Endochronic Simulation for Multiaxial Creep*

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This study replaces the material parameter in the scaling function of the intrinsic time scale, as proposed by Pan et al.(1), with an exponential function. Thus, the endochronic theory can be used to simulate the creep behavior of material under multiaxial loading. Experimental creep-test data obtained by Ohashi et al.(2) on type 304 stainless steel at 650°C under nonproportional repeated loading are used for evaluating the theoretical simulation. In addition, a simple endochronic creep model proposed by Lee(3) and four creep models employed by Kawai(4) are also included herein. It is shown that the experimental results are aptly described by the novel theoretical approach demonstrated from a comparison with the theoretical prediction from models employed by Lee(3) and Kawai(4).

Key Words: Endochronic Theory, Multiaxial Creep, Nonproportional Repeated Loading

1. Introduction

Understanding the material response under multiaxial creep is of importance in designing mechanical or structural systems such as power plant or components of nuclear reactors. Some models may be simple for analyzing the material response under complex creep loading conditions; however, poor predictions of the material behavior are observed when compared with experimental findings. As generally known, the prediction accuracy for real engineering problems relies on the modelling of the components, the component materials, and loading conditions. Owing to the rapid progress in computer technology, some realistic and complex models can be implemented to satisfy an increasing demand to accurately simulate the response of real materials.

Endochronic theory, as originally proposed by Valanis(5), is a different approach to describe the elasto-plastic behavior of history-dependent materials. The theory is based on irreversible thermodynamics and employs the concept of internal variables. The key concept of the theory is the intrinsic time measure which is originally defined in terms of a total strain tensor(5). The definition of the intrinsic time measure has led to difficulties in cases where the history of deformation involves unloading. This was subsequently modified by reformulating the intrinsic time measure in terms of the plastic strain tensor(5). After its modification, the theory has been successfully applied in describing material responses under diverse loading conditions (Wu and Yip(6), Valanis and Fan(7), Valanis and Lee(8), Watanabe and Atluri(9), Valanis and Read(10), Im and Atluri(11), Wu et al.(12), Imai and Xie(13), Fan and Peng(14), Peng and Ponter(15), Wu et al.(16), Lee(17), Pan et al.(18), Yeh et al.(19), and Pan and Lee(20)).

The investigation of creep using endochronic theory was first reported by Watanabe and Atluri(21). They introduced a formulation of the scaling function of the intrinsic time scale, as was initially suggested by Valanis(22). The material behavior of uniaxial creep for type-304 stainless steel at elevated temperature was discussed in their study. Lee(23) used a simple power form to formulate the strain-rate sensitivity function and the internal friction of material remained constant, the endochronic theory was extended to describe the material response under multiaxial creep.

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Experimental creep data of type 304 stainless steel at 650°C tested by Ohashi et al. under repeated multiaxial loading were compared with the theoretical simulation by him. However, in his study, only a part of the repeated multiaxial loading was discussed and the simulated results were not satisfactory when compared with the experimental data. Wu and Ho proposed a different formulation of the scaling function of the intrinsic time scale. The endochronic theory was used to investigate the transient creep with carefully monitored precreep loading stage, either loaded at a prescribed constant strain-rate or at a constant stress-rate. The uniaxial viscoplasticity-creep interaction of annealed 304 stainless steel was investigated experimentally and theoretically in their study. Lee used a different formulation of the intrinsic time measure to investigate the transient creep of metals for variable temperature. The creep under step-up and step-down temperatures with a constant axial stress for SUS 304 stainless steel were theoretically studied in his research. Pan et al. introduced a novel formulation of the scaling function of the intrinsic time scale. The newly formulated scaling function and the rate-sensitivity function, which was proposed by Pan and Chern, were used in the endochronic theory for simulating the rate-dependent elastoplastic behaviors of materials. Several cases of rate-dependent elastoplastic material responses were discussed separately in their study. Experimental data found in literature of 304 stainless steels and Ti-7Al-2Cb-1Ta titanium alloy for rate-dependent elastoplastic responses were used for comparison. It has been shown that the theory satisfactorily simulated the experimental results.

In this paper, we extend the theoretical approach, as derived by Pan et al., to describe the creep behavior of material subjected to multiaxial loading. The material parameter of the scaling functions, as proposed by Pan et al., is replaced by an exponential function. Thus, the anisotropic hardening of the creep strain for multiaxial loading can be described. Experimental data for creep-test under multiaxial repeated stress changes of type 304 stainless steel at 650°C tested by Ohashi are used for examining the theoretical simulation. The corresponding experimental results are creep-tests under (i) pure tension, (ii) pure torsion, and (iii) multiaxial nonproportional repeated loading conditions. Furthermore, the theoretical results obtained by Lee and theoretical results by using four different creep models employed by Kawai are also included herein. It is shown that, through comparison with the experimental data and theoretical results obtained by Lee and Kawai, material response under multiaxial creep can be adequately simulated by new approach.

2. Endochronic Constitutive Equations for Multiaxial Creep

By using a group of exponential decaying functions to form the kernel function \( \rho(x) \), the increment of the deviatoric stress tensor \( ds \) of endochronic theory for homogeneous and initially isotropic materials under creep is expressed by:

\[
\frac{ds}{dz} + \alpha_r s_r = 2C_r \frac{de}{dz}, \quad r = 1, 2, \cdots, n
\]

and

\[
ds = \sum_{r=1}^{n} \frac{ds}{dz} = 2\rho(0)\frac{de}{dz} - \sum_{r=1}^{n} \alpha_r s_r dz.
\]

in which \( C_r \) and \( \alpha_r \) are material parameters, \( z \) is the intrinsic time scale, and \( e^c \) is the deviatoric creep strain tensor, which is defined as

\[
de^c = de - \frac{ds}{2\mu_0}
\]

in which \( e \) denotes the deviatoric strain tensor, and \( \mu_0 \) is the elastic shear modulus. The intrinsic time measure \( \xi \) is defined as

\[
de^c = \frac{k\|de^c\|}{\sqrt{f(\xi)}}
\]

in which \( \| \cdot \| \) is the Euclidean norm, and \( k \) is the rate-sensitivity function. The quantity of \( k \) is treated as unity for describing the material response of rate-independent effect. The intrinsic time scale \( z \), as proposed by Valanis, is expressed as

\[
dz^2 = \left( \frac{de^c}{f(\xi)} \right)^2 + g^2 dt^2
\]

where \( g \) is a scaling function which can be used to describe the material behavior under creep, \( t \) is the Newtonian time, and \( f(\xi) \) is a material function which can be expressed as

\[
f(\xi) = 1 - Ce^{-\beta \xi}, \quad \text{for} \ C < 1
\]

in which \( C \) and \( \beta \) are material parameters.

For describing the material behavior under creep, Watanabe and Atluri proposed a formulation of the scaling function \( g \) to investigate the material behavior under creep, which is

\[
g = f(\xi) \left( \frac{\|s^0\| - r}{\|s^0\|} \right)^{1-n}
\]

in which \( r \) is the back stress tensor, \( s^0 \) is the initial yield stress in shear, and \( B \) and \( m \) are material parameters. The uniaxial creep-plasticity interaction for type-304 stainless steel at elevated temperature was investigated by them. For describing the material behavior under viscoplasticity-creep interaction, not only the scaling function \( g \) must be formulated, the rate-sensitivity function \( k \) must be used. Wu and Ho proposed the scaling function \( g \) by

\[
g = B \sqrt{1 - k \frac{\|s^0\| - r}{\|s^0\|}}
\]

where \( s^0 \) is the yield stress at a reference plastic strain.
rate (usually the lowest strain-rate in the test), and
B is a scaling function which is defined as
\[ B = \frac{b g^2}{\| \bar{\xi}_0 - \bar{\zeta} \|} \left( \frac{k \sum \alpha_r \bar{s}_r}{\sigma(0) f} \right)^m \]  \hspace{1cm} (9)

in which b and m are material parameters. Substitution of Eq.(8) into Eq.(5) yields
\[ \eta = Bd\eta. \]  \hspace{1cm} (10)

When the material function \( f(\xi) \) equals to one, the magnitude of b can be shown to be the initial creep-rate in their study. The material behavior of uniaxial viscoplasticity-creep interaction for 304 stainless steel was also experimentally investigated by Wu and Ho\(^{(24)}\). Note that the formulation of the rate-sensitivity function \( k \) proposed by Wu and Yip\(^{(27)}\) was used by Wu and Ho\(^{(24)}\) in their investigation. For describing the rate-dependent material behaviors (viscoplasticity, creep, relaxation and their interactions), Pan et al.\(^{(1)}\) presented a novel formulation of the scaling function \( g \), which is
\[ g = \sqrt{b} \left( \frac{k \sum \alpha_r \bar{s}_r}{\sigma(0) f} \sum \frac{e^{-\alpha_r t}}{e^{-\alpha_r t}} \right)^{2m} \frac{\Sigma^2}{f^2} \]  \hspace{1cm} (11)

where b and m are material parameters. The formulation of the rate-sensitivity function \( k \) proposed by Pan and Chern\(^{(26)}\) was used in Eq.(11) in their study. The formulation of Eq.(11) came from the formulation of Eq.(8). For considering the formulation of \( g \) without any yield condition, two terms of \( \sum \alpha_r \bar{s}_r \) and \( \sigma(0) \) are used in stead of \( \| \bar{\xi}_0 - \bar{\zeta} \| \) and \( \bar{s}_g \). In addition, the term \( \frac{\Sigma^2}{f^2} \) used in Eq.(11) is that if the Eq. (11) substitutes into Eq.(5), the quantity \( B \) (Eq.(12)) can be obtained a more simpler form than that obtained by Wu and Ho\(^{(24)}\) (Eq.(9)). The formulation of the rate-sensitivity function \( k \) proposed by Pan and Chern\(^{(26)}\) was used in Eq.(11) in their study. Substituting Eq.(11) into Eq.(5) leads to an equation same as Eq.(10). However, the quantity \( B \) is
\[ B = \frac{b}{\sigma(0) f} \left( \frac{k \sum \alpha_r \bar{s}_r}{\sigma(0) f} \sum \frac{e^{-\alpha_r t}}{e^{-\alpha_r t}} \right)^m \]  \hspace{1cm} (12)

Wu and Ho\(^{(24)}\) discovered that the initial creep strain-rate is a continuation of the preloading strain-rate for strain-controlled or stress-controlled uniaxial test. It can be expressed as
\[ (\xi)_{\text{last}} = (\xi)_{\text{initial}} \]  \hspace{1cm} (13)

where \( (\xi)_{\text{last}} \) is the uniaxial last preloading strain-rate before creep, and \( (\xi)_{\text{initial}} \) is the uniaxial initial creep strain-rate. This concept has also been used in the work of Pan et al.\(^{(1)}\).

Due to lack of the experimental data for the multiaxial viscoplasticity-creep interaction, the norm of the initial creep strain-rate is assumed here to be a continuation of the norm of the last strain-rate before creep in this study, which can be expressed as
\[ \| \xi \|_{\text{last}} = \| \xi \|_{\text{initial}} \]  \hspace{1cm} (14)

where \( (\xi)_{\text{last}} \) is the last preloading strain-rate tensor before creep, and \( (\xi)_{\text{initial}} \) is the initial creep strain-rate tensor. Based on this proposal, the quantity b can be determined, which is related to the norm of the last strain-rate before creep. It has been shown by Pan et al.\(^{(1)}\) that the simulated result by using Eq.(11) is correlated well with the experimental data for uniaxial creep tested by Wu and Ho\(^{(24)}\). However, if the multiaxial creep is considered, poor simulation has been found by using Eq.(11). Therefore, to simulate the anisotropic hardening of the creep strain behavior under multiaxial loading, we replace the material parameter m in Eq.(11) by a function \( \beta(1-e^{-m}) \), the scaling g is expressed to be
\[ g = \sqrt{b} \left( \frac{k \sum \alpha_r \bar{s}_r}{\sigma(0) f} \sum \frac{e^{-\alpha_r t}}{e^{-\alpha_r t}} \right)^{2m(1-e^{-m})} \frac{\Sigma^2}{f^2} \]  \hspace{1cm} (15)

where m is the material parameter. Substitution of Eq.(15) into Eq.(5) yields an equation same as Eq.(10) where the quantity B is
\[ B = b \left( \frac{k \sum \alpha_r \bar{s}_r}{\sigma(0) f} \sum \frac{e^{-\alpha_r t}}{e^{-\alpha_r t}} \right)^{2m(1-e^{-m})} \]  \hspace{1cm} (16)

3. Determination of the Material Parameter b for Combined Axial-Torsional Creep

Based on the experimental data tested by Ohashi et al.\(^{(1)}\), the increments of the total and deviatoric stress tensors and total strain tensor for a combined axial-torsional creep, are expressed to be
\[ \Delta \sigma = \left[ \begin{array}{c} \Delta \sigma_{xx} \\ \Delta \sigma_{yy} \\ \Delta \sigma_{zz} \\ \Delta \sigma_{xy} \\ \Delta \sigma_{yz} \\ \Delta \sigma_{zx} \end{array} \right] \quad \Delta \varepsilon = \left[ \begin{array}{c} \Delta \varepsilon_{xx} \\ \Delta \varepsilon_{yy} \\ \Delta \varepsilon_{zz} \\ \Delta \varepsilon_{xy} \\ \Delta \varepsilon_{yz} \\ \Delta \varepsilon_{zx} \end{array} \right] \]  \hspace{1cm} (17.a, b)

in which \( \Delta \varepsilon \) is the axial creep strain and \( \varepsilon_t \) is the torsional shear creep strain. If the assumption of incompressible condition is satisfied under creep, one writes
\[ \Delta \varepsilon_{\text{aa}} = 3K \Delta \varepsilon_{\text{aa}} \]  \hspace{1cm} (18)

where \( \sigma_{\text{aa}} \) and \( \varepsilon_{\text{aa}} \) are the trace of stress and strain tensors, and K is the elastic bulk modulus. Substituting of Eqs.(17.a) and (17.b) into Eq.(18) yields
\[ \Delta \varepsilon = -2 \Delta \varepsilon_{\text{xx}} \]  \hspace{1cm} (19)

Thus, the deviatoric creep strain tensor \( \Delta \varepsilon_{\text{xx}} \) is equal to the creep strain tensor \( \varepsilon_{\text{xx}} \). By substituting Eqs.(3)\(^{(1)}\), (17.a), (17.b) and (19) into Eq.(2)\(^{(1)}\), one writes
\[ \Delta \varepsilon_{\text{xx}} = \frac{2 \alpha_r (\xi_{\text{ii}})}{2 \alpha_r (\xi_{\text{ii}})} \Delta \xi_{\text{xx}} \]  \hspace{1cm} (20)

and
\[ d\xi^r = \frac{\sum_{r=1}^{n} \alpha_r (s_{1r})}{2 \rho(0)} dz. \]

By substituting Eq.(17.b) into Eq.(14), one writes

\[ \dot{\xi}^r = \left( \dot{\xi}_r \right)_{\text{inn}}. \]

Substitution of Eqs.(19), (20) and (21) into Eq.(22) leads to

\[ \dot{\xi} = \frac{\left( \sum_{r=1}^{n} \alpha_r (s_{1r}) \right)^{1/2}}{2 \rho(0)} \left( \frac{\sum_{r=1}^{n} \alpha_r (s_{1r}) r^2}{2 \rho(0)} \right)^{1/2} \]

From Eqs.(10) and (16), the quantity of \( \dot{\xi} \) is

\[ \dot{\xi} = \dot{\xi}_0 \left( \frac{k \sum_{r=1}^{n} \alpha_r (s_{1r}) \sum_{r=1}^{n} e^{-\alpha_r t}}{\rho(0) \psi} \right)^{1/(1-\epsilon^{m})} \]

The quantities of \( \sum_{r=1}^{n} \alpha_r (s_{1r}) \) are known values which are related to the last stress condition before creep, \( \xi \) is also the known magnitude of the last intrinsic time measure before creep. Once the quantity of \( \left( \dot{\xi}_r \right)_{\text{inn}} \) is known, the magnitude of \( \dot{\xi} \) can be determined from Eqs.(23) and (24).

4. Comparison and Discussion

In this section, we compare the theoretical simulation with the experimental data tested by Ohashi et al.\(^{[23]}\) In our study, the kernel function of the theory is considered to be composed of three terms of exponentially decaying function. Therefore, the material parameters of the theory can be determined according to the method proposed by Fan\(^{[28]}\). Due to lack of the experimental result for the pre creep loading stage for the experimental data tested by Ohashi et al.\(^{[23]}\), material behavior of the same material at the same elevated temperature for combined axial-torsional loading strain-path tested by Inoue et al.\(^{[29]}\) are considered as the material response for the pre creep loading. Material parameters were determined by Pan and Chern\(^{[26]}\) to be: \( \mu_0 = 42667 \) MPa, \( K = 112022 \) MPa, \( C_1 = 5.6 \times 10^5 \) MPa, \( \alpha_1 = 7680 \), \( C_2 = 6.95 \times 10^5 \) MPa, \( \alpha_2 = 510 \), \( C_3 = 1.6 \times 10^5 \) MPa, \( \alpha_3 = 110 \), \( C = 0.235 \), \( \beta = 9.2 \). Note that the magnitude of the rate-sensitivity function \( k \) is treated as unity for the creep-plasticity interaction in this case. In addition, a different endochronic creep model proposed by Lee\(^{[23]}\) and four creep models (prototypal, kinematic, isotropic and combined hardening creep models) employed by Kawai\(^{[24]}\) are also included in this study.

Ohashi et al.\(^{[19]}\) conducted creep tests of the thin-walled tubular specimens for type 304 stainless steel at the temperature 650° under pure tension, pure torsion and combined axial-torsional loading conditions. Figure 1 displays the creep stress-time curves for combined axial-torsional loading condition where \( \sigma \) is the axial stress, \( \tau \) is the shear stress, and \( T \) is the creep time. In each cycle, the specimen was initially loaded in pure torsion to \( \sigma_A \); then, held the in constant for \( T \) hours followed by unloading; then, the specimen was loaded along the controlled direction \( \theta \) shown in Fig. 1 to \( \sigma_B \); then, held the stress in constant for \( T \) hours followed by unloading. This cycle of square-wave type was repeated five times under the condition of \( T = 8 \) hours and \( |\sigma_A| = |\sigma_B| = 137.3 \) MPa. The controlled direction \( \theta \) are 30, 60, 90, 120, 150, and 180 degree. Based on the uniaxial creep stress-time curve, the material parameter \( m \) are determined to be 16.6. Figure 2 presents the creep strains vs. time curves for the creep tests under pure tension and pure torsion. The experimental curves indicate the hold stress for creep are \( \sigma = 117.7 \) and 137.3 MPa for pure tension and \( \sqrt{3} \tau = 156.9 \) MPa for pure.
tortion. Figures 3–7 present the axial creep strain $\varepsilon^c$ and shear creep strain $\gamma^c$ vs. time under repeated combined axial-torsional loading for the controlled loading angle $\theta = 30, 60, 90, 120$ and 150 degree, respectively. The axial and shear stress trajectories are depicted in the insert figure of each figure. These figures also contain theoretical results obtained by the endochronic creep model proposed by Lee\textsuperscript{63} and four creep models employed by Kawai\textsuperscript{64}. Figure 8 demonstrates the shear creep strain variation under repeated combined axial-torsional loading for the controlled loading angle of $\theta = 180$ degree. According to Fig. 1, the loading condition for this case is the alternating torsion. It is seen that the amplitude of $\gamma^c/\sqrt{3}$ decreases gradually with the increase of the number of cycles. This is due to the magnitude of the term $2\beta(1-e^{-\mu t})$ of the scaling function $g$ in Eq. (15). Based on the mathematical characteristic of the term $2\beta(1-e^{-\mu t})$, i.e. $2\beta(1-e^{-\mu t}) = 0$ for $\xi = 0$ and $2\beta(1 - e^{-\nu t}) = 2\beta$ for $\xi = \infty$, the amplitude of $\gamma^c/\sqrt{3}$ will finally reach a constant. It is observed from Figs. 3–8 that the proposed endochronic approach correlates
well with the experimental results better than the theoretical simulations reported by Lee and Kawai.

5. Conclusion

By using a function to replace the material parameter \( m \) in the scaling function, as proposed by Pan et al., the endochronic theory can be used to investigate the creep behavior of material under multiaxial loading, especially, the anisotropic hardening of the creep strain. By assuming that the norm of the initial creep strain-rate tensor is a continuation of the norm of the last strain-rate tensor before creep, the material parameter \( b \) in the scaling function can be determined. Creep-test data of the thin-walled tubular specimens for type 304 stainless steel at the temperature 650° under pure tension, pure torsion and combined axial-torsional loading conditions, as obtained by Ohashi et al., are used to evaluate the theoretical simulation. In addition, theoretical results obtained by a simple endochronic creep model proposed by Lee and four different creep models employed by Kawai are also included in this paper. It is demonstrated, through comparison with experimental data and predictions by creep models used by Lee and Kawai, that the prediction by our approach are the best correlation with the experimental results.

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