Dynamic Stress Concentration in a Strip Plate with Fillet*

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A method of measuring the maximum dynamic stress concentration factor α_{Dmax} in a fillet has not yet established. The reflection and transmission of stress wave during the propagation through a fillet result in complex stress states. We propose a new method for analyzing the maximum dynamic stress concentration factor α_{Dmax} in a fillet utilizing the stress concentration factor α_s and the reflection coefficient A obtained by a strain gauge on the incident strip plate. In this experiment, a square wave (a stress wave with a plateau of constant stress for long duration) is used. A strain gauge measures the reflection coefficient A . The factor α_{Dmax} in the fillet is analyzed using a dynamic photoelastic method. It is confirmed that α_{Dmax} is approximated by the following equation: $\alpha_{exDmax} = (1+A)\alpha_s$. As a result, the maximum dynamic stress concentration factor is obtained experimentally using the static stress concentration factor and the reflection coefficient.

Key Words: Stress Concentration, Stress Wave, Experimental Stress Analysis, Impact Strength, High Polymer Materials.

1. Introduction

Studies on the stress concentration at the discontinuities such as holes and notches have been done so far. These studies have provided information under static loading. However, few cases of dynamic loading have been discussed^{(1),(2)}.

The purpose of this study is then to investigate the behavior of discontinuity under dynamic loading. For the purpose, the dynamic stress concentration factor and the reflection coefficient at the fillet are obtained using a square wave. In a previous work, the rising part of the square wave was considered as the dynamic loading, and the plateau of the wave as the static loading on condition that the plateau lasted for long time.

The strip plate with the fillet is one of the basic

models of actual structures. When the stress wave travels into another strip plate with bigger cross sectional area, the reflection and the transmission take place due to the difference of acoustic impedance at the fillet part, even if the material of two strips is same^{(3),(4)}.

Static and dynamic photoelastic methods are employed to measure the stress concentration factor. Strain gauge method is used to obtain the reflection coefficient.

2. Stress Concentration Factor in the Strip Plate with Fillet

2.1 Shape and material of specimen

Figure 1 shows the shape of the strip plate with a fillet and the strip plate for nominal stress. The fillet is shown as the shadowed part in the Fig. 1(a). The strip plate is composed of DAP (diallylphtalate), an isotropic photoelastic resin. In the Fig. 1(a), ρ , c_1 and c_2 represent the density and propagation velocities of stress wave in the strip plate I and II respectively, and E is the Young's modulus. The DAP had the Young's modulus of 3.09 GPa, Poisson's ratio of 0.39, and density of $1.21 \times 10^3 \text{ kg/mm}^3$. The thickness of the plate was 6 mm. The radius of the fillet was changed

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Fig. 1 The shape of the strip plate with fillet

i.e. the value of r/B was changed from 0.5 to 2.0, while the width of plate I was kept constant 10 mm.

In the case of the dynamic experiment, the width and the longitudinal length of the plate II were long to be considered as semi-infinite to eliminate the effect of reflection from the plate II.

2.2 Generating of square stress wave and measuring of nominal stress

A compressed air gun was used to generate square stress waves⁽⁶⁾. A striker made of acrylic resin had an outer diameter of 10 mm and an inner diameter of 8 mm, an length of 300 mm, and a hemispherical shell is attached at an end. Cranz-Shardin optical system used to record the dynamic fringe patterns was composed of 9 micro-flash lamps with a flashing time of 1 μ s, and 9 cameras. The starting point of the square wave propagation time is set as the moment when the square stress wave arrive at flash-lamp trigger gauge I in Fig. 1.

The square stress wave with the rise time $T_r = 45 \mu$ s, and a duration $T = 330 \mu$ s was generated in this study. Figure 2 shows the dynamic fringe patterns of propagating of square stress wave in strip plate I. Figure 3 shows the waveform of the square stress wave obtained by strain gauge II in the strip plate I. Thus, the stress waveform obtained by the photoelastic fringes and the strain waveform obtained by the strain gauge agree well, which shows correspondence between strain and stress. In all the following impact experiments, this square wave was used. In this study, the height of plateau of the square wave was investigated. A study of dynamic behavior depending on the rise time will be performed in the future.

Fig. 2 The square wave obtained by dynamic photoelastic method

Fig. 3 The waveform of the square stress obtained by strain gauge

2.3 Static and dynamic stress concentration factors in the strip plate with fillet

Studies on the static stress concentration at the fillet have been done. However, these studies have made clear only a few cases^{(6),(7)}. A subject of static and dynamic stress concentration factors at the fillet except previous experimental cases was performed by

static and dynamic photoelasticity experiment. Figure 4 shows typical photoelastic fringe patterns at the fillet with $r/B=0.5$ by static load. In the static experiment, the stress concentration factor is defined by the ratio of the maximum stress σ_{smax} at the fillet to the nominal stress σ_{s0} generated in the plate I. Here, uniformly distributed compressive stress in the strip plate I and the maximum fringe order at the fillet are as the nominal stress N_{s0} and N_{smax} , respectively. The number of photoelastic fringe orders N is meaning same as the stress σ . Therefore, static stress concentration factor is defined by following equation.

$$\alpha_s = N_{smax} / N_{s0} = \sigma_{smax} / \sigma_{s0} \quad (1)$$

In the case of dynamic stress concentration factor, as shown in Fig. 5, nominal stress N_{D0} was measured on the point where the fillet was assumed to exist in strip plate without fillet (having the same mechanical characteristics as the strip plate with fillet and the same cross section as strip plate I). The maximum fringe orders at the fillet by a square stress wave are as $N(T)_{Dmax}$, and dynamic stress concentration factor $\alpha_D(T)$ is defined by following equation

$$\alpha_D(T) = N(T)_{Dmax} / N_{D0} = \sigma(T)_{Dmax} / \sigma_{D0} \quad (2)$$

Examples of dynamic fringe patterns at the fillet with $r/B=0.5$ are shown in Fig. 6. Figure 7 shows the dynamic and static stress concentration at the fillet with $r/B=0.5, 1.0$ and 2.0 as a function of time. In the

Fig. 4 The photograph of static fringe pattern at the fillet ($r/B=0.5$)

Fig. 5 Definition of dynamic stress concentration factor

case of $r/B=0.5$, the static stress concentration factor α_s was 1.52, and the maximum dynamic stress concentration factor α_{Dmax} was 2.75 at the time $T=230 \mu s$ after impact, which is later than $Tr=45 \mu s$. The maximum dynamic stress concentration factor is 85% higher than α_s . Furthermore, as time progress, the plateau section of the stress wave and the horizontal vibration of primary wave are superimposed, and the stress concentration tends to increase.

The relation between the stress concentration

Fig. 6 The photograph of dynamic fringe patterns at the fillet ($r/B=0.5$)

Fig. 7 The relation between stress concentration factors and the r/B

factors and the r/B is shown in Fig. 8. The stress concentration factors α_s and α_{Dmax} decrease with the increase in r/B . The α_{Dmax} is 70–90% higher than the α_s , which could be attributed to the duplication of the reflected wave. In general, the dynamic stress concentration factor is nearly equal/lower than static one. However, the opposite result was also reported for the strip plate with discontinuity of a circular hole. In such case, the dynamic stress concentration factor was 12% higher than the static stress concentration factor^{(8),(9)}.

Here, the propagation velocity c_1 of the stress wave in the incident side of strip plate I is the one dimensional propagation velocity of the stress wave propagating in a narrow strip plate. On the other hand, the propagation velocity c_2 of the stress in the transmitted side of strip plate II is the two dimensional propagation velocity of the stress wave propagating in a thin plate. The relationship of the two velocities is $c_1 < c_2^{(3),(4)}$. Thus, the relationship

Fig. 8 The relation between the stress concentration factors and the r/B

Fig. 9 The dynamic and static stress distributions along the free boundary of the fillet ($r/B=0.5$)

between the acoustic impedances ρc is $\rho c_1 < \rho c_2$, cause of the difference in acoustic impedance, a reflected wave with the same polarity as the incident wave is generated at the fillet^{(5),(10)}.

Figure 9 shows dynamic stress distribution along the free boundary of the fillet with $r/B=0.5$ obtained from Fig. 6(E) together with the static stress distribution. All stresses used in this Fig. 9 are normalized by nominal stresses. In dynamic photoelastic method, the plateau height of the incident wave σ_{Imax} is the same as nominal stress σ_{D0} . In this Fig. 9, dynamic stress state is higher than nominal stress σ_{D0} in the plate I caused by superimposition of the σ_{Imax} and the σ_{Rmax} . When dynamic stress concentration factor is maximum, the dynamic stress distribution is similar to the static one on all shape of fillet in this study.

3. Relationship between Static/Dynamic Stress Concentration Factors and the Reflection Coefficient

If dynamic stress concentration factor of a strip plate with the fillet is calculated by the static stress concentration factor and the reflection coefficient, the impact experiment is simply. In the preceding section, it was established that the dynamic stress concentration factor is higher than the static one caused by the superposition of the plateau of the incident square stress wave and the reflected stress wave from the fillet. Furthermore, when dynamic stress concentration factor is maximum, the dynamic stress distribution is similar to the static one. Here, the reflection coefficient at the fillet was measured by strain gauge in order to clarify the relationship between dynamic stress concentration factor and static one.

A strain gauge was pasted on the incident strip plate I 450 mm away from the fillet to avoid superposition of incident and reflected wave. Figure 10 shows typical incident and reflected strain waves in the case of $r/B=0.5$. As can be seen in Fig. 10, ϵ_{Rmax} is the height of the plateau of reflected wave, and the ϵ_{Imax} is the height of the plateau of the incident square

Fig. 10 The incident and reflected wave ($r/B=0.5$)

Fig. 11 The relation between the reflection coefficient A and r/B

Fig. 12 The comparison between the α_{Dmax} and the α_{exDmax} as a function of r/B

stress wave. The reflection coefficient A was calculated using the following equation.

$$A = \varepsilon_{Rmax} / \varepsilon_{Imax} \quad (3)$$

Figure 11 shows the relationship between the reflection coefficient A and r/B . As can be seen in Fig. 11, the reflection coefficient falls from about 0.9 to 0.7 with the increase in r/B . The reflection coefficient A is nearly equal to the difference between the α_{Dmax} and the α_s for the fixed value of r/B .

The factors σ_I and σ_R are superimposed at the strip plate I and the fillet, thus the resulting plateau heights are $\sigma_{Imax} + \sigma_{Rmax}$. In other words, σ_{Imax} and σ_{Rmax} are superimposed in the plate I, which is assuming as static stress state by statically balance. Accordingly, by obtaining the stress concentration factor using $\sigma_{Imax} + \sigma_{Rmax}$ as a new nominal stress, the relationship among static stress concentration factor, dynamic stress concentration factor, and the reflection coefficient is given by following equation.

$$\sigma_{Dmax} / (\sigma_{Imax} + \sigma_{Rmax}) = \alpha_s \quad (4)$$

Substituting the Eqs. (2) and (3) into the Eq. (4), we obtain

$$\alpha_{Dmax} = (1 + A) \times \alpha_s \quad (5)$$

This equation implies that the dynamic stress concentration factor at the fillet can be determined using the reflection coefficient and the static stress concentration factor.

To confirm the validity of the equation above, the comparison between the α_{Dmax} obtained directly by dynamic photoelastic method and the α_{exDmax} calculated by Eq. (5) was done. The result is shown in Fig. 12. The factor α_{exDmax} agrees with the factor α_{Dmax} within the error of 3%. Therefore the maximum stress concentration factor α_{Dmax} at the fillet can be determined by the static stress concentration factor α_s and the reflection coefficient A .

4. Conclusion

The static and the dynamic stress concentration factors at the fillet were determined by photoelastic methods. The comparison between them was carried out. Furthermore, the reflection coefficient A was calculated to investigate the influence of reflected wave on the dynamic stress concentration. The following conclusions are obtained.

- 1) The dynamic stress concentration factor α_D is 70 - 90% higher than static stress concentration factor α_s , and decreases with the increase in r/B .
- 2) The reflection coefficient A decreases from 0.9 - 0.7 with the increase in r/B .
- 3) The maximum dynamic stress concentration factor α_{Dmax} at the fillet is determined by the equation, $\alpha_{Dmax} = (1 + A) \times \alpha_s$.

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