using the generalized Maxwell model which is constructed from parallel connections of elements. The element consists of a series of a spring and a dash pot. The formulation for creep compliance of the matrix is given as

$$J_m(t) = A_{-1}t + A_0 + \sum_{i=1}^{N-1} A_i \exp(a_i t) \quad (27)$$

where a_i and A_i are unknown constants, which are determined from master curve of the matrix with least square method. And associated relaxation modulus is determined from the Laplace inverse transform of following equation:

$$\hat{E}_m(p) = \frac{1}{p^2 \hat{J}_m(p)} \quad (28)$$

The identified creep compliance of the thermo-plastic matrix is shown in Fig. 3 with variation of number of elements N . One-dot chain line, dash line and solid line indicate in case of $N=2$, $N=4$ and $N=8$, respectively. When the number of elements N increases, the curve becomes smooth. The curve for $N=8$ is almost agreed with the master curve, although the curve slightly oscillates. Number of undulations for each curve corresponds to number of elements. If large number of elements is taken, it is supposed that the amplitude of oscillation becomes small. However, the determination of unknown constants of Eq.(27) is more difficult for nonlinearity. $N=8$ is chosen to evaluate the creep compliance of the composite in this study. When the matrix is isotropic, another property is needed. As the variation in the volume of the matrix seems very small during deformation, the Poisson's ratio of the matrix is assumed to be constant and $\nu_m=0.495$.

Fig. 3 Identification of creep compliance of thermo-plastic matrix at glass transition temperature 145°C

Creep compliance of the composite J_{111}^* is shown in Fig. 4 with variation of volume fraction of fiber C_f . The numerical result is obtained from the response for strain ϵ_{11}^* to the stress conditions $\sigma_{11}^* = \sigma_0$ (constant) and $\sigma_{22}^* = \sigma_{33}^* = \sigma_{12}^* = 0$. As the effect of viscoelastic property of matrix on the creep compliance is significant in case of small volume fraction of fiber, the creep compliance increases with time. Although the curves slightly oscillate, the oscillation is caused by the number of elements in the identification of the matrix. Experiment on the creep compliance of the composite is reported in Ref.(12), and the result is plotted in the figure with symbol $\circ$. The result is relatively agreed well with the numerical result for $C_f=0.6$ which corresponds to condition of the experiment.

Creep compliance J_{221}^* is shown in Fig. 5 in similar with J_{111}^* in Fig. 4. The numerical result is obtained from the response for ϵ_{22}^* to same load condition with

Fig. 4 Creep compliance of composite J_{111}^* for transverse load

Fig. 5 Creep compliance of composite J_{221}^* for transverse load

Fig. 6 Creep compliance of composite J_{3333}^* for longitudinal load

Fig. 7 Creep compliance of composite J_{1133}^* for longitudinal load

J_{1111}^* . J_{2211}^* is negative, but the level of the value is similar with J_{1111}^* . Because stiffness of matrix is small in comparison with fiber and the deformation for longitudinal direction of fiber is strongly restricted by fibers.

Creep compliance J_{3333}^* and J_{1133}^* are shown in Figs. 6 and 7, respectively. Uniaxial tension for the fiber direction is supposed, and the responses for macroscopic strains ϵ_{33}^* and ϵ_{11}^* are calculated under the conditions $\sigma_{33}^* = \sigma_0$ (constant) and $\sigma_{11}^* = \sigma_{22}^* = \sigma_{12}^* = 0$. As the elastic modulus of the fiber is very large in comparison with matrix, the time-dependency of the creep compliance J_{3333}^* and J_{1133}^* is small.

Creep compliance J_{1313}^* for longitudinal shear is shown in Fig. 8. In similar with normal components of the creep compliance, macroscopic strain ϵ_{13}^* is calculated under the load conditions $\sigma_{13}^* = \sigma_0$ (constant) and $\sigma_{11}^* = \sigma_{22}^* = \sigma_{33}^* = 0$. As effect restricted by fiber on the

Fig. 8 Creep compliance of composite J_{1313}^* for longitudinal shear load

creep compliance is small for longitudinal shear, the creep compliance increases with time due to creep of matrix with decreasing of volume fraction C_f .

Five properties for creep compliance of carbon-fiber composite are easily obtained by two dimensional analysis as shown in above results.

6. Conclusions

New method to evaluate creep compliance of fiber composites is presented by using the homogenization theory. Macroscopic constitutive equation and microscopic disturbed displacement is formulated by introducing Laplace transform and integral equations. These equations are solved at the same time numerically. The macroscopic strain is determined under suitable condition on the macroscopic stress. As the macroscopic strains are treated as eigenstrain terms in the integral equations for the disturbed displacement, five properties for creep compliance of the carbon-fiber composite are successfully obtained by two dimensional analysis.

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