Determination of the Optical Constants of Caustics under Dynamic Loading Using a Concentrated Load*

Koji SHIMIZU**, Masahiro SUETSUGU***,
Takeshi SENOO**** and Susumu TAKAHASHI**

A new technique is proposed for evaluating the optical constants of caustics under dynamic loading. In this technique, the optical constants are determined from the caustic pattern under a concentrated load and the dynamic value of the concentrated load, which is obtained using strain gages bonded to the surface of the falling weight. The optical constants of caustics c_0 for several polymers are accurately evaluated from the caustic pattern formed by a concentrated load acting on a half-plane, and the dependence of c_0 on the loading rate is determined. The results show that the value of c_0 decreases considerably with increasing loading rate in Plexiglas and Araldite D containing 30% softening material. On the other hand, the values for polycarbonate, Araldite B and Araldite D not containing softening material decrease slightly with increasing loading rate.

Key Words: Experimental Stress Analysis, Fracture Mechanics, Stress Intensity Factor, High Polymer Materials, Caustic Method, Optical Constants, Dynamic Loading, Concentrated Load

1. Introduction

Recently, it is being important to analyze the stress-strain condition and fracture behavior of materials at a crack tip under dynamic loading condition with high speed operation of machines. This is, however, not so easy compared with statically loaded condition because of the complication of dynamic fracture behavior. In the past, attempts have been made for this aim by using the several methods, such as the instrumented Charpy impact test^{(1),(2)}, the split Hopkinson bar technique^{(3),(4)} and photoelastic

method^{(5),(6)}. However, many problems are still remaining as yet. The method of caustics is a very powerful experimental technique to measure the value of stress-intensity factor, K -value. This optical method is widely applied to dynamic fracture problems^{(7)–(10)} since it utilizes directly the information in the vicinity of a crack tip. We have already investigated the dynamic fracture behavior of high polymer⁽¹¹⁾ and ceramic materials at elevated temperatures above 1 000°C^{(12),(13)}. Not only the dynamic fracture toughness, K_{Id} , but also the crack-propagation fracture toughness, K_{Ib} , can be determined by means of the caustic method. It is, however, known that the value of optical constant of caustics, c_0 , is generally affected by a loading rate, and this effect is occasionally considerable in some materials. Yang et al. have been determined the value of c_0 for several polymers under dynamic loading condition using the Fizeau optical interference method and strain gage technique, and they pointed out that the effect of loading rate on c_0 is remarkable in Plexiglas⁽¹⁴⁾. Hence, the dependence of c_0 on loading rate for various materials must be evaluated and the value of c_0 corresponding to the experimental condition to be employed to measure the

* Received 12th September, 1997. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 63, No. 609, A (1997), p. 914–919 (Received 27th June, 1996).

** Department of Mechanical Engineering, Kanto Gakuin University, 4834 Mutsuura, Kanazawa-ku, Yokohama 236, Japan

*** Department of Mechanical Engineering, Suzuka National College of Technology, Shiroko, Suzuka, Mie 510-02, Japan

**** Graduate School of Engineering, Kanto Gakuin University, 4834 Mutsuura, Kanazawa-ku, Yokohama 236, Japan (Current Address: Isuzu Motors Ltd.)

exact K -value by the method of caustics. However, few investigations from this point of view have been reported.

In this study, first, a technique to evaluate the effect of loading rate on c_0 using the experimental procedure applying a concentrated load on a half-plane is proposed. Next, the optical constants of caustics, c_0 , for several polymers under dynamic loading condition are accurately evaluated, and the variation of c_0 in the range from static to dynamic loading is considered.

2. Fundamental Theory of Caustics

We consider to project the convergent light source on the plate under the concentrated load P as shown in Fig. 1. The sign of c_0 for polymers at room temperature is usually negative, and therefore, the caustic pattern formed on the screen is given by the following⁽¹⁵⁾:

$$\begin{aligned} x'_1, x'_2 &= -\lambda(2C_\lambda)^{1/3}(1 \pm \xi)^{1/3} \{ \cos \phi - (1/2) \sin 2\phi \}, \\ y'_1, y'_2 &= -\lambda(2C_\lambda)^{1/3}(1 \pm \xi)^{1/3} \{ \sin \phi + (1/2) \cos 2\phi \} \end{aligned} \quad (1)$$

where λ is the magnification factor of light given by

$$\lambda = (z_0 - z_i)/z_i \quad (2)$$

and C_λ is a constant defined by

$$C_\lambda = -(z_0 c_0 2P)/\pi\lambda. \quad (3)$$

z_0 and z_i are the distances between the specimen and screen, and the focal point and screen, respectively. Initial curve r_0 which is forming the caustic pattern on the reference plane is given by

Fig. 1 Principle of the caustic method

Fig. 2 Caustic pattern on the screen (convergent light, $c_0 < 0$, $\xi = -0.298$)

$$r_{01}, r_{02} = |2C_\lambda(1 \pm \xi)|^{1/3}. \quad (4)$$

Figure 2 illustrates an example of caustic pattern calculated through Eq.(1) for $2C_\lambda=1$ and $\lambda=1$. Caustic pattern actually obtained by experiment is upside down of the picture shown in Fig. 2. The value of ξ , the coefficient of anisotropy effect of materials, is setting as -0.298 . This value has been evaluated for polycarbonate by interferometric technique in our previous study⁽¹⁶⁾. As mentioned above, the light beam transmitted through a specimen of optically anisotropic material (strictly speaking, $\xi \neq 0$) is deviated along the planes of principal stresses σ_1 and σ_2 , and therefore, the double caustic curves shown in Fig. 2 are obtained. The value of concentrated load P can be easily determined from inside diameter of caustic pattern, D_{in} , and/or outside one, D_{out} , through Eq.(5):

$$P = \frac{0.785}{z_0 |c_0| (1 \pm \xi)} \cdot \frac{1}{\lambda^2} \cdot \left(\frac{D_{in, out}}{2.6} \right)^3, \quad (5)$$

where the diameters D_{in} and D_{out} are corresponding to $(1+\xi)$ and $(1-\xi)$, respectively. The initial curves r_{01} and r_{02} are also expressed as the following equation using D_{in} and D_{out} :

$$r_{01}, r_{02} = D_{in, out} / (2.6\lambda). \quad (6)$$

It is seen from Eq.(5) that the values of $c_0(1+\xi)$ and $c_0(1-\xi)$ can be obtained from the diameters of caustic pattern D_{in} and D_{out} , respectively. Therefore, the values c_0 and ξ are evaluated by the followings:

$$c_0 = (A+B)/2 \quad (7)$$

$$\xi = (A-B)/(A+B), \quad (8)$$

where A and B are the values of $c_0(1+\xi)$ and $c_0(1-\xi)$, respectively. Equation (8) can be also written as follows:

$$\xi = \{1 - (D_{out}/D_{in})^3\} / \{1 + (D_{out}/D_{in})^3\}. \quad (9)$$

Consequently, the value of ξ can be easily and directly determined by measuring the ratio D_{out}/D_{in} from the caustic pattern. Moreover, it is seen from Eqs.(7) and (8) that the values of A and B obtained from D_{in} and D_{out} are equivalent to the constants α and β given by the followings⁽¹⁷⁾:

$$\begin{aligned} A = \alpha &= c_1 - (\mu/E)(n-1), \\ B = \beta &= c_2 - (\mu/E)(n-1), \end{aligned} \quad (10)$$

where c_1 and c_2 are the absolute photoelastic constants, μ is the Poisson's ratio, E is the Young's modulus and n is the refractive index under no load. As stated above, the values of α , β , c_0 and ξ are determined all at once from the caustic pattern which has double caustic curves obtained by the concentrated load. The photoelastic sensitivity, α^* , can be also determined by

$$\alpha^* = (c_1 - c_2)/\lambda^* = (\alpha - \beta)/\lambda^*, \quad (11)$$

where λ^* is the wavelength of light.

The concentrated load P can be obtained by the technique mentioned above, and the dynamic loading

effect on c_0 can be studied utilizing this technique. Namely, the applied dynamic concentrated load is measured by strain gage bonded on the tip of falling weight. On the other hand, the concentrated load P is also measured from the caustic pattern formed by the falling weight. It is considered that the correct value of c_0 under dynamic loading can be determined from these information through Eq.(5), because the position of light on the specimen which forms the caustic pattern on the screen is very close to the position where the applied dynamic load is measured, and these information can be obtained without delay. This is the fundamental conception of the technique for determining c_0 under dynamic loading proposed in this study.

3. Experimental Setup and Test Procedure

The shape of the test specimen is shown in Fig. 3. The plates of Plexiglas (SUMIPEX: Sumitomo Chemical Co., Ltd.), polycarbonate (Type-G: Takiron Co., Ltd.), Araldite B (PSM-9: Photolastic Division Measurements Group) and Araldite D are used as the testing materials. Araldite D is made at our laboratory by mixing ARALDITE CY 230 and HARDENER 951. There are two kinds of materials in Araldite D, namely, containing 30% softening material (THIOKOL LP-3) and without this one. We express these materials as 30% and 0%, respectively, in this study. The specimen thickness t is 6 mm by taking account of the effect of r_0 on P -value reported in the Ref. (18). Values of Young's modulus E and Poisson's ratio μ of tested materials are shown in Table 1.

Schematic diagram of the dynamic experimental arrangement of caustics and drop-weight impact testing apparatus are shown in Figs. 4 and 5, respectively⁽¹²⁾. The convergent light from a He-Ne gas laser source of 2 mW impinges on a specimen, and a sequence of dynamic caustic patterns formed on the screen is recorded with the ultrahigh-speed camera (Imacon 790: Hadland Photonics Ltd.). The high-speed pictures are mainly taken at 10^5 FPS (Frames Per Second) in this study. As shown in Fig. 5, the specimen is dynamically loaded by the falling weight from various heights. The tip of the falling weight has a semicircular shape with the radius R of 2 mm by taking account of the effect of r_0 on P -value with increasing the contact area⁽¹⁸⁾. The strain pulses obtained by a set of strain gages (KFG-2-120: Kyowa Electronic Instruments Co., Ltd.) bonded on two sides of the falling weight are amplified by amplifiers (CDV-230 C: Kyowa Electronic Instruments Co., Ltd.), and registered in digital-storage oscilloscope (DS 6612 C: Iwatsu Electric Co.). The applied dynamic load is determined from these strain

Table 1 Values of Young's modulus E and Poisson's ratio μ

	PMMA	Polycarbonate	Araldite		
			B	D	
				0%	30%
Young's modulus E (GPa)	2.9	2.3	3.3	2.4	0.6
Poisson's ratio μ	0.38	0.36	0.36	—	—

Fig. 3 Shape of the test specimen

Fig. 4 Schematic diagram of the dynamic experimental arrangement

Fig. 5 Drop-weight impact testing apparatus

pulses with the relationship between P and ϵ of falling weight tup. It is considered that the correct value of c_0 under dynamic loading can be evaluated by the technique mentioned above, because the positions where the applied dynamic load is obtained and the caustic pattern is formed are very close.

The value of c_0 under dynamic loading can be also

measured from the caustic pattern and dynamic stress-intensity factor calculated from the applied dynamic load by carrying out a dynamic loading test with a cracked specimen. However, in this case, the positions where the dynamic load is applied and the caustic pattern is formed, namely the vicinity of crack tip, are apart from each other. Therefore, it is difficult to determine the exact value of dynamic stress-intensity factor, because a time lag will be existing between the moments when the applied load is measured and the caustic pattern is recorded. Figure 6 shows the variation of dynamic stress-intensity factor with time obtained by carrying out the dynamic three-point bending test with a single-edge notched specimen of Plexiglas. A notch of 0.3 mm width is cut in this specimen and the thickness t is 6 mm. Two kinds of dynamic stress-intensity factors determined by different methods are shown in Fig. 6. One is the dynamic K -value calculated through the static formula with the dynamic applied load $P(t)$ which is obtained from strain pulses. The other one is the dynamic K -value evaluated from the high-speed photograph of caustic pattern using the value of $c_0 = -1.05 \times 10^{-10} \text{ m}^2/\text{N}$. The value of dynamic fracture

Fig. 6 Stress intensity factors for a crack under dynamic loading by a drop-weight

toughness K_{Id} determined by the caustic method is also shown in Fig. 6 for reference. As is evident from Fig. 6, dynamic K -value which are evaluated from $P(t)$ and caustic pattern are dissimilar at all each other, and it is confirmed that dynamic K -value can not be evaluated from the applied dynamic load $P(t)$ in this case.

Figure 7 shows an example of the variation of strain ϵ and diameter D of the caustic pattern with time obtained by carrying out the dynamic concentrated loading test with a half-plane specimen of Plexiglas shown in Fig. 3. A sequence of caustic patterns is recorded at 10^4 FPS in this case. As shown in Fig. 7, variations of D^3 and ϵ with time are in good agreement with each other, and it is ascertained that the experimental technique using the concentrated load stated above is extremely useful to determine the optical constant of caustics c_0 under dynamic loading condition.

4. Experimental Results and Discussion

Examples of the caustic pattern of several polymers under dynamic loading are shown in Fig. 8. The number in this figure indicates the order of frames and

Fig. 7 Relationship between D and ϵ under dynamic concentrated loading

Order of frames	2	4	6	8	10	12	14	16
PMMA ($z_0=1100\text{mm}$, $z_1=615\text{mm}$)								
Polycarbonate ($z_0=1200\text{mm}$, $z_1=565\text{mm}$)								
Araldite B ($z_0=900\text{mm}$, $z_1=580\text{mm}$)								

Fig. 8 Examples of the caustic pattern under dynamic loading condition (10^5 FPS)

Table 2 Optical constants of the various materials (upper row: dynamic loading, lower row: static loading) $\{\alpha, \beta, c_0: 10^{-10} \text{ m}^2/\text{N}, \alpha^*: \text{mm}/\text{N} (\lambda^*=632.8 \text{ nm})\}$

		α	β	c_0	ξ	α^*
Plexiglas		-0.571	-0.571	-0.571	0	0
		-0.999	-0.999	-0.999	0	0
Polycarbonate		-0.965	-1.542	-1.254	-0.23	0.091
		-1.162	-1.855	-1.509	-0.23	0.11
Araldite	B	-0.988	-1.473	-1.230	-0.197	0.077
		-1.015	-1.542	-1.279	-0.206	0.0832
	D(0%)	-0.948	-1.223	-1.035	-0.182	0.06
		-0.925	-1.359	-1.142	-0.19	0.069
	D(30%)	-1.381	-1.677	-1.529	-0.097	0.047
		-3.229	-3.790	-3.510	-0.080	0.089

Fig. 9 Variations of D and ϵ with time (Araldite B)

the interval of these frames is $10 \mu\text{s}$. As shown in Fig. 8, the diameter D of the caustic pattern increases with time. We can also see that the caustic pattern of Plexiglas, which is an optically inert material $\xi \approx 0$, has a single caustic curve, and the caustic patterns of epoxy resin and polycarbonate, which are optically anisotropic materials, have double caustic curves. Figure 9 illustrates an example of the variations of the diameters of caustic pattern, D_{in} and D_{out} with time. D_{in} and D_{out} are obtained from high-speed picture and ϵ is measured by the strain gages bonded on the falling weight tup. The values of optical constants under dynamic loading determined through these experimental results are listed in Table 2. The strain rate $\dot{\epsilon}$ is in the degree of 10^2 s^{-1} . The values of optical constants under static loading condition are also shown in Table 2. As stated in Ref.(17) in which the values of optical constants were determined using the deviation of light beam due to the stress gradient, the signs of α, β, c_0 and ξ are generally negative under usual condition at room temperature, and $\alpha < \beta$. As is evident from Table 2, the values of ξ under static and dynamic loading conditions are almost the same each other in all polymers tested here. In contrast to this circumstance, the values c_0 and α^* of some polymers

are considerably changed between these conditions. Strain rate dependence of c_0 in various materials is shown in Fig. 10. The strain rate $\dot{\epsilon}$ is calculated at the point where the maximum diameter of caustic pattern is given, namely $\phi = -30^\circ / -150^\circ$ and $r = r_0$, using the equation of stress distribution for the concentrated load acting on a half-plane. It is revealed from Fig. 10 that the value of c_0 is generally decreased with the rise of $\dot{\epsilon}$. This tendency is remarkable in Araldite D(30%) and Plexiglas, whereas it is relatively small in polycarbonate. In Araldite B and Araldite D(0%), the value of c_0 is scarcely affected by the strain rate $\dot{\epsilon}$. Dependence of c_0 on $\dot{\epsilon}$ in the range up to $\dot{\epsilon} \approx 10^{-3} \text{ s}^{-1}$ studied by Yang et al.⁽¹⁴⁾ is also shown in Fig. 10. As shown in this figure, the results reported by Yang et al. are consistent with the results in this study for Plexiglas and polycarbonate. For epoxy resin, we consider to compare the results reported by Yang et al. with Araldite D(0%) in this study because they expressed the tested material as only Araldite D in Ref.(14). As shown in Fig. 10(b), the value c_0 determined by Yang et al. is slightly increased with $\dot{\epsilon}$, whereas the value of c_0 in our study is slightly decreased or almost the constant with $\dot{\epsilon}$. The range of strain rate $\dot{\epsilon}$ in the study by Yang et al. is restricted, and therefore it is unknown that the value of c_0 is continuously increased with $\dot{\epsilon}$ in the range up to $\dot{\epsilon} \approx 10^2 \text{ s}^{-1}$. Though the results reported by Yang et al. and obtained in our study can not be compared with each other in whole range of $\dot{\epsilon}$, it can be considered that the value of c_0 in Araldite D(0%) is scarcely affected by the strain rate $\dot{\epsilon}$. Figure 11 shows the variations of photoelastic sensitivity α^* with $\dot{\epsilon}$ for several polymers. It is shown in Fig. 11 that the value of α^* is generally decreased with the rise of $\dot{\epsilon}$. This tendency is remarkable in Araldite D(30%), and then polycarbonate is following. The values of α^* in Araldite B and Araldite D(0%) are scarcely affected by $\dot{\epsilon}$. It is recognized that the tendency of relation-

Fig. 10 Strain rate dependence of c_0

ship between α^* and $\dot{\epsilon}$ for various polymers shown in Fig. 11 is in good agreement with that one between c_0 and $\dot{\epsilon}$ shown in Fig. 10. From relations (7), (8) and (11), the value of photoelastic sensitivity is found to be $\alpha^* = 2\xi c_0 / \lambda^*$, and therefore the variation of α^* with $\dot{\epsilon}$ is similar to that one of c_0 with $\dot{\epsilon}$ under the condition in which the value of ξ is almost the same under

Fig. 11 Dependence of photoelastic sensitivity α^* on $\dot{\epsilon}$

static and dynamic loading conditions as shown in Table 2. The study concerned with the dependence of α^* on $\dot{\epsilon}$ have been given by Mall et al. for polycarbonate⁽¹⁹⁾ and by Clark et al. for Araldite B and Araldite D(0%)⁽²⁰⁾. The result that the value of α^* under dynamic loading is decreased is also reported in their studies.

5. Conclusions

(1) A technique to determine the optical constant of caustics c_0 under dynamic loading is proposed. The value of c_0 can be exactly measured from a sequence of caustic pattern in high-speed photograph and applied load by carrying out the dynamic concentrated loading test with a half-plane specimen. The optical constants of several polymers are accurately and simply determined by the technique proposed in this study.

(2) The optical constant of caustics c_0 is generally decreased with the rise of strain rate $\dot{\epsilon}$. This tendency is especially remarkable in Araldite D containing a softening material and Plexiglas. The variation of c_0 with $\dot{\epsilon}$ in polycarbonate is relatively small in comparison with that one of Araldite D and Plexiglas. The values of c_0 in Araldite B and Araldite D without a softening material are scarcely affected by $\dot{\epsilon}$. The variation of photoelastic sensitivity α^* with $\dot{\epsilon}$ in these materials is similar to that one of c_0 with $\dot{\epsilon}$.

(3) The value of ξ , the coefficient of anisotropy effect of material, is almost the same under static and dynamic loading conditions for all materials tested in this study.

References

- (1) Kobayashi, T., Miyata, H., Kikusawa, K. and Higashihara, T., Instrumented Impact Test and Evaluation of Fracture Toughness in PMMA, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 55, No. 520, A(1989), p. 2434-2438.

- (2) Suzuki, M. and Nishiura, M., A Comparison between Instrumented Charpy Impact Properties of Reformed Polyvinyl Chloride and Polycarbonate, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 55, No. 512, A(1989), p. 1010-1015.
- (3) Yokoyama, T. and Kishida, K., A Novel Impact Three-point Bend Test Method for Determining Dynamic Fracture-initiation Toughness, *Exp. Mech.*, Vol. 29, No. 2(1989), p. 188-194.
- (4) Duffy, J., Suresh, S., Cho, K. and Bopp, E.R., A Method for Dynamic Fracture Initiation Testing of Ceramics, *Trans. ASME, J., Eng. Mater. Technol.*, Vol. 110, No. 4(1988), p. 325-331.
- (5) Kobayashi, A.S., Seo, K., Jou, J.Y. and Urabe, Y., Dynamic Analysis of Modified Compact-tension Specimens Using Homalite-100 and Polycarbonate Plates, *Exp. Mech.*, Vol. 20, No. 3(1980), p. 73-79.
- (6) Oda, J., Shibahara, M. and Ohkubo, T., On Dynamic Fracture Criterion and Stress Intensity Factors of Cracked Plate Subjected to a Tensile Pulse Wave, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 51, No. 462, A(1985), p. 455-462.
- (7) Kalthoff, J.F., Fracture Behavior under High Rates of Loading, *Eng. Fract. Mech.*, Vol. 23, No. 1(1986), p. 289-298.
- (8) Zehnder, A.T. and Rosakis, A.J., Dynamic Fracture Initiation and Propagation in 4340 Steel under Impact Loading, *Int. J. Fract.*, Vol. 43, (1990), p. 271-285.
- (9) Arakawa, K. and Takahashi, K., Branching of Fast Crack in Epoxy Resin, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 54, No. 500, A(1988), p. 718-722.
- (10) Nishioka, T., Kittaka, H., Uchiyama, H., Murakami, T. and Sakakura, K., Measurement of Stress Intensity Factors during Dynamic Crack Propagation and Arrest Using a Laser Caustic Method, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 55, No. 520, A(1989), p. 2410-2415.
- (11) Shimizu, K., Saito, H., Suetsugu, M. and Takahashi, S., A Study on the Dynamic Fracture Behavior of Plexiglas Plates by the Caustic Method, *Trans. JSNDI*, Vol. 5, (1992), p. 41-52.
- (12) Suetsugu, M., Shimizu, K. and Takahashi, S., Analysis of the Dynamic Fracture Behavior of Partially Stabilized Zirconia at Elevated Temperatures by the Method of Caustics, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 59, No. 559, A(1993), p. 693-700.
- (13) Suetsugu, M., Shimizu, K. and Takahashi, S., Evaluation of Dynamic Fracture Behavior of Silicon Nitride Ceramics at Room and Elevated Temperatures by Caustics, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 61, No. 583, A(1995), p. 545-552.
- (14) Yang, H.T., Sakurada, Y. and Takahashi, K., Strain Rate Dependence of Stress-Optical Constants of Glassy Polymers, *Rep. Res. Inst. Appl. Mech.*, Kyusyu Univ., Vol. 63, (1987), p. 87-92.
- (15) Shimizu, K., Suetsugu, M., Nakamura, T. and Takahashi, S., Evaluation of Concentrated Load by Caustics and Its Application in the Measurement of Optical Constant, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 63, No. 605, A(1997), p. 87-93.
- (16) Shimizu, K., Shimada, H. and Sasaki, T., On Determination of Stress Intensity Factor by Caustics for the Birefringent Materials, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 46, No. 411, A(1980), p. 1196-1202.
- (17) Shimizu, K. and Takahashi, S., A Technique for Measuring Optical Constants, *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 55, No. 519, A(1989), p. 2342-2347.
- (18) Shimizu, K., Nakamura, T. and Takahashi, S., Evaluation of the Concentrated Load by Caustics and Its Application to the Dynamic Problem, IV Conf. of Asian-Pacific Congr. on Strength Evaluation, (1991), p. 1537-1542.
- (19) Mall, S., Kobayashi, A.S. and Urabe, Y., Dynamic Photoelastic and Dynamic Finite Element Analyses of Polycarbonate Dynamic Tear Test Specimens, *Fract. Mech.*, ASTM STP 677, (1979), p. 498-510.
- (20) Clark, A.B.J. and Sanford, R.J., A Comparison of Static and Dynamic Properties of Photoelastic Materials, *Proc. SESA*, Vol. 20, No. 1(1963), p. 148-151.