Ultrasonic Measurement of Axial Stress in Short Bolts with Consideration of Nonlinear Deformation*

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A precise method for estimating the axial stress is proposed for the short and highly stressed bolts used for automobile. These bolts show non-linear extension, because of local plastic deformation as well as non-uniform stress distribution in bolt cross section. The proposed method consists of (a) precise measurement of the time-of-flight of longitudinal and transverse waves using a digital pulse-echo technique, (b) to establish the calibration curves which take into account of nonlinear extension due to non-uniform stress distribution based on simultaneous measurement of the axial elongation, time-of-flight of longitudinal and transverse waves and (c) evaluation of the axial stress with the change in the time-of-flight and the assumed length of equivalent stressed length. The axial stress is estimated within 1% error for one type bolts and within 2% error for another type bolts.

Key Words: Experimental Stress Analysis, Ultrasonic Evaluation, Nondestructive Testing, Stress Measurement, Bolt Axial Stress

1. Introduction

Measurement of the axial stress in the bolted joint is an urgent need for automobiles in order to ensure safety and to reduce weight.

The conventional ultrasonic meter is not useful because the most bolts used for automobiles are short (≤ 50 mm) and highly stressed.

Namely,

① The change in the time-of-flight of ultrasonic wave caused by the axial stress is so little that an extremely high precision time-of-flight measurement is required to achieve the practical stress measurement.

② Nonlinear deformation is not negligible for short bolts, because the deformation of the bolt head and the threaded parts is not perfectly elastic, therefore

the conventional linear method of estimating the axial stress will result in a considerable margin of error.

③ The variations in the grip length, the eccentricity, bending and the condition of the bearing surface play important roles for these bolts. Therefore we must take account of these factors and establish some means of compensating the difference of stress environment in actual stress measurement and in establishing the calibration curve.

In this study, the following methods are applied to solve these problems, and an accurate measurement method for short and highly stressed bolts is developed.

(a) A digital signal acquisition and signal processing is applied for the waveform of the back wall echoes in order to improve the precision of the time-of-flight measurement.

(b) The effects of the nonlinear bolt deformation (nonlinear increase in the equivalent stressed length) on estimated stress is established, thus the error resulting from a linear estimation is reduced;

(c) A compensation method is developed for taking account of the difference in actual stress measurement and in establishing the calibration curve

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by changing fastening condition.

2. Principles of Measurement

2.1 Equivalent stressed model

As shown in Fig. 1, a bolt under an axial force (F) is modeled as a cylinder of equivalent length (L_e) and cross section (A_e). The fraction of equivalent length (β) and the average axial stress (σ) are given by;

$$\beta = \frac{L_e}{L} \quad (1)$$

$$\sigma = \frac{F}{A_e} \quad (2)$$

The velocities of longitudinal and transverse waves depend on σ :

$$\text{Longitudinal wave: } V_L = V_{L0}(1 + \alpha_L \sigma) \quad (3)$$

$$\text{Transverse wave: } V_T = V_{T0}(1 + \alpha_T \sigma) \quad (4)$$

where V_{L0} and V_{T0} are the wave velocities in the stress-free condition, and α_L and α_T are the acoustoelastic constants for longitudinal and transverse waves, (Hereafter, subscripts L and T will be omitted if the equations for the longitudinal wave and the transverse wave are identical forms.)

When a small element dx of L_e is elongated to $(1 + \sigma/E)dx$ under the axial stress, the time dt needed to propagate through the elongated element is expressed by the following equation.

$$dt = \left(1 + \frac{\sigma}{E}\right) dx/V \quad (5)$$

Thus, the time-of-flight required for the ultrasonic wave over the bolt's length is expressed by the following equation with the time-of-flight of the unstressed portion ($L - L_e$):

$$T = 2 \frac{L_0}{V_0} \left[1 + \beta \frac{\left(\frac{1}{E} - \alpha\right) \sigma}{1 + \alpha \sigma} \right] \quad (6)$$

The change in the time-of-flight resulting from the stress is given by the following equation.

$$\Delta T = 2\beta \frac{L_0}{V_0} \frac{\left(\frac{1}{E} - \alpha\right) \sigma}{1 + \alpha \sigma} \quad (7)$$

(The change of time-of-flight due to stress is expressed

Fig. 1 Equivalent model of stressed bolt

sed " ΔT " hereafter.)

In this study, the relationship between T and σ is formulated for short and highly stressed bolts which are deformed nonlinearly. In other words, the relationship between β and stress is derived.

2.2 Axial stress measurement method

2.2.1 Difference of time-of-flight method^{(1),(2)}

The relationship between σ and ΔT , the calibration curve, is established in advance of the determination of the actual stress in the bolted joint. Once the calibration curve is established, the stress is determined with ΔT measured. In this method, the stress environment in the actual measurement is assumed to be identical with that of the calibration curve measurement. The relationship between the stress and ΔT is expressed by the linear function.

$$\Delta T = 2\beta \frac{L_0}{V_0} \alpha_b \sigma \quad (8)$$

where,

$$\alpha_b = \frac{1}{E} - \alpha \quad (9)$$

This method is called the " ΔT -method" hereafter.

2.2.2 Velocity ratio method^{(3),(4)}

The velocity ratio method uses the ratio of time-of-flight between the longitudinal wave and the transverse wave (T_T/T_L). Once the relation is established, the actual stress in the bolted joint is determined with T_T/T_L measured. While this method can eliminate the measurement of time-of-flight under stress free condition, an extremely accurate time measurement is necessary because T_T/T_L changes only slightly for applied stress.

In this study, this method is used to estimate the change of β in the actual stress measurement and in the calibration curve.

T is usually expressed by the following equation.

$$\Delta T \approx 2 \frac{L_0}{V_0} (1 - \beta \alpha \sigma) \quad (10)$$

Also the ratio of time-of-flight between the longitudinal wave and the transverse wave is given by the following equation

$$\frac{T_T}{T_L} \approx \frac{V_{L0}}{V_{T0}} [1 + \beta(\alpha_L - \alpha_T)\sigma] \quad (11)$$

In this study, however, the stress is calculated with T based on Eq.(6) because the linearized equation have some error. (This method is called the " VR -method" hereafter.)

2.3 Estimation process of axial stress

Figure 2 shows the estimation process of the axial stress. In this process, firstly the calibration curves which depend on a change of β ($\Delta\beta$) with ΔT -method and VR -method are measured. In next, the change of β is estimated from the difference ($\Delta\sigma$) of the stress determined with ΔT -method and VR -method. Then

Fig. 2 Estimation process of axial stress

the stress (σ) is corrected based on the calculated minimized $\Delta\sigma$ which is estimated from $\Delta\beta$, in order to reduce the margin of error caused by the difference between the stress environment in the actual stress measurement and that in the calibration curve measurement.

2.3.1 Measurement of displacement With the axial displacement measurement, L_e , β and their dependence on the stress are determined.

2.3.2 Calibration curves Calibration curves of the ΔT -method (ΔT vs. σ) and VR -method (T_T/T_L vs. σ) are established. Then, the acoustoelastic constant α is identified with these calibration curve. The nonlinearity of the calibration curve is assumed to be the sum of the nonlinear deformation

Table 1 Chemical composition of JIS-SCM440 bolts [mass%]

C	Si	Mn	P	S	Cr	Mo
0.4	0.25	0.70	<0.03	<0.03	1.1	0.25

Table 2 Mechanical properties of JIS-SCM440 bolts

Hardness	tensile stress	yield stress	Young's modulus
Hv = 370	1100MPa	1000MPa	210.9GPa

effects and the acoustoelastic effect.

Then, the effects of a change in β are simulated based on Eq. (6) and (7) on these calibration curves.

After verifying that these two calibration curves (ΔT vs. σ , T_T/T_L vs. σ) provide a fine correlation when β is designated as the parameter between the bolts (i.e. each calibration curve is expressed with β as a parameter), these calibration curves for each bolt are saved as independent data does not represent the average value. It is also verified from these data (=hypothetical calibration curves) that there is a clear correlation between the change in β ($\Delta\beta$) and the difference of the stress values ($\Delta\sigma$) as calculated from the two axial tension values.

In VR -method, any variation in the initial bolt length is corrected by T under the stress free conditions, in order to exclude the margin of error produced by the difference in the length of the unstressed portion.

2.3.3 Correction of axial stress During the actual measurement of the bolted joint, the stress values are calculated with to the ΔT -method, VR -method. The difference ($\Delta\sigma$) between these two values is determined. Then, the relationship between hypothetical calibration curves previously saved in a PC is used to estimate the value of β which minimizes the difference ($\Delta\sigma$). Based on the β value, the stress value is determined from the calibration curve.

3. Experimentation Method

3.1 The bolts

Two types of bolts were used in the experiment. Dimension, shape, the chemical composition and the mechanical properties of the bolts are shown in Fig. 3 and Tables 1 and 2.

The bolts were finished by cylindrical grinding to achieve the geometrical tolerance illustrated in Fig. 3. Bolt, nut and the edge portion of the clamped object were made smooth by lapping. (Hereafter, a long and thin bolt is called "Bolt-A" and a short and thick bolt, "Bolt-B".)

3.2 Measurement of load and displacement

A compression is applied on a jig⁽⁵⁾ that applies the tension to the bolt as shown in Fig. 4. The

called "ZCM" hereafter.)

3.3.3.b Cross-correlation method A cross-correlation function of an echo $x(t)$ and another echo $y(t)$ shown in Fig. 6 is defined by Eq.(12).

$$R_{xy} = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt \quad (12)$$

Time delay (τ), which maximizes R_{xy} , is given.

$$y(t) = \pm ax(t - T) \quad (13)$$

where T is the time-of-flight of the echo y in relation to x which has no disperse and attenuation a .

The cross-correlation function in the discrete system was defined by Eq.(14), and k which maximizes R_{xy} was determined.

$$R_{xy} = \frac{1}{N} \sum_{n=0}^{N-k-1} x(n)y(n+k) \quad (14)$$

To determine k up to the fraction of the sampling interval, R_{xy} was approximated by a quadratic function of k , and k_m which gave an extreme value was used. (This method is called "CCM" hereafter.)

To verify the accuracy of these two method, the time-of-flight was measured under stress free conditions. As a result, the time resolution (95% reliability) was 0.006 ns for ZCM and 0.004 ns for CCM for the longitudinal wave, and 0.02 ns for ZCM and 0.01 ns for CCM for the transverse wave. In other words, the measurement error (the accuracy that excluded such error factors as the tensile environment and variations in the clamped object) of these two methods was 1×10^{-6} or less in terms of relative error.

For comparison, the measurement using commercially available pulse echo instrument were also taken.

The instrument determines the time-of-flight with the peak interval between the first and the second back wall echoes which were averaged 10 times and has a minimum sampling interval of 10 ns. This method had the time resolution of 0.3 - 0.4 ns under stress free condition. (The method is called "PE" hereafter.)

4. Experimentation Results and Discussion

Changes in the time-of-flight due to the axial stress for five units of Bolt-B are shown in Fig. 7. Variations in the time-of-flight depend on the initial length. Variations in the slope provide a fine correlation with the difference in the grip length actually measured before applying the load, and can be roughly approximated by Eq.(6).

The relationship, calibration curve of ΔT -method, between the axial stress (σ) and the difference of the time-of-flight (ΔT) is shown in Fig. 8 for one piece of Bolt-B. Slight nonlinearity is observed the time-of-flight of the longitudinal wave and the transverse wave. This is the result of the acoustoelastic

Fig. 7 Variation of time-of-flight for different bolts

Fig. 8 Change in time-of-flight with applied stress

effect [the term multiplied by β in Eq.(6) and (7)] and the nonlinear deformation of the bolt. The curvature as defined by Eq.(15) was 8.1×10^{-6} for Bolt-A and 2.3×10^{-5} for Bolt-B.

$$\zeta = \frac{\frac{d^2 \Delta T}{d\sigma^2}}{\left[1 + \left(\frac{d\Delta T}{d\sigma}\right)^2\right]^{3/2}} \quad (15)$$

The values of the curvature are 2.2 times and 8.5 times bigger than the values due to the acoustoelastic effect, which was assumed to deform linearly.

The time-of-flight ratio (T_T/T_L) between the longitudinal wave and the transverse wave at various stress on the Bolt-A is shown in Fig. 9. Although the slopes of the curves are almost agree within 0.02% or less (95% reliability), a 6 to 8% variation of T_T/T_L is observed due to the difference in the initial length. The length of the nondeformed portion was estimated from the difference in the time-of-flight under stress free condition, and the length was corrected to conditions identical with the calibration curves in order to exclude the effects of the difference in the initial

Fig. 9 Variation of time-of-flight ratio with stress

Fig. 10 Comparison of error on measurement methods

length. The curvature in this case was calculated by substituting T_T/T_L for ΔT in Eq.(15); the curvature was 2.8×10^{-10} for Bolt-A and 9.2×10^{-10} for Bolt-B. These values were 2.4 times and 8.9 times greater than the curvature caused by the acoustoelastic effect.

4.1 Comparison of measurement technique

For a comparison of the measurement accuracy between PE and CCM, the averaged relative error are shown in Fig. 10, where it is defined as the difference between the calibration curves and actual stress in the range from 30 to 70% of the yield stress.

Nearly the same results were obtained for ZCM and CCM.

4.2 Error factors for calibration curve

In order to confirm the reliability of the approximate orders, Fig. 11 shows the relationship between the approximate orders of calibration curve and the relative error. It also shows the relationship between the approximate orders and $1-R^2$, which was determined by substituting the contribution ratio (R^2) for the correlation coefficient (r). The relationship shows the nonlinearly effects of above calibration curves on the error. When Bolt-B is calibrated by conventional linear approximation, i.e. by Eq.(8), it is shown that

Fig. 11 Comparison of error on approximate orders

an error of 1% or more will be produced even when a perfect measurement is taken. Since the approximate orders of the calibration curve have a significant effect on the accuracy of the measurement method, the choice of the method will always require the verification of the margin of error based on the actual measurement.

The effect of the linear approximation according to Eq.(11) of VR-method on the error was also checked; and it was confirmed that a similar margin of error (1% or more with the Bolt-B) would result.

To clearly understand the cause of the error, a nondimensional comparison of the effect between the nonlinear deformation and the acoustoelastic effect on the nonlinearity of calibration curves was made as shown in Fig. 12.

δ'' and $\Delta T''$ as defined in Fig. 12 were the values assumed to change linearly according to changes in stress, and was calculated from the linear approximation curve of each measurement data. In other words, the slope of each factor shows the magnitude of effect on the nonlinearity.

While the effects of nonlinear deformation and the acoustoelastic constant were nearly the same in the case of the Bolt-A, while Bolt-B, the effects of nonlinear deformation is 90% or more. It was also found that the sum of two factors was approximately equal to the nonlinearity of the calibration curves. As a result of these effects, β was found to have increased by 4% at 70% of the yield stress.

Moreover, the results of FEM analysis are shown in Fig. 13 (equistress curve) and Fig. 14 (example of elastoplastic FEM analysis on the Bolt-B). Figure 13 shows that only a small portion of the bolt is uniformly stressed in the axial direction. Figure 14 shows qualitatively that the clamped parts by the Bolt-B had been subjected extremely high stress locally when

Fig. 12 Effect of nonlinear deformation on ΔT -variation

Fig. 13 Equistress curves by FEM analysis

compared with the stress in the bolt shaft. Namely some plastic deformation had already appeared for the bolt shaft (Stress less than 500 MPa).

For a short and highly stressed bolt, we should establish the calibration curve take into consideration of the nonlinearity. The calibration curves based on the actual measurement at various grip lengths and the calibration curves simulated by letting $\Delta\beta=0$ were compared to determine whether the calibration curves of the T_T/T_L based on Eq. (6) and the calibra-

Fig. 14 Elastoplastic FEM analysis in Bolt-B

Fig. 15 Difference of estimated stresses from ΔT -method and T_T/T_L -method on β -variation

tion curves of ΔT based on Eq. (7) could express the change in β . The result shows that the error is 0.2% or less (95% reliability) in the stress range from 30% to 70% of the yield stress in terms of the stress for Bolt-A and 0.8% or less (95% reliability) for Bolt-B.

4.3 Correction

The relationship between the change in β ($\Delta\beta$) and the difference ($\Delta\sigma$) between the stress values as are shown in Fig. 15. Since the change is extremely small, the time-of-flight measurement with extremely high precision will be required to carry out an effective correction. However, the time-of-flight

Fig. 16 Effect of correction in estimated stress

resolution of 0.1 ns or less will be sufficient to estimate a change of 0.5% or less in β . Thus, both ZCM and CCM are fully capable of making an adequate correction. When compared with the change as determined under the assumption of linear deformation as also shown in Fig. 15, the change in $\Delta\sigma$ against $\Delta\beta$ shows a reverse slope, indicating that the change will increase and the correction accuracy will improve.

The validity of this correction, the corrected stress value based on the change in β ($\Delta\beta$) which was estimated in the same tensile environment as that during the moment of calibration curves measurement were compared with the actually measured stress value; and the comparison is shown in Fig. 16. The data before the correction was the stress calculated on the assumption that $\Delta\beta=0$. With Bolt-A, the correc-

tion had the tendency of making the stress value closer to the actual measured value. The correction is clearly significant effect, achieving an error margin of nearly 1%. On the other hand for Bolt-B, the correction was proven significant but variations are large and the margin of error was 20 times or more large than the theoretical error (approximate error of calibration curve, etc.). Non uniform stress distribution is the main reason for the error, but details will have to be investigated in future study efforts.

5. Conclusion

The effectiveness of the acoustoelastic stress measurement has shown for short and highly stressed bolts used for automobiles with precision-machining, the measurement error of 1 to 2% has been achieved by combining (a) a highly precise measurement of time-of-flight by Zero-Crossing Method or Cross-Correlation Method with the digital signal acquisition of signal processing, (b) establishing of optimum calibration curves, and (c) the correction of the stress for variations in the actual measurement environment.

To apply this method for actual stress measurement, we should take account of the eccentricity, variations of the clamped object.

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