Recent Progress in Creep Rupture Analysis of Unidirectional Composites Reinforced with Long Brittle Fibers

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The present paper deals with state of the art in creep rupture analysis of unidirectional composites reinforced with long brittle fibers. Experiments, numerical simulations and analytical models related with the creep rupture analysis are treated mainly on metal matrix composites, which can be mathematically modeled more easily because of the assumption of global load sharing. It is described that matrix creep and statistical fiber fracture play important roles in understanding and predicting the creep rupture of such composites appropriately. It is emphasized from the mechanics point of view that two kinds of stress relaxation, i.e., relaxation of matrix normal stress and that of interfacial shear stress around fiber breaks, are the key to the creep rupture in short and long terms, respectively.

**Key Words**: Creep, Composite Material, Stress Relaxation, Rupture, Long Fiber, Fiber Break, Statistical Fiber Strength

1. Introduction

Can unidirectional composites reinforced with noncreeping long fibers be ruptured in longitudinal creep at constant stress? One may be likely to give a negative reply to this question, since the noncreeping fibers seem to prevent the composites from creeping in the fiber direction. According to experiments, however, the composites can be ruptured in longitudinal creep if applied stress is relatively high. A challenging subject has been thus posed, and experimental and theoretical studies on the subject have been done vigorously in recent works, as reviewed in the present paper.

It is easy to guess that matrix viscosity is essential to the creep rupture mentioned above. The creep rupture is however complicated because stochastic fiber fracture is coupled with it somehow. It is then necessary to find dominant mechanisms responsible for such creep rupture induced by matrix viscosity and stochastic fiber fracture. The following two mechanisms, which are peculiar to unidirectional fiber composites and of interest from the mechanics point of view, have been pointed out for the creep rupture so far.

Firstly, matrix normal stress is relaxed and transferred to fiber stress, so that fiber breaks can be induced to result in the overall rupture of composites. As was shown by Du and McMeeking\(1\), this mechanism of creep rupture can be simulated by combining the McLean\(2\) and the Curtin\(3,4\) model, which describe creep elongation and load-carrying capacity of unidirectional long fiber composites, respectively. Secondly, stress in broken fibers is relaxed as a result of matrix shear creep around fiber breaks, so that stress in intact fibers is increased to induce further fiber breaks. This mechanism of creep rupture was first studied for polymer matrix composites by Lifshitz and Rotem\(5\) using the Lapalase transformation for linear viscoelastic problems and Rosen's\(6\) model for fracture of unidirectional composites. The mechanism was then discussed for metal matrix composites numerically by Goda\(7\) as well as Du and McMeeking\(1\) and analytically by Iyenger and Curtin\(8\)
as well as Ohno et al.\textsuperscript{(9)}

It seems that the works mentioned above, most of which have been done in the last ten years, have contributed to better understanding of the creep rupture of unidirectional composites reinforced with long brittle fibers. With this point as background, the present paper reviews studies on experiments, numerical simulations and analytical models concerned with the creep rupture, so that state of the art in the creep rupture analysis is described. Emphasis is put on metal matrix composites, which can be mathematically modeled more easily because of the assumption of global load sharing than polymer matrix composites.

2. Experimental Features

A representative class of unidirectional metal matrix composites reinforced with long fibers are SiC fiber/titanium alloy composites\textsuperscript{(10)(11)}. Creep experiments of such composites were done by Ohno et al.\textsuperscript{(12)(13)}, Schwenker et al.\textsuperscript{(14)}, Weber et al.\textsuperscript{(15)} and so on.

Figure 1 shows an example of the experiments done by Ohno et al.\textsuperscript{(13)}. The composite tested in this example consisted of continuous SiC fibers, SCS-6\textsuperscript{(10)}, and a meta stable beta titanium alloy, Beta 21S. The following features are seen from the figure: Strain \( \varepsilon \) increases very rapidly just after initial loading, but as time \( t \) elapses strain rate decreases and becomes nearly constant. Moreover, creep rupture takes place when applied stress \( \sigma \) is relatively high; however, acceleration of creep rate hardly occurs even just before rupture. Here it is noticed that the SCS-6 fibers do not creep below about 800\textdegree C\textsuperscript{(10)(20)}, and that applied stresses in the experiments are fairly low in comparison with the tensile strength, which was reported to be about 1600 MPa at 500\textdegree C\textsuperscript{(21)}.

Figure 2 illustrates the events of acoustic emission captured in the creep experiment at \( \sigma = 1200 \) MPa shown in Fig. 1. Only strong events with peak amplitude greater than 70 dB are indicated in the figure.

Such strong events, which occurred mainly in the experiments resulting in rupture, are attributable to fiber breakage, cracking in the fiber coating, or debonding at the fiber/matrix interface. It is thus suggested that fiber damage contributed to the creep rupture in the experiments.

Longitudinal creep curves of SCS-6/Ti-6Al-4V tested at 427\textdegree C by Schwenker et al.\textsuperscript{(16)} are shown in Fig. 3. It is seen that creep rupture with little tertiary creep strain occurred at relatively high stresses in their experiments, too. Such creep rupture, which was observed in longitudinal creep experiments of B/Al composites as well in spite of creep elongation of the fibers\textsuperscript{(22)(23)}, can be a consequence of the brittle fracture of fibers, because tertiary creep was observed in off-axis creep accompanied by no fiber breaks\textsuperscript{(12)(22)}.

3. Creep Elongation Due to Stress Transfer from Matrix to Fibers

A preliminary work to understand the creep rupture phenomenon described in the preceding section was done by McLean\textsuperscript{(2)}. His model deals with a composite in which elastic long fibers with no breaks are embedded in a creeping matrix. When the composite is subjected to constant stress \( \sigma \) applied in the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Creep curves of SCS-6/Beta21S at 500\textdegree C\textsuperscript{(13)}}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Creep curve and AE events in creep of SCS-6/Beta21S at 1200 MPa at 500\textdegree C\textsuperscript{(13)}}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Creep strain versus time relations of SCS-6/Ti-6Al-4V at 427\textdegree C\textsuperscript{(14)}}
\end{figure}
fiber direction, longitudinal creep deformation in the matrix is constrained by the fibers; thus, matrix stress $\sigma_m$ in the fiber direction is decreased and transferred to fiber stress $\sigma_f$, and consequently longitudinal strain $\varepsilon$ of the composite increases with time $t$. For the composite, $\sigma$ and $\varepsilon$ satisfy

$$\sigma = V_f \sigma_f + (1 - V_f) \sigma_m, \quad \varepsilon = \varepsilon_f = \varepsilon_m,$$

where $V_f$ indicates the fiber volume fraction, and $\varepsilon_f$ and $\varepsilon_m$ denote longitudinal strains in the fiber and matrix phases, respectively. Here and from now on, the subscripts $f$ and $m$ stand for fiber and matrix, respectively.

To formulate a simple creep model based on the above mechanism of stress transfer, McLean assumed the following constitutive equations for the fibers and the matrix, respectively:

$$\varepsilon_f = \frac{\sigma_f}{E_f}, \quad \varepsilon_m = \frac{\sigma_m}{E_m} + B \delta_m^n,$$

where the superposed dot denotes the differentiation with respect to time $t$, $E_f$ and $E_m$ indicate Young’s moduli of the fibers and the matrix, respectively, and $B$ and $n$ are creep constants of the matrix. Then, using Eqs. (1) to (4), he obtained the time-dependent change of $\varepsilon$ under constant applied stress $\sigma$ as follows:

$$\varepsilon(t) = \frac{\sigma E_f}{V_f E_f} \left[ 1 - V_f \left( \frac{E_m}{E_c} \right)^{(n-1)/n} \right]_{(n-1)/n}^{(n-1)/n} + \frac{(n-1) V_f E_f E_m B t}{E_c},$$

where $E_c$ denotes the modulus based on the rule of mixtures, i.e.,

$$E_c = (1 - V_f) E_m + V_f E_f.$$

According to Eq. (5), $\varepsilon$ increases with time from initial elastic strain $\sigma / E_c$ and approaches asymptotically

$$\varepsilon = \frac{\sigma}{V_f E_f},$$

as illustrated schematically in Fig. 4. This asymptotic strain represents the state in which $\sigma_m$ is completely relaxed. It is obvious that the composite no longer elongates if $\sigma_m$ is completely relaxed. It is needless to say that Eq. (5) can valid if no fiber break occurs, or if the effect of fiber breaks on $\varepsilon(t)$ is negligible.

Let us point out that McLean’s model considers only longitudinal stress and strain but gives fairly good agreements to axisymmetric finite element analysis (23), and that the model was extended by taking account of the matrix/fiber interface layer (26) and by using internal state variables representing damage evolution in composites (27).

4. Creep Rupture Due to Stress Transfer from Matrix to Fibers

Now we consider a composite consisting of brittle fibers and a creeping matrix. Such brittle fibers usually break statistically. Then, as matrix stress is transferred to fiber stress as a result of matrix creep, fiber breaks may occur one after another with the increase of fiber stress. Consequently the composite can be ruptured in creep at constant applied stress. Du and McMeeking (1) as well as Fabeny and Curtin (28) studied this mechanism of creep rupture by combining McLean’s model described in the preceding section with Curtin’s model concerned with the load-carrying capacity of long fibers in unidirectional composites.

4.1 Curtin model

Curtin (29, 30) assumed that the broken fibers are subjected to constant interfacial shear stress $\tau_a$ around the fiber breaks (Fig. 5); then, the broken fibers recover the stress in intact fibers, $E_c \varepsilon$, at the following distance from the fiber breaks, as was first expressed by Kelly and Tyson (31):

$$\delta_a = \frac{D E_c \varepsilon}{4 \tau_a},$$

where $D$ denotes the diameter of fibers. He assumed

![Fig. 4](image1)

**Fig. 4** Schematic illustration of the creep curve based on McLean's model

![Fig. 5](image2)

**Fig. 5** Fiber breaks in a unidirectional composite subjected to overall strain $\varepsilon$
also that the stress released by fiber breaks is shared equally by all the intact fibers. This assumption, which is called global load sharing, is valid to predicting the strength of ceramic matrix and metal matrix composites with weak interface[22-30,31]. He then showed that overall strain \( \varepsilon \) induces the following fiber stress averaged on a cross section AB perpendicular to the fiber direction (Fig. 5):

\[
\sigma_f = (1-q)E_e\varepsilon + q\frac{E_e\varepsilon}{2},
\]

(9)

In this equation, \( q \) denotes the probability that a fiber is broken within the stress recovery distance \( \delta_0 \) from the cross section AB, and if \( \delta_0 \) is much smaller than fiber length the Weibull statistics provides \( q \) with

\[
q = \frac{2\delta_0}{L_0} \left( \frac{E_e\varepsilon}{\delta_0} \right)^m,
\]

(10)

where \( m \) is the Weibull modulus, and \( \delta_0 \) is the reference strength corresponding to a fiber length \( L_0 \). Curtin thus obtained the maximum of \( \sigma_f \) with respect to \( \varepsilon \), and he evaluated the load-carrying capacity of fibers to be equal to the maximum

\[
S_{\text{max}} = S_e \left( \frac{m+1}{m+2} \right)^{1/(m+1)},
\]

(11)

where \( S_e \) indicates a characteristic strength such that[22,23]

\[
S_e = \left( \frac{2\delta_0 E_e L_0}{D} \right)^{1/(m+1)}.
\]

(12)

Incidentally, the second term in the right hand side in Eq. (9), which represents the contribution of the fibers broken within \( \delta_0 \) from the cross section AB, was disregarded in the classical model of Rosen.

4.2 Curtin–McLean model

Du and McMeeking[24] first combined Curtin’s model mentioned above with McLean’s model described in Sec. 3 in order to predict the rupture time in creep. They thus derived the differential equation

\[
\dot{\varepsilon} = \frac{B}{1 + \frac{\sigma - V_s E_e \varepsilon}{V_f E_e \varepsilon}} \left[ 1 - \frac{1}{2} \left( \frac{E_e\varepsilon}{S_e} \right)^{m+1} \right]^n,
\]

(13)

which was obtained subsequently by Fabeny and Curtin[28].

Du and McMeeking found further that the rupture time obtained by integrating the above equation numerically is approximated fairly well by the following analytical model: They approximated the rupture time with the time at which the fiber stress expressed by McLean’s model with no fiber brakes reaches the load-carrying capacity of fibers, \( S_{\text{max}} \), derived by Curtin. This approximated creep rupture time is obtained by substituting Eq. (5) into \( E_e\varepsilon(t_f) = S_{\text{max}} \):

\[
t_f = \frac{n - 1}{V_f E_e E_m B} \left[ 1 - \frac{1}{2} \left( \frac{E_e\varepsilon}{S_e} \right)^{m+1} \right] - \left( \frac{E_e\varepsilon}{S_e} \right)^{n-1}.
\]

(14)

It is obvious that the above equation is effective for

\[
\sigma > V_f S_{\text{max}}.
\]

(15)

In other words, \( V_f S_{\text{max}} \) is the lower limit of \( \sigma \) for creep rupture in Curtin–McLean’s model. This rupture condition is clear if we consider the state in which matrix stress \( \sigma_m \) is completely relaxed; in this state, fiber stress is equal to \( \sigma/V_f \), as seen from Eq. (1), so that creep rupture takes place if \( \sigma/V_f \) is larger than the load-carrying capacity of fibers, \( S_{\text{max}} \).

The validity of Curtin–McLean’s model was discussed by Weber et al.[27] They performed creep experiments of a Ti-6Al-4V alloy reinforced with continuous, aligned SiC fibers. The test temperature was 600°C. They employed short and long specimens with gauge lengths of 25 mm and 100 mm, respectively. Rupture times obtained in the creep experiments, along with results of tensile tests, are plotted in Fig. 6. In the figure, the solid and dashed lines indicate the numerical and analytical predictions based on Eqs. (13) and (14), respectively. For the predictions, two sets of fiber strength data were used to determine the Weibull parameters of fibers; i.e., by extracting fibers from as-processed and heat-treated specimens, \( \delta_0 = 1.47 \) GPa and \( \delta_0 = 1.29 \) GPa were obtained, respectively, though the value of \( m \) was almost the same (\( m \approx 5 \)). It is seen from the figure that Eqs. (13) and (14) give nearly the same predictions if applied stress is not very large, and that the predictions are fairly close to the experiments. It is however noticed that the creep experiments in the figure were of short term because the rupture times were less than 100 hours.

5. Stress Relaxation in Broken Fibers

The preceding section dealt with Curtin–McLean’s model expressing the creep rupture induced by the stress transfer from the matrix to the fibers. In
the model, the relaxation of matrix normal stress $\sigma_n$ was taken into account, but interfacial shear stress $\tau$ acting on each broken fiber was taken to be constant. In fact, $\tau$ can decrease with time, since the stress in broken fibers can be relaxed as a result of matrix shear creep around fiber breaks (Fig. 7), as was first shown by Lifshitz and Rotem\(^7\). Here it is noted that $\tau$ is proportional to the axial gradient of fiber stress $\sigma_z$ because of the equilibrium equation

$$\tau = -\frac{D}{4} \frac{\partial \sigma_z}{\partial z}, \quad (16)$$

where $z$ denotes the axial coordinate of fibers.

The studies on the stress relaxation in broken fibers in creeping matrices are traced back to Lifshitz and Rotem\(^7\). They studied creep rupture of GFRPs by assuming a linear viscoelastic theory; i.e., using the Laplace transformation, they analyzed the stress relaxation in broken fibers embedded in a linear viscoelastic matrix, and then they evaluated creep rupture time by extending the rupture model of Rosen\(^8\). They showed especially that the time-dependent extension of the stress recovery length $\delta$ satisfies the relation

$$\delta(t) \sim \sqrt{J(t)}, \quad (17)$$

where $J(t)$ denotes the shear creep compliance of the matrix material. This result was discussed subsequently in more detailed analysis for polymer-matrix composites by Lagoudas et al.\(^9\) and Mason et al.\(^10\); the former analyzed the time-dependent evolution of stress profiles in neighboring intact fibers by assuming a power-law creep compliance, and the latter clarified the effect of nonlinear, matrix viscosity on the stress profiles around a fiber break.

6. Numerical Analysis of Creep Rupture

The stress relaxation in broken fibers was pointed out also in numerical analyses of metal matrix composites\(^11\).

Goda\(^7\) performed finite element analysis of creep of a monolayer composite plate in which boron fibers were aligned in an aluminum alloy matrix. In the analysis, the fibers and the matrix were modeled respectively with two node bar elements and four node rectangular elements (Fig. 8), and a time-dependent Weibull distribution of fiber strength\(^12\) was assumed. He thus found that matrix stress $\sigma_n$ relaxes very quickly just after initial loading; nevertheless, creep rupture takes place eventually without noticeable acceleration of creep rate (Figs. 9(a) to (c)). To clarify the mechanism for such creep rupture, he performed another analysis, in which only one fiber break is allowed to occur at the center of a specimen under initial loading. This analysis revealed the following (Fig. 10): The stress recovery parts in the broken fiber extend with time as a result of the relaxation of matrix shear stress around the fiber break, so that stress in the intact fibers near the broken fiber is increased. Goda thus concluded that the creep rupture shown in Figs. 9(a) to (c) is attributable to the relaxation of stress in broken fibers as well as to that of matrix normal stress.

Incidentally, let us point out in Figs. 9(a) to (c) that the instantaneous changes of strain due to fiber breaks were significant because of a small number of fibers in the analysis; in other words, if an infinite or a large number of fibers are considered, the increase of strain due to fiber breaks becomes continuous.
To analyze the creep rupture of unidirectional metal matrix composites, Du and McMeeking considered a cell in which a broken fiber of length $L$ is embedded in a creeping matrix (Fig. 11). They assumed that the cell is surrounded by intact fibers with uniform strain equal to overall strain $\varepsilon(t)$, and that shear deformation rate in the matrix is expressed as follows:

$$\frac{\dot{\varepsilon}}{G_m} + 3B\sigma^{-1} = \frac{\dot{u}_r(z, t) - \dot{u}_f(z, t)}{w},$$

where $G_m$, $B$ and $n$ are material constants, $\sigma$ the equivalent stress of Mises, $u_r$ the displacement on the cell surface, $u_f$ the displacement of the fiber, and $w$ the spacing between fiber surfaces. For the hexagonal array of fibers, $w$ is equal to

$$w = D(aV_f^{1/2} - 1),$$

where $a = (\pi/2\sqrt{3})^{1/2}$. Then, using further Eqs. (3) and (16), they derived the 3rd order differential equation

$$\frac{1}{w} \left( \frac{\partial u_r}{\partial t} - \frac{\partial u_f}{\partial t} \right) = \frac{D E_f}{4 G_m} \frac{\partial^3 u_f}{\partial z^3},$$

$$-\frac{3}{4} B D^{n-1} E_f \frac{\partial^3 u_f}{\partial z^3}.$$ 

They solved numerically the above equation by discretizing it with the finite differences of $z$ and $t$ and by allowing the interfacial slip occurring at shear stress $\tau_0$ at $t=0$.  

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First they analyzed the relaxation of stress in the broken fiber under the condition of constant overall strain. They thus found the following: As shown for example in Fig. 12, the stress in the broken fiber relaxes with time though very slowly in comparison with matrix normal stress \( \sigma_m \), and the stress profile in the broken fiber is approximated bilinearly fairly well even when the profile is affected considerably by matrix creep. Then they simulated creep rupture of composites by taking into account the effect of additional fiber breaks in an approximated manner. It thus turned out that Curtin-McLean’s model, in which only the relaxation of matrix normal stress is considered (Sec. 4.2), is not appropriate for long term creep in which the stress relaxation in broken fibers becomes significant (see Fig. 16 shown later).

7. Analytical Model for Stress Relaxation in Broken Fibers

As described in the two preceding sections, the stress relaxation in broken fibers plays an important role in simulating the creep rupture of unidirectional composites reinforced with long brittle fibers. This section deals with a model which can express analytically the stress relaxation in broken fibers.

Ohno et al.\(^{(9)}\) considered the same cylindrical cell as that employed by Du and McMeeking\(^{(11)}\). To formulate a simple model, however, Ohno et al. assumed further that the stress profile in the broken fiber in the cell is approximated bilinearly. Then, on the basis of energy balance in the cell, they derived the following evolution equation of interface shear stress \( \tau \), which is \( z \)-independent in the stress recovery part of \( 0 \leq z \leq \delta \):

\[
\begin{align*}
\dot{\tau} = & \left( A_f E_f \varepsilon^2 - A_m \varepsilon^2 + \frac{A_f E_f \tau \varepsilon}{G_m} - A_m \varepsilon^2 \right) \frac{\tau \dot{\varepsilon}}{2G_m},
\end{align*}
\]

where \( \dot{\varepsilon} \) denotes the shear creep rate of the matrix \( (=3B\sigma^{-1}z) \), and \( A_f \) and \( A_m \) indicate the cross areas of the fiber and matrix phases in the cell, respectively:

\[
\begin{align*}
A_f = & \frac{\pi}{4}D^2, \\
A_m = & \frac{\pi}{4}[(D+2w)^2-D^2].
\end{align*}
\]

For the cell subjected to constant overall strain \( \varepsilon_0 \) applied instantaneously at \( t = 0 \), Eq.(21) can be integrated analytically if the initial value of \( \tau \), i.e., the interfacial sliding stress \( \tau_0 \) is low, and if matrix normal stress \( \sigma_m \) is completely relaxed:

\[
\tau(t) = \tau_0 \left[ 1 + \frac{1}{(n+1)} \left( \frac{t}{t_o} \right)^{1/(n+1)} \right],
\]

where \( t_o \) denotes the relaxation time of \( \tau \) defined as \( t_o = -\tau_0/(\dot{\tau}(0)) \), i.e.,

\[
\tau = \frac{A_f E_f \varepsilon^2}{3^{(n+1)/2} A_m D^2}\frac{2G_m}{1}\frac{\varepsilon}{\varepsilon_0}
\tag{25}
\]

The stress recovery length \( \delta \) then has the time-dependent extension

\[
\delta(t) = \frac{D E_f \varepsilon}{4 \varepsilon_0} \left[ 1 + \frac{(n+1) t}{t_o} \right]^{1/(n+1)} \cdot
\tag{26}
\]

Figure 13 evaluates the analytical expression of \( \delta(t) \) above on the basis of the numerical analysis of Du and McMeeking discussed in the preceding section. In the figure, the solid lines indicate the time-dependent changes of \( \delta \) given by Eq.(26) whereas the solid circles denote the variation of \( \delta \) obtained by approximating bilinearly their numerical result shown in Fig. 12. It is seen that Eq.(26) gives a good agreement to the numerical analysis.

Figure 14 compares the change of \( \tau \) based on Eq. (26) with the relaxation of matrix normal stress \( \sigma_m \) under constant overall strain \( \varepsilon_0 \); the latter is expressed using Eq.(4) with \( \varepsilon_m = \varepsilon_0 \) as follows:

\[
\sigma_m(t) = E_m \varepsilon_0 \left[ 1 + \frac{1}{(n+1)} \left( \frac{t}{t_o} \right)^{-1/(n+1)} \right],
\]

where \( t_o \) denotes the relaxation time of \( \sigma_m \) defined as \( t_o = -\sigma_m(0)/\dot{\sigma}_m(0) \), i.e.,

\[
t_o = \frac{1}{B E_m \varepsilon_0}. \tag{28}
\]

Here it is noticed that Eqs.(24) and (27) satisfy

\[
\begin{align*}
\text{Series A, Vol. 41, No. 2, 1998}
\end{align*}
\]

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\[ d \ln \tau / dt = -1/(n+1) \] and \[ d \ln \sigma / dt = -1/(n-1) \] at \( t \) much larger than \( t_c \) and \( t_e \), respectively. It is also noticed that since the relaxation times \( t_c \) and \( t_e \) have the ratio

\[ \frac{t_t}{t_e} = \frac{A_t E_t}{3 A_n E_n} \left( \frac{E_n \sigma_0}{\sqrt{3} \Delta \alpha} \right)^{n+1}, \]  

we have \( t_t > t_e \) if \( A_t E_t \gg A_n E_n \) or \( (E_n \sigma_0/\sqrt{3} \Delta \alpha)^{n+1} \gg 1 \). Consequently, as illustrated schematically in Fig. 14, usually \( \tau \) relaxes much more slowly than \( \sigma_n \); in other words, the stress relaxation in broken fibers becomes significant in long term creep.

Miyake et al. verified the feature above experimentally. They used Raman spectroscopy to measure the stress relaxation in a broken fiber in a single carbon fiber/epoxy model composite subjected to constant overall strain. They thus found that the stress profile in the broken fiber changed a little after 1000 h whereas matrix normal stress relaxed to about a quarter of the initial value in 200 h. This agrees qualitatively with the relaxation curves illustrated in Fig. 14.

Simple models for the relaxation of \( \tau \) around fiber breaks were presented in other works: Ohno and Yamakawa obtained an analytical expression similar to Eq.(24) by imposing the deformation compatibility at broken fiber ends. Iyenger and Curtin, on the other hand, derived a relaxation equation of \( \tau \) by assuming that the stress recovery parts in broken fibers move rigidly, and they solved the relaxation equation analytically with respect to the three integers of \( n = 1, 2 \) and 3.

8. Analytical Estimation of Creep Rupture Time in Long Term Creep

This section deals with the analytical estimation of rupture time \( t_c \) in long term creep, in which the stress relaxation in broken fibers becomes noticeable. The problem of estimating \( t_c \) in long term creep is complicated, because the fiber breaks which occurred at different times in the progress of creep have different amounts of relaxation of \( \tau \), as illustrated in Fig. 15; i.e., \( \tau \) relaxes more significantly at older breaks, and if a break has just occurred \( \tau \) is equal to

\[ \delta(t) = \frac{D \varepsilon}{4 \pi (t)}, \]  

Consequently, Curtin’s model described in Sec. 4.1 becomes available, so that Eqs.(9) and (10) with \( \delta_n \) replaced by \( \delta(t) \) above give

\[ \sigma = E \varepsilon(t) \left[ 1 - \frac{\eta}{2 \tau(t)} \left( \frac{E \varepsilon(t)}{E \varepsilon} \right)^{n+1} \right] \]  

where \( E \varepsilon \) was defined by Eq.(12).

Ohno et al. assumed further that the relaxation function \( \tau(t) \) in Eq.(31) is approximated with the analytical solution (24) at the initial strain \( \varepsilon' \). They thus derived the overall strain \( \varepsilon \) versus time \( t \) relation

\[ t = \frac{t_t}{n+1} \left[ \frac{\left( 1 - \frac{\sigma}{V_r E \varepsilon} \right)}{S_r \tanh \left( \frac{S_r}{E \varepsilon} \right)} \right]^{n+1} - 1, \]

Then, substituting Eq.(32) into the rupture condition \( \varepsilon = \infty \), they obtained the strain and time at rupture as follows:

\[ \varepsilon_r = \frac{m+2}{m+1} \frac{\sigma}{V_r E \varepsilon}, \]

\[ t_r = \frac{t_t}{n+1} \left[ \left( \frac{V_r S_{\text{max}}}{\sigma} \right)^{n+1} - 1 \right]. \]

where \( S_{\text{max}} \) indicates the load carrying capacity of fibers expressed by Eq.(11). It is obvious from Eq. (35) that the above relations are effective for \( \sigma < V_r S_{\text{max}} \).

Figure 16 illustrates the prediction of Eq.(35), along with that of Eq.(14) and the numerical results of Du and McMeeking described in Sec. 6. It is seen from the figure that Eqs.(14) and (35) are responsible for short and long term creep ruptures, and that the predictions are fairly close to the numerical results in spite of the drastic simplifications introduced to derive them analytically. Figure 17 compares the analytical predictions of Eqs.(14) and (35) with the experiments shown in Fig. 1. As seen from the figure, especially the prediction for long term creep gives an overestimation to the experiments when \( V_r \) is taken to be the

Fig. 15 Distribution of interfacial shear stress \( \tau \) around fiber breaks in long term creep
Fig. 16 Relations between applied stress and creep rupture time \((n=3, m=5, E_f/E_m=3, z_i/S_i=0.01)\)\(^{49}\); short term predictions by Eq. (14), long term predictions by Eq. (35), and numerical simulations of Du and McMeeking\(^{41}\)

Fig. 17 Creep rupture time of SCS-6/Beta21S at 500°C\(^{49}\); experiments\(^{18}\); short term prediction by Eq. (14), and long term predictions by Eq. (35)

measured value of 0.35; however, the overprediction is reduced to some extent if \(V_f\) is taken to be 0.31 by taking account of the degradation of fibers near the edges of specimens\(^{49}\). Such a damage of fibers was observed by Schwenker et al.\(^{14}\)

9. Concluding Remarks

This paper described state of the art in creep rupture analysis of unidirectional composites consisting of long brittle fibers and creeping matrices. Studies on experiments, numerical simulations and analytical models related with the creep rupture were reviewed by treating mainly metal matrix composites. Main findings in the studies reviewed in the paper are as follows: (1) Matrix creep and statistical fiber fracture play important roles in the creep rupture, and two kinds of stress relaxation, i.e., relaxation of matrix normal stress and that of interfacial shear stress around fiber breaks, can occur to induce the creep rupture in short and long terms, respectively. (2) Now analytical models are available to understand and predict the creep rupture resulting from the two kinds of stress relaxation.

Models based on the assumption of global load sharing were dealt with in the present paper by targeting mainly metal matrix composites with weak interface. Generally speaking, however, it is necessary to consider local load sharing around fiber breaks especially for polymer matrix composites. Moreover, the time-dependent development of interfacial debonding, which was pointed out in creep experiments of carbon fiber/epoxy microcomposites by Phoenix et al.\(^{43}\) and Otani et al.\(^{44}\), seems to be substantial in analyzing the creep rupture. Besides the time-dependent damage of fibers resulting from slow crack growth can be important\(^{44}\).

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Nobutada Ohno has been a Professor in the Department of Mechanical Engineering at Nagoya University since 1994. Previously, he was an Associate Professor there (1988–1994) and taught at Toyohashi University of Technology (1980–1988). He has published extensively on constitutive modeling of inelastic behavior of metallic materials and inelastic analysis of structures. Recently, he has been involved in studies on experiments and analysis of time-dependent behavior of composite materials. He is an Editorial Advisory Board Member of International Journal of Plasticity and an Editorial Board Member of Materials Science Research International. He was born in September 17, 1950.