Study on Factors Controlling Low Cycle Fatigue Life*

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In this paper, an effective and convenient method for estimation of low cycle fatigue life based on the small-crack growth law was proposed, and the factors that control fatigue life were clarified. The main results obtained are as follows: (1) Low cycle fatigue life is controlled by three factors: crack initiation length, crack propagation limit length and relative crack growth rate. (2) Fracture in pearlitic spheroidal graphite cast iron (PDI) starts from a microshrinkage that exists on the neighboring surface in the case of a cylindrical specimen; thus, the crack initiation length in PDI is larger than that in its matrix material (0.85% carbon steel; JIS SK5). (3) Preexisting cracks originating from defects shorten the crack propagation limit length. (4) The relative crack growth rate is accelerated by the increase in crack opening displacement range near the crack tip.

Key Words: Fatigue, Low Cycle Fatigue, Life Prediction, Crack Propagation, Crack Opening Displacement, Spheroidal Graphite Cast Iron, Microshrinkage, Small-Crack Growth Law, Crack Initiation

1. Introduction

Various investigations have been conducted on fatigue, which is the major cause of failure troubles in machines or structures. Recently, from the point of optimizing the utilization of resources, making materials thinner and lighter are required. Therefore, it is necessary to examine the strength of materials against low cycle fatigue. Since the proposal of the low cycle fatigue life law by Manson in 1953 and Coffin in 1954, various studies have been carried out on low cycle fatigue. It is well known that the Manson-Coffin law is used for achieving reliable design.

At present, in a damage-allowable design, the evaluation of safety in the finite life range is neces-

sary, and the evaluation of crack propagation process based on fracture mechanics is important. In a high-stress region, when the condition of small-scale yielding is not satisfied, a number of elasto-plastic fracture mechanics parameters such as J integral have been proposed. Elasto-plastic fracture mechanics parameter is an effective parameter for crack growth rate determination. One of us carried out a series of research concerning the propagation of small cracks, and found that the crack growth rate of a plane specimen is determined uniquely by the term $\sigma_a^n l^{(1)}$. Then, the validity of the small-crack growth law was confirmed by its application to the results obtained using various kinds of materials under various stress conditions⁽²⁾⁻⁽⁴⁾.

$$\frac{dl}{dN} = C \sigma_a^n \cdot l, \quad (1)$$

where dl/dN is crack growth rate, σ_a is the nominal stress amplitude, l is the crack length, and C and n are constants.

In general, fatigue life in metals is divided into the two: crack initiation life and crack propagation life (crack propagation life is further divided into three lives). Since most of the fatigue life is consumed by

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the growth of a small crack, the crack growth rate is considered as a factor that controls fatigue life. Estimations of fatigue life by means of fracture mechanics have been reported⁽⁴⁾⁻⁽⁷⁾. Because there is no known method for calculating crack initiation life of a plane specimen⁽⁸⁾, the ratio of crack propagation life to total fatigue life⁽⁴⁾ or the crack initiation life obtained from experimental results⁽⁵⁾⁻⁽⁷⁾ is used for an accurate estimation of fatigue life. However, this method of fatigue life estimation differs depending on the material used or its shape because the ratio of crack propagation life to total fatigue life differs.

In this study, an evaluation of low cycle fatigue life based on the small-crack growth law was made, focusing on crack growth rate during stable crack propagation. Then, an effective and convenient method for estimating low cycle fatigue life was proposed, and the factors that control fatigue life were clarified by relating crack initiation life to the crack initiation length.

Plastic-strain-controlled low cycle fatigue tests were also carried out on pearlitic spheroidal graphite cast iron (JIS FCD 700; PDI) and its matrix material (0.85% carbon steel; JIS SK5). The effect of defects on fatigue life and the factors that control fatigue life were examined from a microscopic point of view.

In addition, the present results serve as guidance in the development of new materials, and contribute toward clarification of the physical background between fracture ductility and fatigue life.

2. Relationship between Small-Crack Growth Law and Factor Controlling Low Cycle Fatigue Life

In order to evaluate low cycle fatigue life N_f of plane specimens, it is advantageous to divide the fatigue life into four stages:

- (1) the period before the initiation of grain-sized crack (crack initiation life: N_i)
- (2) the propagation period of microcrack (crack propagation period in which propagation behavior is affected by the microstructure: N_{p1})
- (3) the stable propagation period of crack (N_{p2})
- (4) unstable propagation period up to failure (N_{p3})

Generally, the period from stages (2) to (4) is called propagation life N_p .

Figure 1(a) shows the relationship between crack length l and the number of cycles N for an annealed 0.15% carbon steel under axial direction loading. Low cycle fatigue tests were carried out using cylindrical specimens ($l_0/d_0=1.5^{(9)}$) and the machine was set in such a way that the plastic strain range $\Delta\varepsilon_p$ is constant ($\Delta\varepsilon_p=0.01$). As seen from the figure, the relationship between crack length l and

number of cycles N is not simple. Therefore, it is difficult to directly estimate the factors that control crack growth.

The logarithm of crack length $\ln l$ is used and plotted against the number of cycles N . Figure 1(b)

Fig. 1 Schematic illustration of crack growth curve

shows the relationship between $\ln l$ and N for an annealed 0.15% carbon steel. In this figure, the four stages mentioned above are seen clearly. Figure 1(c) shows a schematic illustration of the crack growth curve of Fig. 1(b). Here, l_0 and l_f are crack lengths at $N=N_i$ and $N=N_f$, respectively. Also, it is known that $\ln l$ is almost directly proportional to N in the stable propagation period of crack $N_{p2}^{(3),(4)}$. The crack lengths at the lowest and the highest limit of N_{p2} are given as l_1 and l_2 , respectively. Using Eq. (1), the fatigue life can be predicted approximately as follows.

$$\int_{l_1}^{l_2} \frac{dl}{l} = \int_{N_1}^{N_2} C \sigma_a^n dN \quad (2)$$

$$\ln \frac{l_2}{l_1} = C \sigma_a^n (N_2 - N_1) \quad (3)$$

Here, $N_2 - N_1$ is the stable propagation period of crack N_{p2} and is different from N_f . Such a tendency is distinct in ductile materials in which the crack initiates from a slip band. Then, the ratio α (N_{p2} to N_f) is used for the estimation of N_f . The fatigue life N_f is represented by the following equation.

$$N_f \approx \ln \frac{l_2}{l_1} / \alpha C \sigma_a^n \quad (4)$$

In Eq. (4), however, the problem of the determination of α is left unsolved.

On the other hand, most of the fatigue life in spheroidal graphite cast iron or eutectoid steel is consumed by the growth of a small crack⁽³⁾. In this case, $\ln l$ is almost directly proportional to N from the initiation of crack to the fracture, as shown by a broken line in Fig. 1(c). When the straight line is fitted to crack growth curve and the crack lengths at $N=0$ and $N=N_f$ are given as l_0^* and l_f^* , respectively, the next relationships nearly hold: $l_1 = l_0 = l_0^*$, $l_2 = l_f = l_f^*$. That is, Eq. (4) is rewritten as follows:

$$N_f \approx \ln \frac{l_f^*}{l_0^*} / C \sigma_a^n \quad (5)$$

Because the nonlinear parts are considered in l_0^* and l_f^* , Eq. (5) holds for the case in which $\ln l$ is not directly proportional to N from the initiation of crack to the fracture. Therefore, if l_0^* and l_f^* can be predicted, then Eq. (5) is an effective and convenient method for estimating low cycle fatigue life.

From Eqs. (5) and (1), fatigue life N_f is rewritten as follows:

$$N_f \approx \ln \frac{l_f^*}{l_0^*} \frac{1}{\left(\frac{dl/dN}{l} \right)} \quad (6)$$

That is, fatigue life N_f can be obtained by multiplying the logarithm of the ratio of crack propagation limit length l_f^* to crack initiation length l_0^* , $\ln(l_f^*/l_0^*)$, by the reciprocal of relative crack growth rate $(dl/dN)/l$. In this case, stress amplitude σ_a is used for estimating low cycle fatigue life. On the other hand, σ_a and plastic strain range $\Delta \epsilon_p$ show one-to-one corre-

spondence^{(3),(7)}; thus, $\Delta \epsilon_p$ can also be used. Therefore, for a material in which most of its fatigue life is consumed by the growth of a small crack, except in a special case, the fatigue life is controlled by three factors.

- Crack initiation length: l_0^*
- Crack propagation limit length: l_f^*
- Relative crack growth rate: $(dl/dN)/l$.

Here, l_0^* and l_f^* do not change with materials and plastic strain range, and the relative crack growth rate $(dl/dN)/l$ is constant regardless of crack length. Therefore, the above three factors can be considered as three constants. Various phenomena can be rationally explained using these three factors.

In this study, we deal with spheroidal graphite cast iron which is considered to be a defective material, and investigate the difference in fatigue life between spheroidal graphite cast iron and its matrix material by using the three factors.

3. Materials, Specimens and Experimental Procedures

The spheroidal graphite cast iron used was pearlitic (PDI). Carbon steel (SK5, 0.85%) was used as the matrix material. SK5 was thermally refined so that it had the same degree of hardness as the matrix in PDI. It was normalized at 1173 K for 1 hour. The microstructures, chemical composition and mechanical properties are shown in Fig. 2, Table 1 and 2,

Table 1 Chemical composition (wt. %)

	C	Si	Mn	P	S	Mg	Cu
PDI	3.68	2.07	0.355	0.021	0.007	0.038	0.599
SK5	0.85	0.28	0.34	0.015	0.005	—	—

Table 2 Mechanical properties (MPa, %)

	$\sigma_{0.2}$	σ_B	ψ	H_b	H_v (1/20)
PDI	463	813	6.6	259	*287
SK5	515	967	18.3	—	288

ψ : Reduction of area *: Hardness of matrix

Fig. 2 Microstructures

Table 3 Characteristics of spheroidal graphite

	D_m (μm)	n_g ($1/\text{mm}^2$)	f_g (%)	h_g (%)
PDI	28.9	131	8.38	81.2

D_m : Average graphite diameter n_g : Nodule count

f_g : Area fraction of graphite h_g : Nodularity

Fig. 3 Shape and dimension of specimen

respectively. Table 3 shows some characteristics of spheroidal graphite which are measured by the image processing analysis technique. The threshold value of graphite is $12\ \mu\text{m}$.

Figure 3 shows the shape and dimensions of the specimen. In this study, cylindrical specimens were used. The diameter d_0 and length l_0 of the central part are 6 mm and 9 mm, respectively. The aspect ratio l_0/d_0 of the central part is 1.5. We previously suggested that an aspect ratio of 1.5 is necessary to obtain true fatigue behavior⁽⁹⁾. The specimens were finished with diamond paste. Before testing, the central part of the specimens was etched lightly in 3% nital.

Fatigue tests were carried out using a hydraulic servo testing machine with a capacity of 49 kN and frequency ranging from 0.03 to 0.1 Hz. Strain was measured by an axial direction extensometer and the machine was set in such a way that the plastic strain range is constant.

Plastic replicas were used to measure the crack length and to observe successive fatigue damage on the specimen surface. Also, SEM was used to observe fractured sections.

4. Experimental Results and Discussion

4.1 Evaluation of fatigue life of PDI based on the small-crack growth law

Figure 4 shows the relationship between plastic strain range $\Delta\epsilon_p$ and number of cycles to failure N_f obtained for PDI and SK5. These experimental data satisfy the following Manson-Coffin law and the slopes of the two lines are almost the same.

$$\Delta\epsilon_p \cdot N_f^{0.59} = 0.157 \quad (\text{PDI}) \quad (7)$$

$$\Delta\epsilon_p \cdot N_f^{0.60} = 0.743 \quad (\text{SK5}) \quad (8)$$

From Fig. 4, the fatigue life of PDI is shorter than that of the matrix material by approximately one-tenth.

Figure 5 shows a comparison of the crack growth

Fig. 4 Low cycle fatigue curves in PDI and SK5

Fig. 5 Comparison of the crack growth curves between PDI and SK5

curves between PDI and SK5 in the case of $\Delta\epsilon_p = 0.01$. In the same manner as SK5, most of the fatigue life of PDI is consumed by the growth of a small crack, and the small-crack growth law holds⁽³⁾. Thus, logarithm of crack length $\ln l$ is almost directly proportional to number of cycles N . However, the variation range of the crack growth curve for PDI is larger than that for SK5. From Fig. 5, the three factors that control low cycle fatigue life are obtained by using the average value of the crack growth curves. In the case of $\Delta\epsilon_p = 0.01$, the three factors l_0^* , l_f^* and $(dl/dN)/l$ are as follows: $l_0^* = 0.09\ \text{mm}$, $l_f^* = 0.9\ \text{mm}$ and $(dl/dN)/l = 1.3 \times 10^{-2}$ for PDI; and $l_0^* = 0.006\ \text{mm}$, $l_f^* = 2.0\ \text{mm}$ and $(dl/dN)/l = 3.6 \times 10^{-3}$ for SK5.

Figure 6 shows a comparison of the average crack growth curves between PDI and SK5. Using Eq. (6), the fatigue life of PDI and SK5 is predicted as follows:

$$N_f = \ln\left(\frac{0.9}{0.09}\right) \left(\frac{1}{1.3 \times 10^{-2}}\right) = 2.3 \times 76.9$$

$$= 177 \quad (\text{PDI}) \quad (9)$$

$$N_f = \ln\left(\frac{2.0}{0.006}\right) \left(\frac{1}{3.6 \times 10^{-3}}\right) = 5.8 \times 278$$

$$= 1612 \quad (\text{SK5}). \quad (10)$$

Fig. 6 Comparison of the average crack growth curves between PDI and SK5

Fig. 7 Fracture origin in spheroidal graphite cast iron stated in previous studies (High cycle fatigue)

The prediction obtained using Eq.(6) shows good agreement with the experimental results. Accordingly, it is seen that the cause of the marked decrease in the fatigue life of PDI is explained by the three factors. In the next section, the difference between PDI and SK5 in terms of the three factors is investigated.

4.2 Investigation of the three factors

4.2.1 Crack initiation length: l_0^* Crack initiation length l_0^* is the value obtained by fitting a straight line to the crack growth curve shown in Fig. 1(b) and extrapolating it to $N=0$. In PDI and SK5, the next relationship nearly holds: $l_1 = l_0 = l_0^*$. Using our definition, crack initiation length l_0^* in PDI is about 90 μm and l_0^* in SK5 is about 6 μm . l_0^* in PDI is larger than that in SK5 and varies markedly.

In general, because the fracture starts from a defect, it is assumed that crack initiation length in a defective material is larger than that in a matrix material. It has been considered that, in spheroidal graphite cast iron, the kinds of defects in fracture origin are graphite, microshrinkage and dross. Since the effect of defects on low cycle fatigue life is large, various investigations have been conducted on them. Figures 7 and 8 show the fracture origin in spheroidal graphite cast iron stated in previous studies for high cycle fatigue⁽¹⁰⁾⁻⁽³³⁾ and low cycle fatigue^{(3),(34)-(38)}, respectively. In high cycle fatigue, it had been believed that the fracture started from graphite until the late 1970's. Recently, however, it has been considered that the fracture almost starts from microshrinkage or dross (containing abnormal graphite) regardless of the matrix used. On the other hand, in low

Fig. 8 Fracture origin in spheroidal graphite cast iron stated in previous studies (Low cycle fatigue)

cycle fatigue, the fracture origin differs with the material used and the researchers conducting the study. It is considered, for example, that fracture starts from graphite⁽³⁷⁾. It is also considered that fracture starts from different defects according to the strength of materials^{(34)-(36),(38)}.

In this study, we observed a fractured section by using SEM in order to specify the kind of defect in fracture origin.

Figure 9 shows an SEM micrograph of fracture origin in PDI observed on a cylindrical specimen ($\Delta\epsilon_p = 0.02$). The main crack initiates from a microshrinkage that exists on the neighboring surface over the whole plastic strain range. The length of microshrinkages projected on surface ranges from 100 to 800 μm . Most of them have a length of 250~300 μm . Also, there is no precise relationship between the plastic strain range and the size of microshrinkage. Consequently, the cause of increase in crack initiation

(PDI : $\Delta\epsilon_p = 0.02$, $l_f/d_0 = 1.5$ ⊕ : Fracture origin)

Fig. 9 Fracture origin observed on cylindrical specimen

Fig. 10 Variation of fracture origin by the shape of specimen

length in PDI is the existence of a microshrinkage or dross on the neighboring surface.

The present result is different from previous ones⁽³⁴⁾⁻⁽³⁸⁾. This is due to the effect of the shape of specimens. Namely, hourglass specimens were always used in previous studies. When hourglass specimens are used, the probability that microshrinkages exist on the neighboring surface around the minimum cross section is quite low. Thus, it is considered that fracture starts from graphite in the case of hourglass specimens. Figure 10 shows the variation of fracture origin with the specimen shape. The reason why the fracture origin changes from microshrinkage to graphite is shown in this figure. Here, the distribution of microshrinkages is difficult to determine. Therefore, the distribution obtained by using marine propellers of aluminum bronze castings is referred to Ref. (39).

Figure 11 shows an SEM micrograph of the fracture origin in PDI observed on an hourglass specimen ($\Delta\epsilon_p = 0.005$). The shape of hourglass specimen is $\rho_0/d_0 = 8$ (ρ_0 : radius of curvature of the central part, d_0 : diameter of minimum cross section). It is seen that fracture starts from graphite. However, the diameter of the graphite is larger than that of average graphite.

4.2.2 Crack propagation limit length : l_f^*

Figure 12 shows the shape of cracks on vertical sections of PDI and SK5, which was observed by using the heatbaking method immediately before fracture ($N/N_f = 0.9$). In SK5, an ellipsoidal main crack exists, the length of which is about 2 mm. In PDI, the crack is also ellipsoidal; however, its length is about 1 mm. Thus, l_f^* in PDI is smaller than that in SK5.

Figure 13 shows the variation of crack propagation limits in PDI with a small blind hole ($\Delta\epsilon_p = 0.01$). The diameters of the small blind holes are 0.4mm, 0.6mm and 1.0mm, and the depths of them are equal to their diameters. As seen from the figure, when the diameter exceeds 0.4 mm, crack propagation limit length l_f^* in PDI is about 2 mm and is as large as that in SK5. l_f^* in these specimens with a small blind hole

(PDI : $\Delta\epsilon_p = 0.005$, $\rho_f/d_0 = 8$ ⊕ : Fracture origin)

Fig. 11 Fracture origin observed on hourglass specimen

Fig. 12 Shape of cracks on vertical sections in PDI and SK5 observed by heatbaking method just before fracture

Fig. 13 Variation of crack propagation limits in PDI with a small blind hole

is about two times larger than that in plane specimens. This suggests that fatigue damage in the plane specimen is produced in whole matrix and not in the neighboring main crack only. Then, we observed crack distribution on the head of a main crack tip on PDI specimens.

Figure 14 shows crack distribution at $N/N_f=0.9$. In the plane specimen, many preexisting cracks initiate from microshrinkages and spheroidal graphite in the early stage of total fatigue life ($N/N_f < 0.03$). In the specimen with a small blind hole, a few preexisting cracks initiate at the direction of the maximum shear stress from a main crack tip. Consequently, it is considered that the existence of many preexisting cracks inflicts fatigue damage on a matrix and that the preexisting cracks act so as to lead to connection at the final stage of fatigue.

Fig. 14 Observation of cracks distribution a head of a main crack tip on PDI ($\Delta\epsilon_p=0.01$, $N/N_f=0.9$)

4.2.3 Relative crack growth rate: $(dl/dN)/l$

As shown in Fig. 6, when the plastic strain range $\Delta\epsilon_p$ in PDI with defects is the same as that in SK5, the relative crack growth rate $(dl/dN)/l$ in PDI is about four times faster than that in SK5. On the other hand, a main crack in PDI initiates from a microshrinkage that exists on the neighboring surface, and many preexisting cracks initiate from other microshrinkages and spheroidal graphite. According to the observation of crack propagation behavior in PDI, preexisting cracks are repeatedly connected with a main crack. However, after the connection, crack growth rate is suppressed⁽³⁾. Then, the connection of cracks is not the main factor for accelerating crack growth rate.

Nisitani et al. have clarified that the shape of crack opening near the crack tip is considered to be the main factor that controls the crack growth rate^{(40),(41)}. That is, when the shape of cracks opening near the crack tip are geometrically similar to each other, crack growth rates are nearly equal regardless of crack length or load condition. Then, crack opening displacement ranges $\Delta COD(x)$ between PDI and SK5 are measured and compared.

Figures 15 and 16 show a comparison of $\Delta COD(x)$ between PDI and SK5 in the case of $\Delta\epsilon_p=0.01$. When the crack lengths in PDI and SK5 are the same, $\Delta COD(x)$ in PDI are larger than that in SK5 (see Fig. 15). On the other hand, when the crack growth rates in PDI and SK5 are the same, $\Delta COD(x)$ in both materials are almost the same (see Fig. 16). Therefore,

Fig. 15 Comparison of crack opening displacement ranges between PDI and SK5 ($l = \text{const.}$)

Fig. 16 Comparison of crack opening displacement ranges between PDI and SK5 ($dl/dN = \text{const.}$)

the cause of acceleration of crack growth rate in PDI is that the $\Delta COD(x)$ in PDI are larger than those in SK5 at a constant crack length.

As the cause of increase in $\Delta COD(x)$ in PDI, it is considered that Young's modulus E (166 GPa) in PDI is smaller than that in SK5.

5. Conclusions

An effective and convenient method for the estimation of low cycle fatigue life based on the small crack growth law was proposed, and the factors that control fatigue life were clarified by relating crack initiation life to the crack initiation length. Also, plastic-strain-controlled low cycle fatigue tests were carried out on pearlitic spheroidal graphite cast iron (JIS FCD700; PDI) and its matrix material (0.85 %carbon steel; JIS SK5). The effect of defects on fatigue life and the factors that control fatigue life were examined from a microscopic perspective. The main results obtained are as follows:

(1) Regardless of the presence or absence of defects, low cycle fatigue life is controlled by three factors: crack initiation length l_0^* , crack propagation

limit length l_f^* and relative crack growth rate $(dl/dN)/l$.

(2) Characteristics of low cycle fatigue in PDI are as follows: crack initiation length is larger, crack propagation limit length is smaller and relative crack growth rate is faster than those in SK5.

(3) Fracture in PDI starts from a microshrinkage that exists on the neighboring surface in the case of a cylindrical specimen; thus, the crack initiation length in PDI is about 100 μm . It is larger than the average diameter of graphite.

(4) Preexisting cracks originating from defects are one of the causes for shortening the crack propagation limit length.

(5) The relative crack growth rate is accelerated by the increase in crack opening displacement range near the crack tip.

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