Multipurpose Optimization Problem of Material Composition for Thermal Stress Relaxation Type of a Functionally Graded Circular Plate*

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In this paper, one-dimensional transient heat conduction and thermal bending problems under an axisymmetric condition are developed theoretically for a functionally graded circular plate due to a uniform heat supply from its surfaces. We construct the approach for the multipurpose optimization of the nonhomogeneous material composition in a functionally graded circular plate considering the relaxation of the thermal stress distribution and the improvement of the heat resistance. We deal with this multipurpose optimization using the weighting method. Using these analytical solutions for the temperature field and the associated thermoelastic one, and the approach for the multipurpose optimization of the nonhomogeneous material composition, numerical calculations of the optimal material composition are carried out for a titanium alloy (Ti-6 Al-4 V)/zirconium oxide (ZrO_2) functionally graded circular plate subjected to a uniform heat supply from one side of its surface. The trade-off relationship between the maximum absolute stress ratio and the maximum heat flux is elucidated. It is shown that this approach is useful for the multipurpose optimal design of nonhomogeneous material composition considering both the securement of strength against thermal stress and the improvement of heat resistance against thermal load.

Key Words: Thermoelasticity, Thermal Stress, Thermal Bending, Functionally Graded Materials, Circular Plate, Material Design, Material Composition, Multipurpose Optimization

1. Introduction

The development of functionally graded materials (FGMs) for the purpose of the thermal stress relaxation with high heat resistance has been expected as one of the applications to the aerospace engineering field such as the thermal protection wall in the spaceplane and the combustion chamber member in the scramjet engine. When designing the material composition in FGMs under the environment in use, both the securement of strength against thermal stress and the improvement of heat resistance against thermal load are the principle purposes to be achieved⁽¹⁾. However, how much these two purposes can be achieved simultaneously has not fully been investigated

for the design of material composition in FGMs.

This paper is intended as an investigation for a multipurpose optimization problem of the material composition in FGMs. We investigate the optimal material composition which enables to achieve two purposes simultaneously which are the securement of strength against thermal stress and the improvement of heat resistance against thermal load. In this paper, we deal with a functionally graded circular plate as an analytical model. We assume that the circular plate has nonhomogeneous material properties in the thickness direction and is subjected to a uniform heat supply from its surfaces. To begin with, we treat the theoretical development of one-dimensional transient heat conduction and thermal bending problems under an axisymmetric condition. Next, we construct the approach for the multipurpose optimization of the material composition in FGMs. Furthermore, we carry out numerical calculations in the case that a titanium alloy/zirconium oxide functionally graded

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The coefficients A_i, B_i, A'_i and B'_i are the solutions of following simultaneous equations of $2N$ variables, respectively.

$$\begin{cases} [a_{ik}]\{x_k\} = -\{c_i\}/\mu_j^2 \\ [e_{ik}]\{x'_k\} = \{c_i\} \end{cases} \quad (10)$$

where

$$\begin{cases} \{x_k\} = \{A_1, B_1, \dots, A_N, B_N\}^T \\ \{x'_k\} = \{A'_1, B'_1, \dots, A'_N, B'_N\}^T \end{cases} \quad (11)$$

The non-zero elements in the coefficient matrices $[a_{ik}]$, $[e_{ik}]$ and the constant vector $\{c_i\}$ are shown as follows:

$$\left. \begin{aligned} a_{11} &= H_a \\ a_{12} &= -\beta_{1j} \\ a_{2i,2i-1} &= \cos \beta_{ij} \bar{l}_i \\ a_{2i,2i} &= \sin \beta_{ij} \bar{l}_i \\ a_{2i,2i+1} &= -1 \\ a_{2i+1,2i-1} &= -\bar{\lambda}_i \beta_{ij} \sin \beta_{ij} \bar{l}_i \\ a_{2i+1,2i} &= \bar{\lambda}_i \beta_{ij} \cos \beta_{ij} \bar{l}_i \\ a_{2i+1,2i+2} &= -\bar{\lambda}_{i+1} \beta_{i+1,j} \\ a_{2N,2N-1} &= H_b \cos \beta_{Nj} \bar{l}_N - \beta_{Nj} \sin \beta_{Nj} \bar{l}_N \\ a_{2N,2N} &= \beta_{Nj} \cos \beta_{Nj} \bar{l}_N + H_b \sin \beta_{Nj} \bar{l}_N \\ e_{11} &= H_a \\ e_{12} &= -1 \\ e_{2i,2i-1} &= 1 \\ e_{2i,2i} &= \bar{l}_i \\ e_{2i,2i+1} &= -1 \\ e_{2i+1,2i} &= \bar{\lambda}_i \\ e_{2i+1,2i+2} &= -\bar{\lambda}_{i+1} \\ e_{2N,2N-1} &= H_b \\ e_{2N,2N} &= 1 + H_b \bar{l}_N \\ c_1 &= H_a \bar{T}_a \\ c_{2N} &= H_b \bar{T}_b \end{aligned} \right\} \quad i=1, \dots, N-1 \quad (12)$$

Solving the simultaneous equations (10) with the aid of the Cramer formula, the coefficients A_i and B_i are expressed by the following equations

$$\begin{cases} A_i = -\bar{A}_i / \{\mu_j^2 \Delta(\mu_j)\} \\ B_i = -\bar{B}_i / \{\mu_j^2 \Delta(\mu_j)\} \\ \Delta(\mu_j) = \det a_{ik} \end{cases} \quad (13)$$

where $\bar{A}_i$ and $\bar{B}_i$ are defined as the determinants of matrices similar to the coefficient matrix $[a_{ik}]$, in which the $(2i-1)$ th column and the $2i$ th column are exchanged by the constant vector $\{c_i\}$, respectively. Furthermore, eigenvalue μ_j is the j th positive root of the transcendental equation

$$\Delta(\mu) = 0, \quad (14)$$

and $\Delta'(\mu_j)$ is defined by

$$\Delta'(\mu_j) = \lim_{\mu \rightarrow \mu_j} \frac{d\Delta(\mu)}{d\mu}. \quad (15)$$

2.2 Thermal bending analysis

We treat the theoretical development of a thermal bending problem for a nonhomogeneous circular plate using the classical plate theory⁽⁴⁾. We treat this problem based on the Kirchhoff-Love hypothesis and

take the axisymmetric thermal condition into account. Denoting the in-plane and out-of-plane displacements at the midplane of the plate as u_0 and w , respectively, the strain distributions in i th layer ε_{rri} and $\varepsilon_{\theta\theta i}$ are expressed in the dimensionless forms by

$$\begin{cases} \bar{\varepsilon}_{rri} = \frac{d\bar{u}_0}{d\bar{r}} - \bar{z} \frac{d^2 \bar{w}}{d\bar{r}^2} \\ \bar{\varepsilon}_{\theta\theta i} = \frac{\bar{u}_0}{\bar{r}} - \bar{z} \frac{1}{\bar{r}} \frac{d\bar{w}}{d\bar{r}} \end{cases} \quad (16)$$

Denoting the stress distribution, the longitudinal elastic modulus, the Poisson ratio and the coefficient of linear thermal expansion in i th layer as σ_{rri} , $\sigma_{\theta\theta i}$, E_i , ν_i and α_i , respectively, the equations of the relationship between stress and strain distributions in i th layer are expressed in the dimensionless forms by

$$\begin{cases} \bar{\varepsilon}_{rri} = \frac{\bar{\sigma}_{rri} - \nu_i \bar{\sigma}_{\theta\theta i}}{E_i} + \bar{\alpha}_i \bar{T}_i \\ \bar{\varepsilon}_{\theta\theta i} = \frac{\bar{\sigma}_{\theta\theta i} - \nu_i \bar{\sigma}_{rri}}{E_i} + \bar{\alpha}_i \bar{T}_i \end{cases} \quad (17)$$

Therefore, the equations of equilibrium for the circular plate are expressed by

$$\begin{aligned} \frac{d}{d\bar{r}} \left[\bar{r} \frac{d}{d\bar{r}} \left\{ \frac{1}{\bar{r}} \frac{d}{d\bar{r}} (\bar{r} \bar{u}_0) \right\} \right] \\ = \frac{1}{\bar{A}_1 \bar{D}_1 - \bar{B}_1^2} \left\{ \bar{D}_1 \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{N}^T}{d\bar{r}} \right) \right. \\ \left. - \bar{B}_1 \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{M}^T}{d\bar{r}} \right) \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{d}{d\bar{r}} \left[\bar{r} \frac{d}{d\bar{r}} \left\{ \frac{1}{\bar{r}} \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{w}}{d\bar{r}} \right) \right\} \right] \\ = \frac{1}{\bar{A}_1 \bar{D}_1 - \bar{B}_1^2} \left\{ \bar{B}_1 \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{N}^T}{d\bar{r}} \right) \right. \\ \left. - \bar{A}_1 \frac{d}{d\bar{r}} \left(\bar{r} \frac{d\bar{M}^T}{d\bar{r}} \right) \right\}, \end{aligned} \quad (19)$$

where $\bar{N}^T$, $\bar{M}^T$, $\bar{A}_1$, $\bar{B}_1$ and $\bar{D}_1$ denote the dimensionless forms of the thermal resultant force N^T , the thermal resultant moment M^T and the rigidities of plate A_1 , B_1 and D_1 , respectively, and which are defined by

$$\begin{cases} \bar{N}^T \\ \bar{M}^T \end{cases} = \sum_{i=1}^N \frac{\bar{E}_i \bar{\alpha}_i}{1 - \nu_i} \int_{\bar{h}_{i-1}}^{\bar{h}_i} \bar{T}_i \begin{cases} 1 \\ \bar{z} \end{cases} d\bar{z}, \quad (20)$$

$$\begin{cases} \bar{A}_1 \\ \bar{B}_1 \\ \bar{D}_1 \end{cases} = \sum_{i=1}^N \frac{\bar{E}_i}{1 - \nu_i^2} \begin{cases} \frac{\bar{h}_i - \bar{h}_{i-1}}{2} \\ \frac{\bar{h}_i^2 - \bar{h}_{i-1}^2}{2} \\ \frac{\bar{h}_i^3 - \bar{h}_{i-1}^3}{3} \end{cases}. \quad (21)$$

In Eqs.(20) and (21), $\bar{h}_i$ denotes the dimensionless form of the distance between upper surface in i th layer and the midplane of the plate h_i and which is defined by

$$\bar{h}_i = \sum_{p=1}^i \bar{l}_p - \frac{\bar{H}}{2}. \quad (22)$$

Operating the integral on the equations of equilibrium (18) and (19), the solutions of thermal deformation and thermal stress distributions for the circular plate are obtained by

$$\bar{u}_0 = \frac{1}{\bar{A}_1 \bar{D}_1 - \bar{B}_1^2} (\bar{D}_1 \bar{N}^T - \bar{B}_1 \bar{M}^T) \frac{\bar{r}}{2} + c_2 \bar{r}, \quad (23)$$

$$\bar{w} = \frac{1}{\bar{A}_1 \bar{D}_1 - \bar{B}_1^2} (\bar{B}_1 \bar{N}^T - \bar{A}_1 \bar{M}^T) \frac{\bar{r}^2}{4} + c'_1 \bar{r}^2 + c'_2, \quad (24)$$

$$\begin{aligned} \bar{\sigma}_i (= \bar{\sigma}_{rri} = \bar{\sigma}_{\theta\theta i}) \\ = \frac{E_i}{1 - \nu_i^2} \left[\frac{1}{2} \frac{1}{\bar{A}_1 \bar{D}_1 - \bar{B}_1^2} \{ (\bar{D}_1 \bar{N}^T - \bar{B}_1 \bar{M}^T) \right. \\ \left. - \bar{z} (\bar{B}_1 \bar{N}^T - \bar{A}_1 \bar{M}^T) \} + c_2 - 2 \bar{z} c'_1 - \bar{a}_i \bar{T}_i \right], \end{aligned} \quad (25)$$

where c_2 , c'_1 and c'_2 are the unknown constants determined by the mechanical boundary conditions.

Therefore, we assume the simply supported edge condition which the displacement in the radial direction is free at the end edge of the circular plate. Denoting $\bar{N}_r$ and $\bar{M}_r$ as the dimensionless forms of the force resultant N_r and the moment resultant M_r , the mechanical boundary conditions for the circular plate are given by

$$\bar{r} = 1: \bar{N}_r = 0, \quad \bar{M}_r = 0, \quad \bar{w} = 0, \quad (26)$$

and $\bar{N}_r$, $\bar{M}_r$ are defined by

$$\begin{Bmatrix} \bar{N}_r \\ \bar{M}_r \end{Bmatrix} = \sum_{i=1}^N \int_{\bar{h}_{i-1}}^{\bar{h}_i} \bar{\sigma}_i \begin{Bmatrix} 1 \\ \bar{z} \end{Bmatrix} d\bar{z}. \quad (27)$$

By solving Eqs. (26) with Eqs. (23)–(25) and (27), the unknown constants c_2 , c'_1 and c'_2 are obtained.

The dimensionless quantities introduced into Eqs. (15)–(26) are defined as follows:

$$\left. \begin{aligned} (\bar{r}, \bar{z}, \bar{h}_i, \bar{H}) &= (r, z, h_i, H)/R \\ (\bar{u}_0, \bar{w}) &= (u_0, w)/(\alpha_0 T_0 R) \\ (\bar{\varepsilon}_{rri}, \bar{\varepsilon}_{\theta\theta i}) &= (\varepsilon_{rri}, \varepsilon_{\theta\theta i})/(\alpha_0 T_0) \\ (\bar{\sigma}_i, \bar{\sigma}_{rri}, \bar{\sigma}_{\theta\theta i}) &= (\sigma_i, \sigma_{rri}, \sigma_{\theta\theta i})/(\alpha_0 T_0 E_0) \\ \bar{E}_i &= E_i/E_0 \\ \bar{a}_i &= a_i/\alpha_0 \\ \bar{A}_1 &= A_1/(E_0 R) \\ \bar{B}_1 &= B_1/(E_0 R^2) \\ \bar{D}_1 &= D_1/(E_0 R^3) \\ (\bar{N}^T, \bar{N}_r) &= (N^T, N_r)/(\alpha_0 T_0 E_0 R) \\ (\bar{M}^T, \bar{M}_r) &= (M^T, M_r)/(\alpha_0 T_0 E_0 R^2) \end{aligned} \right\}, \quad (28)$$

where α_0 and E_0 are the typical values of the linear thermal expansion and longitudinal elastic modulus, respectively.

2.3 Optimization problem of material composition

Now we treat the multipurpose optimization of the material composition in order to achieve the two purposes which are the securement of strength against thermal stress and the improvement of heat resistance against thermal load.

First, we assume that the volume fraction in the thickness direction shown in Fig. 2 is given by

$$V_m = \begin{cases} \left(\frac{1}{2} - \frac{z}{H} \right)^{1/M} & 0 < M \leq 1 \\ 1 - \left(\frac{z}{H} + \frac{1}{2} \right)^M & M \geq 1 \end{cases}, \quad (29)$$

$$V_c = 1 - V_m$$

Fig. 2 Variation of volume fraction V_m

where V_m , V_c and M denote the volume fractions for metal and ceramic phases and the material composition parameter, respectively.

Next, we adopt a maximum absolute stress ratio in the whole time and space as an index of strength against thermal stress

$$\begin{aligned} f_{\max} &= \max(|\bar{\sigma}_i / \bar{\sigma}_{Bi}|) \\ \left\{ \begin{aligned} \bar{\sigma}_{Bi} &= \bar{\sigma}_{ci} & \bar{\sigma}_i < 0 \\ \bar{\sigma}_{Bi} &= \bar{\sigma}_{ti} & \bar{\sigma}_i \geq 0 \end{aligned} \right\}, \end{aligned} \quad (30)$$

where σ_{ci} and σ_{ti} denote strength against compression and tension in i th layer.

Moreover, we adopt the following two typical values of the heat flux at transient or steady state as an index of heat resistance.

(1) Maximum value of heat flux which transfers through the plate

$$\bar{q}_{t \max} = \bar{K} (\bar{T}_b - \bar{T}_a), \quad (31)$$

$$\frac{1}{\bar{K}} = \frac{1}{H a \bar{\lambda}_1} + \frac{1}{H b \bar{\lambda}_N} + \sum_{i=1}^N \frac{\bar{l}_i}{\bar{\lambda}_i}. \quad (32)$$

$\bar{K}$ denotes the dimensionless form of the overall heat transfer coefficient $K^{(5),(6)}$, where $1/K$ is the thermal resistance.

(2) Maximum value of heat flux which flows in from the surface of the plate subjected to the heat supply

$$\begin{aligned} \bar{q}_{i \max} &= \bar{\lambda}_N \{ B'_N \\ &+ \sum_{j=1}^{\infty} \frac{2}{\mu_j \mathcal{A}'(\mu_j)} (-\bar{A}_N \beta_{Nj} \sin \beta_{Nj} \bar{l}_N \\ &+ \bar{B}_N \beta_{Nj} \cos \beta_{Nj} \bar{l}_N) \}. \end{aligned} \quad (33)$$

Furthermore, we adopt the following scalar function $F(M)$ as an objective function for the material composition optimization taking both strength and heat resistance into account

$$\begin{aligned} F(M) &= \frac{1}{\sqrt{w_1^2 + w_2^2 + w_3^2}} (w_1 f_{\max} + w_2 Q_{t \max} \\ &+ w_3 Q_{i \max}), \end{aligned} \quad (34)$$

where

$$\begin{aligned} Q_{t \max} &= \bar{q}_{t \max} / \bar{q}_{t0} \\ Q_{i \max} &= \bar{q}_{i \max} / \bar{q}_{i0} \end{aligned} \quad (35)$$

$Q_{t \max}$ is the maximum ratio of heat flux in transmission and $Q_{i \max}$ is the one in inflow, respectively. As

the typical values in normalization of $\bar{q}_{t0}$ and $\bar{q}_{i0}$, we adopt the maximum values of heat flux in the dimensionless forms for a homogeneous metal medium of same thickness ratio $\bar{H}$ as the present functionally graded circular plate under the condition of $H_a = H_b = \infty$. Moreover, w_i ($0 \leq w_i \leq 1$, $i=1, 2, 3$) denote the weight coefficients⁽⁷⁾ for the index of strength or heat resistance.

The dimensionless quantities introduced into Eqs. (30)–(35) are defined by

$$\left. \begin{aligned} (\bar{\sigma}, \bar{\sigma}_{Bi}) &= (\sigma, \sigma_{Bi}) / (\alpha_0 T_0 E_0) \\ (\bar{q}_{t \max}, \bar{q}_{i \max}, \bar{q}_{t0}, \bar{q}_{i0}) &= (q_{t \max}, q_{i \max}, q_{t0}, q_{i0}) \cdot (\lambda_0 T_0) / R \\ \bar{K} &= K \cdot R / \lambda_0 \end{aligned} \right\} \quad (36)$$

The objective function Eq. (34) with the weight coefficients w_i ($i=1, 2, 3$) varied parametrically is minimized with respect to the decision variable of the material composition M with the aid of the modified Powell method⁽⁸⁾.

3. Numerical Results and Discussion

We carried out the numerical simulations on the multipurpose optimization of the material composition considering both strength and heat resistance in a functionally graded circular plate by using the theoretical formulations and the optimization approach of the material composition described in Chapter 2.

In this numerical calculation, the results for a titanium alloy (Ti-6 Al-4 V)/zirconium oxide (ZrO₂) FGM are illustrated. Table 1 indicates the material properties used in the present calculation. Material properties of the nonhomogeneous medium are estimated by the Kerner mixture law. For the typical values $\lambda_0, c_0, \rho_0, \alpha_0$ and E_0 in normalization of the material properties, ones of titanium alloy are used.

Numerical parameters, the initial actual temperature of the plate, the actual temperature of surrounding mediums, the dimensionless heat transfer coefficients, the typical temperature change, the thickness ratio, the number of layers and the thickness of each layer are given as follows:

$$\left. \begin{aligned} \theta_0 &= 300 \text{ K} \\ \theta_a &= 300 \text{ K} \\ \theta_b &= 800 \text{ K} \\ H_a &= \infty \\ H_b &= \infty \\ T_0 &= 500 \text{ K} \\ \bar{H} &= 0.3 \\ N &= 20 \\ \bar{l}_i &= \bar{H}/N \quad i=1, \dots, N \end{aligned} \right\} \quad (37)$$

Furthermore, we assume the weight coefficients w_i ($i=1, 2, 3$) involved in the objective function Eq. (34) as the following two cases

$$\text{Case 1: } w_1, w_2 \geq 0, \quad w_3 = 0, \quad (38)$$

Table 1 Material properties

Properties	Ti-6Al-4V	ZrO ₂
Thermal conductivity [W/mK]	6.2	1.7834
Specific heat [kJ/kgK]	0.5377	0.4614
Density [kg/m ³]	4.420×10 ³	3.657×10 ³
Coefficient of linear thermal expansion [1/K]	8.8559×10 ⁻⁶	8.7830×10 ⁻⁶
Young's modulus [GPa]	1.0575×10 ²	1.1638×10 ²
Poisson ratio	0.2984	0.3333
Tensile strength [MPa]	0.9974×10 ³	0.1456×10 ³
Compressive strength [MPa]	0.9974×10 ³	0.3128×10 ⁴

(Material composition optimization considering maximum absolute stress ratio and maximum ratio of heat flux in transmission).

$$\text{Case 2: } w_1, w_3 \geq 0, \quad w_2 = 0, \quad (39)$$

(Material composition optimization considering maximum absolute stress ratio and maximum ratio of heat flux in inflow).

The following numerical results are illustrated for Case 1 and Case 2. Tables 2(a) and (b) indicate the results for each value of the weight coefficient, which are the weight ratio, the optimal material composition parameter M_{opt} , the maximum absolute stress ratio $f_{\max}$, the maximum heat flux ratios $Q_{t \max}$ and $Q_{i \max}$, the inverse of maximum absolute stress ratio $1/f_{\max}$ and the thermal resistance in the dimensionless form $1/\bar{K}$. The inverse of the maximum absolute stress ratio $1/f_{\max}$ can be considered as a safety factor against thermal fracture when choosing the maximum stress in the circular plate for the allowable stress. The dimensionless form of thermal resistance $1/\bar{K}$ can also be considered as a factor of the heat resistance. The values in the second line in Tables 2(a) and (b) correspond to the results of the material composition optimization considering the factor of the maximum absolute stress ratio $f_{\max}$ only ($w_2/w_1=0$ or $w_3/w_1=0$). The numerical results for lower weight coefficient ratio in w_1/w_2 or w_1/w_3 correspond to the material composition design made much of the maximum heat flux ratios.

Figures 3(a) and (b) indicate the variations of optimal material composition parameter M_{opt} with the weight coefficient ratios for the two cases. The optimal material composition parameter M_{opt} is equal to 2.0891 when the range of the weight coefficient ratio w_2/w_1 is less equal than about 0.8 for Case 1 and w_1/w_3 is larger equal than about 0.4 for Case 2. The volume fraction of zirconium oxide phase V_c is more than one of titanium alloy phase V_m in this material composition. Furthermore, the optimal material composition parameter M_{opt} decreases with a decline in the weight coefficient ratio w_1/w_2 or w_1/w_3 , then the material composition varies into the distribution which contains rich volume fraction of zirconium oxide phase

Table 2 Results of optimization for weight coefficients

(a) Case 1

w_2/w_1	w_1/w_2	M_{opt}	f_{max}	Q_{tmax}	Q_{imax}	$1/f_{max}$	$1/R$
0.0		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.2		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.4		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.6		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.8		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
1.0	1.0000	1.6842	0.28303	0.60452	0.39910	3.5332	0.49626
	0.83333	1.1917	0.34517	0.54784	0.38102	2.8971	0.54761
	0.71429	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.62500	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.55556	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.50000	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.45455	0.95005	0.39806	0.51190	0.36795	2.5122	0.58606
	0.41667	0.89442	0.42183	0.50156	0.36371	2.3706	0.59814
	0.38462	0.84372	0.44607	0.49186	0.35987	2.2418	0.60994
	0.35714	0.79642	0.47117	0.48256	0.35630	2.1224	0.62169
	0.33333	0.75110	0.49765	0.47343	0.35290	2.0094	0.63368
	0.31250	0.70614	0.52644	0.46414	0.34956	1.8996	0.64636
	0.29412	0.65889	0.55946	0.45414	0.34607	1.7874	0.66059
	0.27778	0.60169	0.60311	0.44169	0.34192	1.6581	0.67921
	0.26316	0.52741	0.66454	0.42497	0.33657	1.5048	0.70594
	0.25000	0.52741	0.66454	0.42497	0.33657	1.5048	0.70594
	0.23810	0.52741	0.66454	0.42497	0.33657	1.5048	0.70594
	0.22727	0.52741	0.66454	0.42497	0.33657	1.5048	0.70594
	0.21739	0.48927	0.70457	0.41613	0.33381	1.4193	0.72094

(b) Case 2

w_3/w_1	w_1/w_3	M_{opt}	f_{max}	Q_{tmax}	Q_{imax}	$1/f_{max}$	$1/R$
0.00		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.40		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
0.80		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
1.00		2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
	0.8333	2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
	0.6250	2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
	0.5000	2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
	0.4167	2.0891	0.24898	0.64130	0.41061	4.0164	0.46780
	0.3571	2.0032	0.25501	0.63410	0.40834	3.9214	0.47312
	0.3125	1.6967	0.28178	0.60577	0.39949	3.5489	0.49524
	0.2778	1.1923	0.34507	0.54792	0.38105	2.8980	0.54753
	0.2500	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.2273	1.0000	0.37896	0.52091	0.37178	2.6388	0.57592
	0.2083	0.99167	0.38201	0.51943	0.37114	2.6177	0.57756
	0.1923	0.95537	0.39593	0.51287	0.36836	2.5257	0.58495
	0.1786	0.92249	0.40949	0.50681	0.36585	2.4421	0.59194
	0.1667	0.89102	0.42337	0.50092	0.36346	2.3620	0.59891
	0.1562	0.86164	0.43720	0.49532	0.36123	2.2873	0.60568
	0.1471	0.83538	0.45032	0.49023	0.35924	2.2206	0.61196
	0.1389	0.80961	0.46392	0.48518	0.35729	2.1555	0.61834
	0.1316	0.78468	0.47779	0.48022	0.35542	2.0930	0.62473
	0.1250	0.76177	0.49120	0.47560	0.35370	2.0358	0.63079
	0.1190	0.73982	0.50464	0.47112	0.35206	1.9816	0.63679
	0.1136	0.71812	0.51852	0.46664	0.35044	1.9286	0.64290
	0.1087	0.69767	0.53215	0.46237	0.34893	1.8792	0.64884
	0.1042	0.67726	0.54629	0.45806	0.34742	1.8305	0.65494
	0.1000	0.65634	0.56133	0.45359	0.34589	1.7815	0.66139
	0.09615	0.63583	0.57659	0.44917	0.34439	1.7343	0.66791
	0.09259	0.61448	0.59302	0.44451	0.34284	1.6863	0.67491
	0.08929	0.59034	0.61220	0.43918	0.34110	1.6335	0.68310
	0.08621	0.55993	0.63714	0.43237	0.33891	1.5695	0.69386
	0.08333	0.52798	0.66405	0.42510	0.33662	1.5059	0.70573
	0.08065	0.52743	0.66453	0.42497	0.33658	1.5048	0.70594
	0.07812	0.52740	0.66454	0.42497	0.33657	1.5048	0.70595
	0.07576	0.52743	0.66452	0.42497	0.33658	1.5048	0.70594
	0.07353	0.52741	0.66454	0.42497	0.33657	1.5048	0.70594
	0.07143	0.50001	0.69308	0.41864	0.33460	1.4428	0.71662

(a) Case 1

(b) Case 2

Fig. 3 Variations of optimal parameter M_{opt} with weight coefficient ratio

(a) Case 1

(b) Case 2

Fig. 4 Variations of maximum absolute stress ratio f_{max} with weight coefficient ratio

V_c .

Figures 4(a) and (b) show the variations of the maximum absolute stress ratio $f_{\max}$ with the weight coefficient ratio for the two cases. In both cases, the maximum absolute stress ratio increases with a decline in the weight coefficient ratio w_1/w_2 or w_1/w_3 . The rate of increase in the maximum absolute stress ratio $f_{\max}$ with the variation of the weight coefficient ratio for Case 1 is lower than one for Case 2.

Figures 5(a), (b) and Figs. 6(a), (b) show the variations of the maximum heat flux ratios $Q_{t\max}$ and $Q_{i\max}$ with the weight coefficient ratio for the two cases. The maximum heat flux ratios $Q_{t\max}$ and $Q_{i\max}$ decrease with a decline in the weight coefficient ratio w_1/w_2 or w_1/w_3 in both cases. The variation of the maximum ratio of heat flux in inflow $Q_{i\max}$ with the weight coefficient ratio is smaller than one of the maximum ratio of heat flux in transmission $Q_{t\max}$.

Figures 7(a) and (b) indicate the variations of the inverse of maximum absolute stress ratio $1/f_{\max}$ and the dimensionless form of thermal resistance $1/\bar{K}$ with the weight coefficient ratio for the two cases. The inverse of the maximum absolute stress ratio $1/f_{\max}$ decreases with a decline in the weight coefficient ratio w_1/w_2 . On the other hand, the dimensionless form of thermal resistance $1/\bar{K}$ increases with a decline in the weight coefficient ratio w_1/w_3 . The trade-

Fig. 5 Variations of maximum ratio of heat flux in transmission $Q_{t\max}$ with weight coefficient ratio

Fig. 6 Variations of maximum ratio of heat flux in inflow $Q_{i\max}$ with weight coefficient ratio

Fig. 7 Variations of inverse of absolute stress ratio $1/f_{\max}$ and dimensionless form of heat resistance $1/\bar{K}$ with weight coefficient ratio

Fig. 8 Relationship of absolute stress ratio $f_{\max}$ and maximum ratios of heat flux $Q_{t \max}$ and $Q_{i \max}$ (Case 1)

off relationship between the inverse of the maximum absolute stress ratio $1/f_{\max}$ and the dimensionless form of thermal resistance $1/\bar{K}$ can be seen. In decreasing the weight coefficient ratio w_1/w_2 or w_1/w_3 , on one hand, the thermal resistance of circular plate is improved, on the other hand, the allowance of strength against thermal fracture declines.

Figure 8 shows the relationship among the maximum absolute stress ratio $f_{\max}$ and the maximum ratios of heat flux $Q_{t \max}$ and $Q_{i \max}$ for the three cases of the weight coefficient ratios in Case 1. In the case which the weight coefficient ratio is $w_2/w_1=0.0$, the maximum absolute stress ratio $f_{\max}$ shows the least value of all cases. However, the maximum ratios of the heat flux $Q_{t \max}$ and $Q_{i \max}$ show the largest values of all cases. This case corresponds to the material composition which is made most of the securement of strength against thermal stress.

Furthermore, the cases of $w_1/w_2=0.5, 0.25$ correspond to the material compositions which are made more of the improvement of heat resistance against thermal load than the case of $w_2/w_1=0.0$. Consequently, we can hold down the rise in the maximum absolute stress ratio $f_{\max}$ and decrease the maximum ratio of heat flux ratio in transmission $Q_{t \max}$ by the appropriate choice of the weight coefficient ratio. We can treat the material composition design considering both strength against thermal stress and heat resistance against thermal load by mean of the control in the weight coefficient ratios.

4. Conclusions

In this study, one-dimensional transient heat conduction and thermal bending problems for a function-

ally graded circular plate due to uniform heat supply from its surface are developed theoretically. Secondly, an optimization problem of material composition for a functionally graded circular plate is treated by the weighting method which is one of the methods in multipurpose planning. Furthermore, numerical calculations of the multipurpose optimization of material composition are carried out for the titanium alloy/zirconium oxide functionally graded circular plate in order to achieve the two purposes that are both the securement of strength against thermal stress and the improvement of thermal resistance against thermal load. The trade-off relationship between the maximum absolute stress ratio and the maximum ratios of heat flux which transfers through the plate or which flows in from the surface subjected by heat supply are elucidated from the numerical results. Finally, the effectiveness of the present technique is shown for multipurpose optimal design of material composition considering both securement of strength against thermal stress and improvement of thermal resistance against thermal load.

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