Application of Viscoplasticity Theory Based on Overstress(VBO) to High Temperature Cyclic Deformation of 316FR Steel*

Toshiya NAKAMURA**

The viscoplasticity theory based on overstress (VBO) has been employed to model the uniaxial cyclic deformation of 316FR steel at high temperature. The experiments show that the stress strain response is rate independent in monotonic tension or the initial stage of cyclic straining, while the observed cyclic strain hardening is rate dependent. Also, the subsequent holding of strain results in a particular stress relaxation. These interesting deformation behaviors relating rate effects cannot be well reproduced by the conventional viscoplasticity theory. In the present paper extensions of VBO are introduced to reproduce these behaviors. The first extension admits a rate independent asymptotic solution. The second one produces the time (rate) dependent cyclic hardening. Good agreement is found between the experimental data and the numerical simulations by the present model.

Key Words: Constitutive Equation, Inelasticity, Viscoplasticity Theory Based on Overstress(VBO), 316FR Steel, Cyclic Deformation at High Temperature, Stress Relaxation, Strain Aging

1. Introduction

New viscoplastic or unified constitutive equations have been developed and improved for rate dependent inelastic deformation behavior of metals and alloys within the last two decades, see Miller⁽¹⁾ and Krausz and Krausz⁽²⁾ for good summaries. Such constitutive equations are especially desirable in the stress analysis of high-temperature equipment when creep-plasticity interaction takes place. For example, a precise prediction of stress relaxation is the first step in the evaluation of creep-fatigue damage. But since it is known that the stress relaxation behavior is strongly affected by the preceding deformation history, a comprehensive and rational method is desirable. It is difficult to develop a constitutive model which reproduces such history-dependent deformation behaviors. As well as a combination of "classical" plasticity and creep theories, a unified constitutive equation may be

one possible solution to this problem because in such a material model the state of a solid after any kind of loading history is identified by a set of state variables and thus the deformation history can be naturally reflected in the calculation of stress-strain behaviors. For example, recent stress relaxation tests showed that the stress relaxation behavior strongly depends on the preceding strain rate and Krempl and Nakamura found that the viscoplasticity theory Based on overstress (VBO) can reproduce this interesting relaxation behavior without any modification⁽³⁾. In the framework of the viscoplasticity (unified) theory, the total strain rate is assumed to be a sum of the elastic strain rate and the rate (time) dependent inelastic strain rate where the latter represents both creep and plastic strains. A distinction between creep and plasticity no longer exists and the interaction between plasticity and creep is expected to be described in a natural way.

Viscoplasticity theory based on overstress (VBO) was proposed and has been developed by Krempl and his colleagues. As reviewed by Krempl⁽⁴⁾, VBO has been successfully applied to different materials under a variety of loading conditions; monotonic and cyclic

* Received 7th April, 1998

** Department of Mechanical Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

loading, biaxial proportional and nonproportional loading, and at low and high homologous temperatures. Recent analysis shows that VBO can naturally reproduce the history-dependent stress relaxation behaviors, as mentioned earlier. Furthermore, its application to the thermal ratcheting problem was presented recently⁽⁵⁾, demonstrating its applicability to engineering structural analysis. In this analysis, the total amount of accumulated ratchet strain and the strain range at the postratcheting states, and the rate effects on them are numerically and theoretically discussed in order to obtain their theoretical formulas.

In the present study, VBO is applied to uniaxial cyclic deformation of 316FR steel at 923K (650°C). The 316FR steel is an advanced type of austenitic stainless steel based on type 316 steel, being specially designed for future use in Japanese high-temperature equipment. It is known to show significant strength in high temperature mechanical properties⁽⁶⁾⁻⁽⁹⁾. As will be described in detail in the next section, 316FR steel shows interesting inelastic deformation behaviors under cyclic loading at this temperature:

- The stress strain response is rate independent under monotonic tension (or the first monotonic portion of the cyclic loading) although the subsequent strain holding results in a typical stress relaxation (inconsistency of viscosity).

- The cyclic strain hardening behavior is rate dependent.

These features, particularly the inconsistency of viscosity, do not conform with the conventional VBO or with normal unified models where only one stress component, which is usually called overstress or viscous stress, is responsible for the entire viscous behavior of a material. Indeed, Haupt et al. reported that similar features can be observed in 316L steel at 1023K and that there is a difficulty in applying the Chaboche model to these features⁽¹⁰⁾. The inconsistency in viscosity is known as the dynamic strain aging. A series of experimental and theoretical studies on this phenomenon have been carried out by Kaneko and Oyamada⁽¹¹⁾. Recently Yaguchi and Takahashi proposed a unified model for modified 9Cr-1Mo steel using which the experimentally observed dynamic strain aging is well reproduced⁽¹²⁾.

In the present paper, two extensions of VBO are introduced to reproduce the above mentioned two features. The first extension admits a rate independent asymptotic solution (flow stress). The second one produces the time (rate) dependent cyclic hardening. Good agreement is found between the experimental data and the numerical simulations using the present model.

2. Cyclic Deformation of 316FR Steel at High Temperature

Some experimental observations are described in this section. The 316FR steel is specially designed for future use in Japanese high-temperature equipment and is known to show significant strength in high temperature mechanical properties⁽⁶⁾⁻⁽⁹⁾. The experimental data used in the present study were obtained in the creep-fatigue tests conducted by Asada and coworkers^{(13),(14)}. This series of creep-fatigue tests has been performed at 923K under uniaxial push-pull conditions with several types of strain waveforms. The strain waveforms included fast tension-fast compression, slow tension-fast compression, fast tension-slow compression, and strain holding cycles with a strain rate ranging from 10^{-6} to 10^{-3} s^{-1} and a strain holding period of 600s.

Experimental data are shown in Figs. 1 to 4. Figure 1 shows the stress-strain curves at the first cycle of creep-fatigue tests, and Fig. 2 at the 20th cycle. As seen in Fig. 3, this steel shows a cyclic strain hardening behavior and the cyclic stress-strain

Fig. 1 Stress-strain behavior under cyclic loading (1) : First cycle

Fig. 2 Stress-strain behavior under cyclic loading (2) : 20th cycle

Fig. 3 Cyclic strain hardening: Experimental data and simulations

Fig. 4 Stress relaxation behavior from creep-fatigue test

response saturates within the first 10 to 15 cycles. Thus, the stress strain curves in Fig. 2 at the 20th cycle are the stable ones. From Fig. 1, we see that the stress response is independent of the strain rate in the first monotonic tension or at the first cycle even at the high temperature of 923K. But as observed in Fig. 2, it shows typical rate dependency in the stable response, that is, the slower strain rate results in a smaller stress magnitude. Figure 2 also indicates that the compressive stress magnitude depends on the preceding strain rate in tension; although the compressive strain rates are the same, the slower tensile strain rate results in the smaller compressive stress magnitude. These rate-effects are also shown in Fig. 3 in terms of peak stresses. Figure 4 shows the stress relaxation behavior observed in the strain holding cycle. It is interesting that although the stress response is rate independent at the first cycle as mentioned earlier, we observe a typical stress relaxation from the first cycle. This inconsistency in the viscosity of the material makes the application of the unified theory difficult.

3. Viscoplasticity Theory Based on Overstress (VBO)

3.1 General multiaxial formulation

The viscoplasticity theory based on overstress (VBO) was originally proposed by Krempl and has been developed by Krempl and his colleagues. As reviewed by Krempl⁽⁴⁾, VBO has been applied to several different materials under a variety of loading conditions; monotonic and cyclic loading, biaxial proportional and nonproportional loading, and at low and high homologous temperatures. The model described below represents the current stage of development for small strain⁽⁴⁾.

The total strain rate $\dot{\epsilon}_{ij}$ is the sum of the elastic strain rate $\dot{\epsilon}_{ij}^{el}$ and the inelastic strain rate $\dot{\epsilon}_{ij}^{in}$;

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{el} + \dot{\epsilon}_{ij}^{in}. \quad (1)$$

The rate form of the generalized Hooke's law is assumed for the elastic counterpart

$$\dot{\epsilon}_{ij}^{el} = \frac{1+\nu}{E} \dot{\sigma}_{ij} \quad (2)$$

$$\dot{\epsilon}_{ii}^{el} = \frac{1-2\nu}{E} \dot{\sigma}_{ii}, \quad (3)$$

where σ_{ij} , s_{ij} , e_{ij}^{el} are the stress tensor, its deviator, and the deviatoric elastic strain tensor, respectively. E and ν are Young's modulus and Poisson ratio, respectively. The flow rule is given as

$$\dot{\epsilon}_{ij}^{in} = \dot{\epsilon}_{ij}^{in} \frac{3}{2} \frac{s_{ij} - g_{ij}}{Ek[\Gamma]}, \quad (4)$$

where

$$\Gamma^2 = \frac{3}{2} (s_{ij} - g_{ij})(s_{ij} - g_{ij}). \quad (5)$$

$\dot{\epsilon}_{ij}^{in}$ is the deviatoric inelastic strain rate and incompressibility is assumed for inelastic deformation. g_{ij} is called an equilibrium stress and is a state variable in the model. $s_{ij} - g_{ij}$ is called the overstress tensor and Γ is its equivalent value of the von Mises type. The function $k[\Gamma]$, the viscosity function, is the repository for rate dependence (in the present paper, following Krempl, $f[x]$ denotes a "function of x "). The deviatoric stress rate is

$$\dot{s}_{ij} = 2G(\dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^{in}), \quad (6)$$

where G is the shear modulus and e_{ij} is the deviator of total strain. The growth rule for the equilibrium stress is

$$g_{ij} = \frac{\phi[\Gamma]}{E} \left(\dot{s}_{ij} + \frac{s_{ij} - g_{ij}}{k[\Gamma]} - \frac{\Gamma}{k[\Gamma]} \frac{g_{ij} - f_{ij}}{A} \right) + \left(1 - \frac{\phi[\Gamma]}{E} \right) \dot{f}_{ij}, \quad (7)$$

where A is the rate independent contribution to the stress in the asymptotic state as will be discussed later. An increase in A due to the accumulation of the inelastic strain, as given by Eq.(8), represents the cyclic hardening when the uniaxial push-pull cyclic loading is considered.

$\dot{A} = (B_1 - B_2 A) \dot{\phi}$, (8)
initial value $A(l^{in}=0) = A_0$
where

$$\dot{\phi} = \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^{in} \dot{\epsilon}_{ij}^{in}} = \frac{\Gamma}{E k[\Gamma]}$$

is the equivalent inelastic strain rate and

$$l^{in} = \int_0^t \dot{\phi} dt$$

is the accumulated inelastic strain. The growth law for the kinematic stress f_{ij} , another state variable, is

$$\dot{f}_{ij} = \frac{3}{2} \hat{E}_t e_{ij}^{in} = \frac{\hat{E}_t}{E} \frac{s_{ij} - g_{ij}}{k[\Gamma]}, \quad (9)$$

where $\hat{E}_t = E_t / (1 - E_t/E)$, E_t is the tangent modulus with respect to total strain at the maximum total strain of interest. $\phi[\Gamma]$ is a shape function and models the transition of the stress-strain curve from the elastic to the inelastic range. The viscosity function $k[\Gamma]$ and the shape function $\phi[\Gamma]$ are determined to produce an agreement between the model and the experimental data and will be presented later for the present data together with the extended equations.

3.2 Uniaxial formulation

Since only uniaxial deformation is considered in the present study, the uniaxial formulation of VBO is shown below. The total strain rate $\dot{\epsilon}$ is composed of the elastic strain rate $\dot{\epsilon}^{el}$ and the inelastic strain rate $\dot{\epsilon}^{in}$, where the uniaxial Hooke's Law is employed for the elastic strain rate;

$$\dot{\epsilon} = \dot{\epsilon}^{el} + \dot{\epsilon}^{in} = \frac{1}{E} \dot{\sigma} + \dot{\epsilon}^{in}. \quad (10)$$

The inelastic strain rate is given as

$$\dot{\epsilon}^{in} = \frac{\sigma - g}{E k[\Gamma]}, \quad (11)$$

where the equivalent overstress Γ is now

$$\Gamma = |\sigma - g|. \quad (12)$$

The evolution equations for the equilibrium stress g , A , and the kinematic stress f are as follows.

$$\dot{g} = \phi[\Gamma] \left(\frac{\dot{\sigma}}{E} + \dot{\epsilon}^{in} - \frac{g - f}{A} |\dot{\epsilon}^{in}| \right) + \left(1 - \frac{\phi[\Gamma]}{E} \right) \dot{f} \quad (13)$$

$$\dot{A} = (B_1 - B_2 A) |\dot{\epsilon}^{in}| = (B_1 - B_2 A) \frac{\Gamma}{E k[\Gamma]} \quad (14)$$

$$\dot{f} = \hat{E}_t \dot{\epsilon}^{in}, \quad (15)$$

where the accumulated inelastic strain is now defined as

$$l^{in} = \int_0^t |\dot{\epsilon}^{in}| dt.$$

In Eq.(13), the term

$$\phi[\Gamma] \frac{g - f}{A} |\dot{\epsilon}^{in}|$$

is the dynamic recovery term. It should also be pointed out that since long-term stress relaxation and creep are not analyzed in this study, the static recovery of the equilibrium stress is not been considered. When the model is employed to simulate the long-

Fig. 5 Asymptotic solutions of the standard VBO

term stress relaxation or static creep, the static recovery must be taken into account in the evolution equation of $g^{(15)}$.

Equation (14) can easily be integrated as

$$A = \frac{B_1}{B_2} - \left(\frac{B_1}{B_2} - A_0 \right) \exp[-B_2 l^{in}]. \quad (16)$$

This solution indicates that when $\frac{B_1}{B_2} - A_0 > 0$, the internal variable A monotonically increases due to the accumulation of inelastic strain. This increasing A represents the cyclic hardening when push-pull cyclic loading is considered. And taking the limit $l^{in} \rightarrow \infty$ in Eq.(16), we obtain

$$\lim_{l^{in} \rightarrow \infty} A = \frac{B_1}{B_2}, \quad (17)$$

which represents a saturation of cyclic hardening after a large number of cycles.

3.3 Asymptotic solutions of VBO

Asymptotic solutions were derived for the constant strain rate and large (mathematically infinite) time⁽⁴⁾ and are summarized in this section. The asymptotic state is thought to be corresponding to the fully established inelastic flow in a material. The following relations were derived for the asymptotic limit^{*1}

$$\{\dot{\sigma}\} = \{\dot{g}\} = \{\dot{f}\} \quad (18)$$

and

$$\{g - f\} = A. \quad (19)$$

These equations indicate that when the inelastic strain is sufficiently large (note that we consider only the small strain version of VBO), σ , g and f grow parallel with the tangent modulus of

$$\left\{ \frac{d\sigma}{d\epsilon} \right\} = \left\{ \frac{dg}{d\epsilon} \right\} = \left\{ \frac{df}{d\epsilon} \right\} = E_t \quad (20)$$

This is schematically shown in Fig. 5.

*1 Following Krempl, $\{x\}$ designates the asymptotic limit of x .

4. Viscoplasticity Theory Based on Overstress Applied to 316FR Steel

As mentioned in the previous sections, the stress-strain response of 316FR steel at 923K shows rate independence under the monotonic tension (or the first monotonic portion of the cyclic loading) although the subsequent strain holding results in a particular stress relaxation, and the cyclic strain hardening behavior is rate dependent. These features do not conform with the conventional VBO described above. In this section, two extensions are proposed to reproduce these deformation behaviors and their validity will be examined through numerical simulations.

4.1 Rate independent asymptotic solutions

Although the stress-strain response is rate independent under the monotonic tension, the subsequent strain holding results in a particular stress relaxation. We first investigate a method to reproduce this inconsistency of the viscosity. According to Eq.(19), the asymptotic solution (asymptotic stress response, flow stress) in monotonic tension is⁽⁴⁾

$$\{\sigma\} = \{g + \Gamma\} = \{f + A\} + \{\Gamma\}. \quad (21)$$

The asymptotic solution $\{g - f\} = A$ (Eq.(19)) results from the dynamic recovery term $\phi[\Gamma](g - f)/A \cdot |\dot{\epsilon}^{in}|$, and is equivalent to

$$\left\{ \frac{g - f}{A} \right\} = 1. \quad (22)$$

Taking this into account, we extend the asymptotic solution, Eq.(19), to be *rate dependent* by assuming the following dynamic recovery.

$$\phi[\Gamma] \frac{g - f}{d[A, \Gamma]} |\dot{\epsilon}^{in}|. \quad (23)$$

The new material function $d[A, \Gamma]$ should satisfy $d[A, \Gamma] > 0$ in general. The evolution equation of the equilibrium stress is now

$$\begin{aligned} \dot{g} = & \phi[\Gamma] \left(\frac{\dot{\sigma}}{E} + \dot{\epsilon}^{in} - \frac{g - f}{d[A, \Gamma]} |\dot{\epsilon}^{in}| \right) \\ & + \left(1 - \frac{\phi[\Gamma]}{E} \right) \dot{f}. \end{aligned} \quad (24)$$

The function $d[A, \Gamma]$ controls the rate dependent magnitude of the asymptotic solution and in the present study, as a special case, the following equation is assumed so that it cancels the rate dependency (Γ) from the asymptotic solution^{*2}.

$$d[A, \Gamma] = A - \Gamma \quad (25)$$

Referring to the original asymptotic solution, Eq.(19), the dynamic recovery term Eq.(23) with a material

Fig. 6 Asymptotic solutions of the present VBO

function Eq.(25) is expected to allow the following *rate dependent* asymptotic solution^{*3}.

$$\{g - f\} = A - \{\Gamma\} \quad (26)$$

which yields the *rate independent* asymptotic stress response

$$\{\sigma\} = \{g + \Gamma\} = \{f + A - \Gamma\} + \{\Gamma\} = \{f + A\}. \quad (27)$$

This indicates that the viscosity due to the overstress disappears at least from the asymptotic solution. But since this does not imply $\Gamma \equiv 0$, the subsequent strain holding results in stress relaxation due to the overstress. The schematic illustration is shown in Fig. 6. It should be noted that even though the present model admits the rate independent asymptotic solution, it shows rate dependency in the transient region from the elastic to inelastic stage, as will be shown in Fig. 7.

4.2 Time dependent cyclic hardening

According to Chaboche and Nouailhas⁽¹⁶⁾, the reduction of the peak stress at stabilization is usually observed as a function of the hold time in various stainless steels, and they introduced a static (thermal) recovery term into the evolution equation of the isotropic hardening parameter in their viscoplastic model (Chaboche model).

As was mentioned earlier, experiments show that the cyclic strain hardening of 316FR steel at 923K is rate dependent and that it also depends on the preceding strain rate. Referring to the study by Chaboche and Nouailhas mentioned above, we consider that this feature can be modeled by introducing a static recovery term in the parameter A , which produces cyclic hardening with respect to the accumulated inelastic strain. The evolution equation of A is now assumed as

$$\dot{A} = (B_1 - B_2 A) |\dot{\epsilon}^{in}| - r \left(\frac{A - A_0}{B_3} \right)^{B_4}, \quad (28)$$

^{*2} This simple representation of Eq.(25) was found to be enough to satisfy the condition $d[A, \Gamma] > 0$ within the present study. For a more general formulation, a function such as $d[A, \Gamma] = A \exp[-a\Gamma]$ for example, can be employed which satisfies the condition $d[A, \Gamma] > 0$.

^{*3} Rigorous mathematical proof has not been established in the present study, but the numerical examples shown in Fig. 7 assert its validity.

where A_0 is the initial value of A and B_1, B_2, B_3, B_4 and r are material constants. According to this equation A increases basically due to the accumulation of the inelastic strain with simulating cyclic hardening, but the static recovery makes the evolution of A smaller for the slower deformation process or strain/stress holding. That is, slower strain rate or strain holding period results in a smaller amount of cyclic hardening.

4.3 Material functions and constants

According to Fig. 6, E_t , thus $\hat{E}_t$ is determined by measuring the tangent modulus at the largest strain level to be simulated. Then, since A represents the cyclic hardening, we can assume that $A=A_0$ during the monotonic tensile stage because l^{in} in monotonic tension is much smaller than that accumulated during cyclic loading. Considering this and that the kinematic stress f can be calculated with determined $\hat{E}_t$, the initial value A_0 is determined.

The shape function and the viscosity function, $\phi[\Gamma]$ in Eq. (24) and $k[\Gamma]$ in Eq. (11), respectively, are determined by trial and error, by producing an agreement between the calculated and the experimental results. In this study, $\phi[\Gamma]$ was determined to reproduce the realistic stress-strain curve of monotonic tension, and $k[\Gamma]$ was determined to reproduce the relaxation behavior at the stable cycle. The following functions and related material constants were obtained.

$$\phi[\Gamma] = C_1 \text{ (const.)} \quad (29)$$

$$k[\Gamma] = \frac{\Gamma}{K_1 \sinh\left[\frac{\Gamma}{K_2}\right]} \quad (30)$$

The shape function was assumed to be constant for simplicity, because it was found through numerical works that this simplification of the shape function does not produce a large effect in the stress-strain response simulation. The above k -function yields the following representation for the flow rule (inelastic strain rate)

$$\dot{\epsilon}^{in} = \frac{K_1}{E} \sinh\left[\frac{\sigma - g}{K_2}\right] \quad (31)$$

The material constants of the evolution equation of A (Eq. (28)) were determined by the following procedure. First, the initial value A_0 was already fixed as above. Then, the original evolution equation (Eq. (14) or Eq. (28)) with omitting the static recovery term, was employed to the cyclic loading data of fast tension-fast compression where the effect of static recovery can be neglected. In this manner, in this study, the material constants B_1 and B_2 were determined to reproduce the cyclic strain hardening behavior with a strain rate of 10^{-3} s^{-1} for both tension and compression going stage. In performing these

Table 1 Material constants

E	E_t	K_1	K_2	C_1	B_1	B_2	B_3	B_4	A_0	r^*
138 GPa	3.60 GPa	2.39×10^{-3} MPa s ⁻¹	3.84 MPa	89.7 GPa	2.35 GPa	9.42 GPa	95.0 MPa	14.5	145 MPa	1 MPa s ⁻¹

* r is used only for matching the dimensions.

Fig. 7 Simulation of monotonic tension tests with various strain rates

identifications, we used the fact (known from Eq. (16)) that $B_1/B_2 - A_0$ represents the amount of cyclic strain hardening at the stable cycle and that B_2 controls the transient behavior of cyclic strain hardening. Finally, the material constants B_3 and B_4 were determined from the cyclic strain hardening behavior with a slower strain rate of (tensile strain rate / compressive strain rate) $10^{-5}/10^{-3} \text{ s}^{-1}$.

The determined material constants are shown in Table 1.

4.4 Simulation and comparison with experiment

Figure 7 shows the simulation results of monotonic tension tests for various strain rates. It can be seen that the rate dependency of the stress response completely disappears from the asymptotic solutions. But it should be noted that the particular rate dependency still remains in the "knee" of the stress-strain curves. Further studies should be carried out for more precise modeling.

Stress-strain hysteresis loops under various strain rates are simulated as shown in Figs. 8 and 9. Good agreement can be observed between the experimental data and the corresponding simulation results. The cyclic strain hardening behaviors are shown in Fig. 3. Good agreement is again observed, in particular the rate dependency of the cyclic hardening is well reproduced by the present model.

Finally, Fig. 10 shows the simulation results of strain holding cycles. The figure shows the stress relaxation behavior. It is interesting that stress relaxation is generated although the rate dependency completely disappears in the tension peak of the

Fig. 8 Simulation of push-pull tests (1) : $\dot{\epsilon} = 10^{-3} \text{ s}^{-1}$ in both tension and compression, 1st and 20th cycles

Fig. 9 Simulation of push-pull tests (2) : $\dot{\epsilon} = 10^{-6} \text{ s}^{-1}$ in tension and 10^{-3} s^{-1} in compression, 1st and 20th cycles

Fig. 10 Simulation of strain holding cycle: $\dot{\epsilon} = 10^{-3} \text{ s}^{-1}$ in both tension and compression, strain holding time=600s at the tension peak, 1st and 20th cycles

first cycle. The experimental data indicates that the amount of stress relaxation at the stable cycle is larger than that at the first cycle. The present model cannot reproduce this feature. It is noted that this

does not come from the present extensions but from the original formulations. It should be investigated in a future study.

5. Conclusion

In the present paper, two extensions of the viscoplasticity theory based on overstress (VBO) were introduced in order to reproduce the experimentally observed monotonic and cyclic inelastic behaviors of 316FR steel at 923K. The features focused in the present study cannot be simulated by the conventional VBO or unified theory. The new material function is introduced in the dynamic recovery term of the evolution equation of the equilibrium stress which admits the rate independent asymptotic solution. This does not mean that the viscosity disappears, that is, the subsequent strain holding results in a particular stress relaxation. As such, the inconsistency of the viscosity observed in the experiments can be simulated. Another extension is proposed to generate the time dependent cyclic hardening. This extension allows the reduction of peak stress due to the slow deformation rate. Although some discrepancies still remains between the simulation results and the corresponding experimental ones and will be subjects of future study, in general, numerical simulations show good agreement with the experimental data indicating the validity of the present model.

Acknowledgments

The author is grateful to Professor Erhard Krempl at Rensselaer Polytechnic Institute for helpful communications. Professor Krempl and his colleagues are independently working on a similar idea about the extension on the dynamic recovery of the equilibrium stress.

References

- (1) Miller, A. K., ed., Unified Constitutive Equations for Plastic Deformation and Creep of Engineering Alloys, (1987), Elsevier Applied Science.
- (2) Krausz, A. S. and Krausz, K. ed., Unified Constitutive Laws of Plastic Deformation, (1996), Academic Press.
- (3) Krempl, E. and Nakamura, T., The Influence of the Equilibrium Stress Growth Law Formulation on the Modeling of Recently Observed Relaxation Behaviors, JSME Int. J. Ser. A, Vol. 41, No. 1 (1998), p. 103-111.
- (4) Krempl, E., A Small Strain Viscoplasticity Theory Based on Overstress, in Krausz, A.S. and Krausz, K. ed., Unified Constitutive Laws of Plastic Deformation, (1996), p. 281-318 Academic Press.
- (5) Nakamura, T. and Krempl, E., An Analysis of the

- Two Bar Ratcheting Behavior Using the Viscoplasticity Theory Based on Overstress (VBO), Transactions of the ASME Journal of Pressure Vessel Technology, Vol. 119(1997), p. 306-312.
- (6) Asada, Y., Ueta, M., Kanaoka, T., Sukekawa, M. and Nishida, T., Current Status of The Development of Advanced 316-Steel for FBR Structures, Becht IV, C., Seshadri, R. and Marriott, D. eds., Stress Classification, Robust Methods, and Elevated Temperature Design, ASME PVP-230 (1992), p. 61-65.
 - (7) Ueta, M., Nishida, T., Koto, H., Sukekawa, M. and Taguchi, K., Creep-Fatigue Properties of Advanced 316-Steel for FBR Structures, Rao, K. R. and Asada, Y. eds., Current Perspectives of International Pressure Vessels and Piping Codes and Standards, ASME PVP-Vol. B(1995), p. 423-428.
 - (8) Takahashi, Y., Long-Term Temperature Strength of 316FR Steel, Prager, M. et al. eds., Fitness-For-Service and Decisions for Petroleum and Chemical Equipment, ASME PVP-Vol. 315 (1995), p. 421-427.
 - (9) Aoto, K., Iwata, K. and Wada, Y., Inelastic Stress-Strain Relationship of FBR-Grade Type 316 Stainless Steel at Elevated Temperature, Tanimura, S. and Khan, A.S. eds., Dynamic Plasticity and Structural Behaviors, Proc. Plasticity '95, (1995), p. 577-580.
 - (10) Haupt, A., Munz, D., Scheibe, W., Schinke, B., Schmitt, R. and Sklenicka, V., High Temperature Creep and Cyclic Deformation Behaviour of AISI 316 L(N) Austenitic Steel and its Modelling with Unified Constitutive Equations, Nuclear Engineering and Design, Vol. 162(1996), p. 13-20.
 - (11) Kaneko, K. and Oyamada, T., Proposition of The Two Factors Constitutive Model Considering Viscosity and Aging For Metals (in Japanese), a Material Presented at a Research Meeting on Nonlinear Behavior of Materials, JSME(1995).
 - (12) Yaguchi, M. and Takahashi, Y., Inelastic Behavior of Mod. 9Cr-1Mo Steel and Its Description by Constitutive Model Considering Dynamic Strain Aging, SMiRT 14, Division L, Metallic Material Behavior and Damage(1997), p. 59-66.
 - (13) Motegi, J., Nakai, T., Ishikawa, A. and Asada, Y., Design Consideration on Creep-Fatigue Interaction of 316FR Steel for Actual Service Loading, Proceedings of the 8th International Conference on Pressure Vessel Technology (ICPVT-8), Volume 1, Chaaban, A. ed., Fatigue/Fracture, NDE, Materials and Manufacturing(1996), p. 33-39.
 - (14) Sugiura, T., Nakamura, T. and Asada, Y., Creep-Fatigue Damage Recovery and Life Evaluation Based on Overstress, Prager, M. ed., Fitness-for-Service and Decisions for Petroleum and Chemical Equipment, ASME PVP-Vol. 315(1995), p. 369-375.
 - (15) Majors, P. S. and Krempl, E., The Isotropic Viscoplasticity Theory Based on Overstress Applied to the Modeling of Modified 9wt.% Cr-1wt.% Mo Steel at 538°C, Materials Science and Engineering, Vol. A186(1994), p. 23-34.
 - (16) Chaboche, J.L. and Nouailhas, D., A Unified Constitutive Model for Cyclic Viscoplasticity and Its Applications to Various Stainless Steels, Transactions of the ASME, Journal of Engineering Materials and Technology, Vol. 111(1989), p. 424-430.