The Dynamic J Integral, Separated Dynamic J Integrals and Moving Finite Element Simulations, for Subsonic, Transonic and Supersonic Interfacial Crack Propagation*

Toshihisa NISHIOKA** and Anwer YASIN**

First, the concepts of separated dynamic J integral and separated energy release rate were presented for dynamic interfacial fracture mechanics. The separated dynamic J integral has the physical meaning of the energy flow rate to a propagating interfacial crack tip from each material component. Next, a moving finite element method for dynamic interfacial fracture analysis was developed to make it possible to analyze transonic and supersonic interfacial crack propagation. The moving finite element simulations revealed the shock waves (Mach waves) emanated from transonically and supersonically propagating crack tips. The dynamic J integral and the separated dynamic J integrals showed excellent path independence for all crack-velocity regimes even for supersonic interfacial crack propagation. It was found that, in the subsonic crack-velocity regimes, the compliant material supplies larger fracture energy than the stiff material does. From the energy flow rate to the propagating interfacial crack tip, the theoretical limit of interfacial crack velocity without macro-contact zone was found to be the shear wave velocity of the compliant material.

Key Words: Fracture Mechanics, Computational Mechanics, Dynamic Fracture, Dynamic J Integral, Dynamic Energy Release Rate, Energy Flow, Interfacial Crack Propagation, Dynamic Crack Propagation, Mach Wave, Terminal Crack Velocity

1. Introduction

In recent years, interfacial fracture mechanics has been a focus of much attention. This is due to the fact that the performance of a material with an interface such as composite material, jointed material, etc., heavily depends on the interfacial strength. However, most of studies on interfacial fracture mechanics have been done under static conditions. Therefore, the establishment of dynamic interfacial fracture mechanics is one of the most important issues in the field of fracture mechanics.

In recent experiments, Rosakis and coworkers^{(1),(2)} have observed extremely high velocities of interfacial crack propagation in bimetals, such as transonic regimes with respect to the elastic wave

velocities in the compliant material side. In homogeneous materials, such range of crack velocity is not possible to achieve, from the theoretical points of view.

The singular stress fields for dynamically propagating interfacial cracks have been derived by Yang et al.⁽³⁾ for the subsonic regime. For a crack propagating transonically along an elastic/rigid interface, Liu et al.⁽⁴⁾ have presented the crack-tip singular stress field. They have also shown a singular line emanating from the moving crack tip, due to the transonic nature of crack propagation.

Finite element simulations for dynamic interfacial crack propagation have been done by Nakamura et al.⁽⁵⁾ using the fixed element method with a nodal release technique for four-noded elements. Xu and Needleman⁽⁶⁾ also have used the fixed element method with triangular cohesive surface elements. The cohesive surface relation automatically determines the creation of new fracture surfaces. Thus, this simulation may be classified into the

* Received 3rd August, 1998

** Department of Ocean Mechanical Engineering, Kobe University of Mercantile Marine, 5-1-1 Fukae Minamimachi, Higashinada-ku, Kobe 658-0022, Japan. E-mail: nishioka@cc.kshosen.ac.jp

application-phase simulation^{(7),(8)} in which the crack propagation history is one of direct outcomes of the analysis. In their simulations⁽⁶⁾, the crack velocities that exceed the Rayleigh wave velocity of the compliant material were found.

So far, to the authors' knowledge, no simulations for supersonic interfacial crack propagation have been done. Furthermore, no finite element simulations with the moving finite element method have been developed for dynamic interfacial crack propagation, although it is known that the moving finite element method is more accurate than the fixed element method, for the analysis of dynamic crack propagation in a homogeneous material^{(8),(9)}.

In this paper, first, the path independent dynamic J integral derived by Nishioka and Atluri⁽¹⁰⁾ is extended for dynamic interfacial fracture mechanics. The concepts of separated dynamic J integral and separated energy release rate are presented for a crack propagating along an interface. The separated dynamic J integral has the physical meaning of the energy flow rate to the propagating interfacial crack tip from each material component.

Furthermore, in this paper, the moving isoparametric element method for dynamic interfacial fracture analysis is developed. Using the moving finite element method, numerical simulations are carried out for subsonic, transonic and supersonic interfacial crack propagation in bimaterial plates.

2. Path Independent Dynamic J Integral for Homogeneous Materials

The static J integral⁽¹¹⁾ has played important roles in the advancement of *static fracture mechanics*. From the theoretical and computational points of view, the static J integral has the following salient features: (i) it physically represents the energy release rate; (ii) it has the property of the path-independent integral, which gives a unique value for an arbitrary integral path surrounding the crack tip; (iii) it can be related to the stress intensity factors by arbitrarily shrinking the integral path to the crack tip.

In the case of *dynamic fracture mechanics*, Nishioka and Atluri⁽¹⁰⁾ have derived the dynamic J integral (J') which has the aforementioned three features as:

$$J'_k = \lim_{\Gamma_c \rightarrow 0} \int_{\Gamma_c} [(W + K)n_k - t_i u_{i,k}] dS \quad (1.a)$$

$$= \lim_{\Gamma_c \rightarrow 0} \left\{ \int_{\Gamma + \Gamma_c} [(W + K)n_k - t_i u_{i,k}] dS + \int_{V_{\Gamma} - V_c} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\} \quad (1.b)$$

where W and K are the strain and kinetic energy densities, respectively, and $(\cdot)_{,k} = \partial(\cdot)/\partial X_k$. The integral paths are defined in Fig. 1. Physically, the

Fig. 1 A dynamically propagating crack in a homogeneous material

near-tip region V_c can be considered as the process zone in which micro-processes associated with fracture occur. The physical meaning of the J'_k integrals can be considered as the global-axis components of the energy release rate.

The crack-axis components of the dynamic J integral J'^0_k can be evaluated by the coordinate transformation:

$$J'^0_k = \alpha_{kl} J'_l \quad (2)$$

where α_{kl} is the coordinate transformation tensor. For a two-dimensional case as shown in Fig. 1, the magnitude of the vectorial dynamic J integral J' can be expressed by

$$J' = J'_1 \cos \theta_0 + J'_2 \sin \theta_0. \quad (3)$$

Thus the dynamic energy release rate G can be expressed by

$$G = J' = J'^0_1 \quad (4)$$

where J'^0_1 is the tangential component of the dynamic J integral along the crack axis. The crack-axis components $J'^0_k (k=1, 2)$ can be related to the dynamic stress intensity factors⁽¹⁰⁾ as:

$$J'^0_1 = \frac{1}{2\mu} \{A_I(C)K_I^2 + A_{II}(C)K_{II}^2 + A_{III}(C)K_{III}^2\} \quad (5.a)$$

$$J'^0_2 = -\frac{A_{IV}(C)}{\mu} K_I K_{II}. \quad (5.b)$$

In most numerical analyses, the far-field integrals are usually used to evaluate the values of the dynamic J integral. In this case it is convenient to consider the following expression of the dynamic J integral:

$$J'_k = \int_{\Gamma + \Gamma_c} [(W + K)n_k - t_i u_{i,k}] dS + \int_{V_{\Gamma}} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV. \quad (6)$$

3. Separated Dynamic J Integrals for a Dynamically Propagating Interfacial Crack

Now we consider a dynamically propagating crack along a bimaterial interface (see Fig. 2). The

Fig. 2 A dynamically propagating crack along an interface in a nonhomogeneous material

Fig. 3 Separated near-field paths

interface may be curved or straight. Similarly to the homogeneous case, the moving coordinate system x_k^0 is considered at the propagating interfacial crack tip. The angle θ_0 measured from the global X_1 axis denotes the direction of crack propagation at an instantaneous time t .

The energy flow to the propagating interfacial crack tip can be separated as shown in Fig. 3. The energies from the materials 1 and 2 flow separately into the propagating crack tip through the separated near-field paths $\Gamma_\epsilon^{(m)} (m=1, 2)$. Then the separated dynamic J integrals which have the physical meaning of separated energy release rate components for the materials 1 and 2, can be defined as

$$J_k^{(m)} = \lim_{\Gamma_\epsilon^{(m)} \rightarrow 0} \int_{\Gamma_\epsilon^{(m)}} [(W+K)n_k - t_i u_{i,k}] dS. \quad (7)$$

Fig. 4 Definition of integral paths for an interfacial crack

Using the integral paths defined in Fig. 4 and the divergence theorem, the path independent forms of the separated dynamic J integral components can be derived as

$$J_k^{(m)} = \lim_{\Gamma_\epsilon^{(m)} \rightarrow 0} \left\{ \int_{\Gamma^{(m)}} [(W+K)n_k - t_i u_{i,k}] dS + \int_{V_{\Gamma^{(m)}} - V_{\epsilon^{(m)}}} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\} \quad (m=1, 2) \quad (8)$$

where $\Gamma^{(m)} (m=1, 2)$ are the integral paths along the interface in the sides of the materials 1 and 2, respectively. The path independence of the separated dynamic J integrals can be verified in a similar manner as used for the dynamic J integral derived by Nishioka and Atluri⁽¹⁰⁾.

The crack-axis components of the separated dynamic J integrals can be obtained by the coordinate transformation:

$$J_k^{0(m)} = a_{kl} J_l^{(m)} \quad (m=1, 2). \quad (9)$$

Also the crack-axis components of the separated dynamic J integrals $J_k^{0(m)}$ can be evaluated directly by the following equation, when all components in the integrands of Eqs. (7) and (8) are expressed by the crack-tip coordinate system x_k^0 , or when $\theta_0 = 0$:

$$J_k^{0(m)} = \lim_{\Gamma_\epsilon^{(m)} \rightarrow 0} \int_{\Gamma_\epsilon^{(m)}} [(W+K)n_k - t_i u_{i,k}] dS \quad (10.a)$$

$$= \lim_{\Gamma_\epsilon^{(m)} \rightarrow 0} \left\{ \int_{\Gamma^{(m)}} [(W+K)n_k - t_i u_{i,k}] dS + \int_{V_{\Gamma^{(m)}} - V_{\epsilon^{(m)}}} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\} \quad (m=1, 2) \quad (10.b)$$

Thus the separated energy release rates $G^{(m)} (m=1, 2)$ which are the energy flow rates from the material $m (m=1, 2)$ into the propagating interfacial crack tip per unit crack extension, can be expressed by

$$G^{(m)} = J_1^{0(m)} = J_1^{(m)} \cos \theta_0 + J_2^{(m)} \sin \theta_0, \quad (m=1, 2). \quad (11)$$

Consequently the energy release rate G can be evaluated by

$$G = G^{(1)} + G^{(2)} = J_1^0 = J_1^{0(1)} + J_1^{0(2)} \\ = (J_1^{(1)} + J_1^{(2)}) \cos \theta_0 + (J_2^{(1)} + J_2^{(2)}) \sin \theta_0. \quad (12)$$

4. Energy Flows from Both Material Sides to a Dynamically Propagating Interfacial Crack Tip

The energy release rate G can also be evaluated by using the crack closure integral⁽¹²⁾ as

$$G = \lim_{\Delta a \rightarrow 0} \left\{ \frac{1}{2\Delta a} \int_0^{\Delta a} [t_i(a) \delta_i(a + \Delta a)]_{x_2^0=0} dS \right\} \quad (13)$$

where t_i is the traction force along the line of $a \leq x_1^0 \leq a + \Delta a$, δ_i is the crack opening displacement in the x_i^0 direction.

By using the concept of crack closure integral, the separated energy release rates can also be derived by

$$G^{(m)} = \lim_{\Delta a \rightarrow 0} \left\{ \frac{1}{2\Delta a} \int_0^{\Delta a} [t_i^{(m)}(a)(u_i^{(m)}(a + \Delta a) - u_i^c(a + \Delta a))]_{x_2^0=0} dS \right\} \quad (m=1, 2) \quad (14)$$

Where $u_i^{(m)}$ is the crack-face displacement of the material m ($m=1, 2$), and u_i^c is the x_i^0 displacement component at the crack tip.

Furthermore, the energy release rate can be evaluated from the global energy balance of the whole cracked body (see Fig. 5) during the crack extension of da :

$$G = \frac{1}{B} \frac{dF}{da} = \frac{1}{B} \left(\frac{d\bar{P}}{da} - \frac{d\bar{W}}{da} - \frac{d\bar{T}}{da} \right) \\ = \frac{1}{B \cdot C} \left(\frac{d\bar{P}}{dt} - \frac{d\bar{W}}{dt} - \frac{d\bar{T}}{dt} \right) \quad (15)$$

where B is the thickness of the body, and F , $\bar{P}$, $\bar{W}$ and $\bar{T}$ are the fracture energy, the work done by the applied forces, the strain energy and the kinetic

energy, respectively. When the crack is propagating with its velocity C , the crack extension da during the time increment dt can be replaced by $C \cdot dt$.

Similarly the separated energy release rates $G^{(m)}$ can be evaluated by

$$G^{(m)} = \frac{1}{B \cdot C} \frac{dF^{(m)}}{dt} = \frac{1}{B \cdot C} \left(\frac{d\bar{P}^{(m)}}{dt} + \frac{d\bar{P}_1^{(m)}}{dt} - \frac{d\bar{W}^{(m)}}{dt} - \frac{d\bar{T}^{(m)}}{dt} \right) \quad (m=1, 2) \quad (16)$$

where $\bar{P}^{(m)}$ is the input energy by the applied loads at the material- m side and $\bar{P}_1^{(m)}$ is the work done by the traction of the other-side material through the interface. The rates of these input energies to the material m can be evaluated by

$$\frac{d\bar{P}^{(m)}}{dt} = \int_{\partial V^{(m)} - S_1^{(m)}} t_i \dot{u}_i dS \quad (17)$$

and

$$\frac{d\bar{P}_1^{(m)}}{dt} = \int_{S_1^{(m)}} t_i^{(m)} \dot{u}_i^{(m)} dS \quad (18)$$

where $\partial V^{(m)}$ denotes the whole boundary of the region occupied by the material m , and $S_1^{(m)}$ denotes the whole uncracked interface at the material- m side. These integral paths are defined in Fig. 5. Since the continuity conditions along the uncracked interface can be written as:

$$\dot{u}_i^{(1)} = \dot{u}_i^{(2)} \text{ and } t_i^{(1)} + t_i^{(2)} = 0, \quad (19.a, b)$$

the following relation holds:

$$\frac{d\bar{P}_1^{(1)}}{dt} + \frac{d\bar{P}_1^{(2)}}{dt} = 0. \quad (20)$$

Therefore the sum of the separated energy release rates naturally becomes the total energy release rate G :

$$G^{(1)} + G^{(2)} = \frac{1}{B \cdot C} \left\{ \frac{d(\bar{P}^{(1)} + \bar{P}^{(2)})}{dt} - \frac{d(\bar{W}^{(1)} + \bar{W}^{(2)})}{dt} - \frac{d(\bar{T}^{(1)} + \bar{T}^{(2)})}{dt} \right\} = G. \quad (21)$$

5. Moving Finite Element Method

In numerical simulation of dynamic crack propagation, one must treat a moving-singularity boundary-value problem. Since the moving singularity induces various errors in numerical models, it is critical to develop accurate solution procedures for the advancement of fast fracture mechanics.

Nishioka and coworkers^{(8),(9),(13)} have developed various types of moving finite element procedure. In this study, the moving isoparametric element method⁽¹⁴⁾ is extended to interfacial crack problems as shown in Fig. 6. In this paper, it is assumed that the crack always propagates along the interface.

In the moving element procedure, the mesh pattern for the elements near the crack-tip translates in each time step for which crack growth occurs as illustrated in Fig. 6. Thus, the crack-tip always remains at the center of the moving elements

Fig. 5 A bimaterial with an interfacial crack

Fig. 6 Moving finite element method

throughout the analysis. The regular isoparametric elements surrounding the moving elements are continuously distorted. To simulate a large amount of crack propagation, the mesh pattern around the moving elements is periodically readjusted as also shown in Fig. 6.

The Newmark method is used for the time integration of the equation of motion. The final form to be solved for the moving finite element method can be expressed by

$$(a_0[M]^{(n)} + [K]^{(n)})\{\dot{Q}\}_n^{(n)} = \{F\}_n^{(n)} + [M]^{(n)}(a_0\{\dot{Q}\}_{n-1}^{(n)} + a_2\{\ddot{Q}\}_{n-1}^{(n)} + a_3\{\ddot{Q}\}_{n-1}^{(n)}) \quad (22)$$

where $[M]$, $[K]$, $\{F\}$ are the mass matrix, stiffness matrix, and load vector, respectively. The vectors of nodal displacements, velocities and accelerations are expressed by $\{Q\}$, $\{\dot{Q}\}$, $\{\ddot{Q}\}$, respectively. The subscripts n and $n-1$ denote the nodal quantities at the current step n and at the previous step $n-1$, respectively, while the superscript (n) denotes the nodal quantities of the mesh pattern at the step n . The coefficient a_0 and others are summarized in Eq. (25). As seen in Eq. (22), to obtain the solution at the current step, this method requires the nodal displacements, velocities, and accelerations at the previous step.

After determining the nodal displacement vector at the current time step, the nodal velocities and accelerations can be determined by

$$\{\ddot{Q}\}_n^{(n)} = a_0(\{\dot{Q}\}_n^{(n)} - \{\dot{Q}\}_{n-1}^{(n)}) - a_2\{\ddot{Q}\}_{n-1}^{(n)} - a_3\{\ddot{Q}\}_{n-1}^{(n)} \quad (23)$$

$$\{\dot{Q}\}_n^{(n)} = \{\dot{Q}\}_{n-1}^{(n)} + a_6\{\ddot{Q}\}_{n-1}^{(n)} + a_7\{\ddot{Q}\}_n^{(n)} \quad (24)$$

The coefficients $a_0 \sim a_7$ are given by

$$\begin{aligned} a_0 &= 1/(\beta \Delta t_n^2), \quad a_1 = \delta/(\beta \Delta t_n), \quad a_2 = 1/(\beta \Delta t_n); \\ a_3 &= 1/(2\beta) - 1, \quad a_4 = (\delta/\beta) - 1, \\ a_5 &= (\Delta t_n/2)\{(\delta/\beta) - 2\}, \quad a_6 = \Delta t_n(1 - \delta), \quad a_7 = \delta \Delta t_n \end{aligned} \quad (25)$$

where Δt_n is the time increment, and β and δ are the Newmark's parameters, and $\beta=1/4$ and $\delta=1/2$ are

Fig. 7 Inverse mapping of an isoparametric 8-noded element

chosen to assure the unconditionally stable time integration scheme.

Thus, to obtain the solution at the current step, the displacements, velocities, and accelerations of the previous step at the newly created nodes after shifting or readjusting the mesh pattern (see Fig. 6) are required. These conversions can be done for a new node p as indicated in Fig. 7, as follows:

$$(u_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(u_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (26.a)$$

$$(v_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(v_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (26.b)$$

$$(\dot{u}_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(\dot{u}_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (27.a)$$

$$(\dot{v}_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(\dot{v}_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (27.b)$$

$$(\ddot{u}_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(\ddot{u}_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (28.a)$$

$$(\ddot{v}_p)_{n-1}^{(n)} = \sum_{i=1}^8 N_i^{(n-1)}(\xi_p, \eta_p)(\ddot{v}_i)_{n-1}^{(n-1)}; \quad (p=1, 2, \dots, N) \quad (28.b)$$

in which u and v are the displacement components in the global X_1 and X_2 directions. N is the number of

nodes in the moving elements or in the elements to be readjusted (see Fig. 6).

To calculate the conversion Eqs. (26) to (28), two automatic schemes were developed: One scheme automatically finds an old element that includes a newly created node (see the point p in Fig. 7) after moving or readjusting the element around the propagating crack tip. Another scheme automatically finds the natural coordinates (ξ_p, η_p) in the old element (see Fig. 7), corresponding to the global coordinates (X_{1p}, X_{2p}) of the newly created node. The inverse mapping formulas were obtained by using a computerized symbolic manipulation^{(15),(16)}. For general quadrilateral elements with straight sides, explicit expressions for the inverse mapping formulas were derived⁽¹⁵⁾. For curved eight-noded isoparametric

Fig. 8 Bimaterial plate with an interfacial edge crack

elements, the inverse mapping relations become lengthy irrational equations. To solve the irrational equations, a numerical procedure based on the Regula-Falsi method was developed⁽¹⁶⁾.

An extremely important feature that distinguishes the moving element procedures from the fixed element procedures with various nodal release techniques often used in literature is that the displacement boundary conditions ahead of the crack-tip can be satisfied exactly in the moving element procedures.

6. Numerical Simulations of Dynamic Crack Propagation

In this study, dynamic crack propagation along the interface of a bimaterial plate (see Fig. 8) is considered. The dimensions of the plate are $W=2L=40$ mm. The more compliant material is placed in the upper part of the plate, and denoted by the material 1, while the stiffer one is denoted by the material 2. The plane strain condition is assumed to prevail in the plate. The plate is subject to constrained displacements $\bar{u}_0$ in the X_2 direction at the upper and lower ends of the plate. Then at the time $t=0$, the crack is assumed to propagate with a constant velocity C .

Young's moduli, Poisson's ratios, the mass densities and the shear moduli of both materials are denoted by $E^{(m)}$, $\nu^{(m)}$, $\rho^{(m)}$, $\mu^{(m)}$ ($m=1, 2$), respectively. The velocities of dilatational, shear and Rayleigh waves are given by

$$C_d^{(m)} = \sqrt{\frac{(\kappa^{(m)} + 1)}{(\kappa^{(m)} - 1)} \frac{\mu^{(m)}}{\rho^{(m)}}}, \quad C_s^{(m)} = \sqrt{\frac{\mu^{(m)}}{\rho^{(m)}}},$$

$$C_R^{(m)} = C_s^{(m)} \frac{(0.862 + 1.14\nu^{(m)})}{(1 + \nu^{(m)})}, \quad (m=1, 2)$$

(29.a,b,c)

where $\kappa^{(m)} = 3 - 4\nu^{(m)}$ for plane strain, and $(3 - \nu^{(m)}) / (1 + \nu^{(m)})$ for plane stress.

Fig. 9 Moving finite element mesh at $t=0$.

Figure 9(a) shows the entire finite element mesh at time $t=0$. The eight-noded isoparametric elements were used. The moving elements around the crack tip and the integral paths for the dynamic J integral and the separated dynamic J integrals are shown in Fig. 9(b).

6.1 Dynamic crack propagation in a homogeneous material

In order to verify the moving finite element method, dynamic crack propagation in a homogeneous plate ($E^{(2)}/E^{(1)}=1.0$) with the same dimensions as depicted in Fig. 8 is analyzed by using the same mesh pattern shown in Fig. 9. The crack is assumed to start to propagate from an initial length of $a_0=0.2W$, with the velocity of $C=0.4C_s$. The same problem has been analyzed by using a moving singular element method⁽¹⁷⁾. This problem is analogous to that treated by Nilsson⁽¹⁸⁾ who obtained an analytical solution for the constant velocity propagation of semi-infinite crack in an infinite-width strip ($W=\infty$) with the same finite-height $2L$.

The values of the dynamic J integral in this analysis indicate excellent path independence. Those dynamic J integral values are converted to the mode-I dynamic stress intensity factors K_I by using Eq. (5.a) with $K_{II}=K_{III}=0$. The results are shown in Fig. 10 along with the other solutions. In the figure, the stress intensity factors were normalized by the static stress intensity factor for the semi-infinite crack in the infinite-width strip, namely, $K_{IS}^\infty = u_0 E / \{L^{1/2}(1-\nu)\}$.

It is seen from Fig. 10 that the present results agree with the results obtained by the moving singular element method⁽¹⁷⁾ after certain amount of dynamic crack propagation, and agree with the Nilsson's steady-state solution⁽¹⁸⁾ when the steady-state condition prevails. The discrepancy between the present result in the early stage and the moving singular element result can be attributed to the sizes of time step and the apparent damping matrix due to the convection

Fig. 10 Dynamic stress intensity factors converted from the dynamic J integral values

term in the moving coordinate formulation⁽¹⁷⁾.

In order to check the applicability of the dynamic J integral for extremely high velocity of crack propagation, the crack velocity was increased up to the Rayleigh wave velocity ($C=0.924C_s=1.0C_R$) in the same homogeneous plate. Theoretically the energy release rate for this crack velocity becomes zero⁽¹⁹⁾. Moreover, the crack cannot sustain straight propagation and tends to bifurcate, since the maximum hoop stress around the crack tip with its velocity of $C > 0.6C_s$ no longer appears right ahead of the crack direction⁽¹⁹⁾. Thus in the present case, the crack was forced to propagate straight.

The variation of the dynamic J integral during the crack propagation with $C=1.0C_R$ is shown in Fig. 11. The dynamic J integral values were normalized by the static J integral value at $t=0$, i.e., J_{IS}^0 . The dynamic J integral values are very close to zero (about 1% numerical errors compared with the initial static J value) except in the very early stage. The energy release rates are also calculated by the global energy balance (see Eq. (15)) and plotted in Fig. 11. As seen from the figure, for high velocity of crack propagation, the global energy release rate fluctuates and differs from the theoretical one. One of the reasons for this may be attributed that extremely small increments of time and crack length ($\Delta t=2.9363$ ns and $\Delta a=5.8594$ μ m) were used in this analysis. Thus the differences of the energies during the increment are extremely small and not very accurate. Another reason can be attributed that relatively coarse mesh was used except the area of the moving elements.

Contrary to the global energy release rate, the dynamic J integral gives excellent smooth results that agree with the theoretical value even for the extremely high crack-velocity.

6.2 Dynamic interfacial crack propagation

In a homogeneous material under a remote loading, the velocity of dynamic crack propagation

Fig. 11 Variation of the dynamic J integral for a crack propagating with the Rayleigh wave velocity in a homogeneous material

Table 1 Crack-velocity regimes used in the simulations

Regime	Crack velocity C	$C/C_R^{(1)}$	$C/C_s^{(1)}$	$C/C_d^{(1)}$
Subsonic I	864 m/s	0.433	0.4	0.219
Subsonic II	2052 m/s	1.028	0.95	0.520
Transonic	2592 m/s	1.300	1.2	0.660
Supersonic	4320 m/s	2.160	2.0	1.090

theoretically cannot exceed the Rayleigh wave velocity⁽¹⁹⁾, since the energy release rate becomes zero at the Rayleigh wave velocity, as shown in Fig. 11. However, in a nonhomogeneous material, as stated in the introduction of this paper, the velocity of dynamic interfacial crack propagation may exceed the Rayleigh and shear wave velocities of the more compliant material.

Thus dynamic interfacial crack propagation with its velocity C may be classified into the following regimes, as done for moving load problems⁽²⁰⁾:

- (i) Subsonic Interfacial Crack Propagation I: ($C < C_R^{(1)} < C_s^{(1)} < C_d^{(1)}$)
- (ii) Subsonic Interfacial Crack Propagation II: ($C_R^{(1)} < C < C_s^{(1)} < C_d^{(1)}$)
- (iii) Transonic Interfacial Crack Propagation: ($C_R^{(1)} < C_s^{(1)} < C < C_d^{(1)}$)
- (iv) Supersonic Interfacial Crack Propagation: ($C_R^{(1)} < C_s^{(1)} < C_d^{(1)} < C$)

In this study, the mismatch of the bimaterial is assumed to be $E^{(2)}/E^{(1)} = 3.0$ with $E^{(1)} = 29.4$ GPa. For simplicity, no mismatches in Poisson's ratios and the mass densities are assumed as $\nu^{(1)} = \nu^{(2)} = 0.286$ and $\rho^{(1)} = \rho^{(2)} = 2450$ kg/m³. Thus, the elastic wave velocities are $C_R^{(1)} = 1995$ m/s, $C_s^{(1)} = 2160$ m/s, and $C_d^{(1)} = 3945$ m/s for the material 1, and $C_R^{(2)} = 3456$ m/s, $C_s^{(2)} = 3741$ m/s, and $C_d^{(2)} = 6834$ m/s for the material 2.

The numerical simulations were carried out using the moving finite element method explained in section 5. The crack velocities for various regimes used in the simulations are listed in Table 1.

6.2.1 Subsonic interfacial crack propagation I and II First, for the subsonic regimes I and II, the separated dynamic J integrals are evaluated by Eq. (8) with $V_\epsilon^{(m)} = 0$, and plotted against the integral path number in Figs. 12(a) and (b). Note that $J_k^{(0(m))} = J_k^{(m)}$ since $\theta_0 = 0$ in the present problem. Excellent path independence of the tangential components $J_1^{(0(m))}$ ($m=1, 2$) of the separated dynamic J integrals can be seen. The $J_2^{(0(m))}$ integral values also show excellent path independence although they are not given in this paper. These results indicate that the components of the dynamic J integral are also path independent since the sum of the path independent values is also path independent.

The variations of the separated dynamic J integrals during subsonic interfacial crack propagation are shown in Fig. 13. The dynamic J integral values

Fig. 12 Path independence of separated dynamic J integrals in the subsonic regimes I and II

were normalized by the static J integral value at $t = 0$, i.e., J_{1s}^0 . The energy flow rate from the material-1 side ($J_1^{(0(1))}$) is much larger than that from the material-2 side ($J_1^{(0(2))}$). This means that the compliant material supplies more energy for interfacial fracture.

The normal component of the vectorial dynamic J integral, i.e., the J_2^0 integral can be used as a measure for the mixed-mode state of an interfacial crack tip. Since the relation between the J_2^0 integral and the dynamic stress intensity factors has not been established for an interfacial crack, we look at the relation for a homogeneous case (see Eq. (5.b)). The J_2^0 integral becomes zero for the pure modes I and II, and becomes larger for increasing K_{II} value with a fixed K_I value.

As seen in Fig. 13(a), the $J_2^{(0(2))}$ integral is almost constant during the crack propagation of subsonic regime I while the $J_2^{(0(1))}$ integral is nearly zero. This reveals that the material 1 is fracturing under almost pure opening mode while the material 2 is fracturing

Fig. 13 Variations of the separated dynamic J integrals in the subsonic regimes I and II

Fig. 14 Variations of the dynamic J integral in the subsonic regimes I and II

under a mixed-mode state. In the subsonic regime II, it is seen from Fig. 13(b) that the mixed-mode state is still larger in the material 2.

Figure 14 shows the variations of the dynamic J integrals for various subsonic interfacial crack velocities, together with the dynamic energy release rates calculated by the global energy balance (see Eq. (15)). As seen from the figure, the J integral varies smoothly during higher-velocity crack propagation, while the energy release rate more fluctuates when the crack velocity approaches the regime of subsonic

Fig. 15 Strain energy density distribution in the subsonic regime I

interfacial crack propagation II. Moreover, the global energy release rate differs from the dynamic J integral for subsonic interfacial crack propagation II, and have similar features with those of high-velocity crack propagation in the homogeneous material (see Fig. 11). It should be noted that the energy flow rate into the interfacial crack tip becomes smaller for increasing crack velocity but still finite even in the subsonic regime II.

The strain energy density distributions for the subsonic regime I at $a=0.2732\text{ W}$ ($t=3.391\text{ }\mu\text{s}$) and at $a=0.3464\text{ W}$ ($t=6.782\text{ }\mu\text{s}$) are shown in Figs. 15(a) and (b), respectively. The higher concentration of the strain energy density is seen in the material 1. A large discrepancy of the strain energy densities of

Fig. 16 Strain energy density distribution in the subsonic regime II

both materials can be seen along the uncracked interface. When the time elapses, the unloaded area becomes broader due to the crack propagation.

Figures 16(a) and (b) show the strain energy density distributions for the subsonic regime II at $a=0.2732W$ ($t=1.428\mu s$) and at $a=0.3464W$ ($t=2.855\mu s$). The features of the strain energy distributions are similar to those of the subsonic regime I.

6.2.2 Transonic and supersonic interfacial crack propagation In these regimes, since the shear wave continuously emanated from the dynamically propagating crack tip to the material 1 is slower than the crack velocity, the shear wave is trapped and concentrated along a certain line (see Fig. 17). This singular line may be considered as the front of shock

Fig. 17 Angles of shock waves

Fig. 18 Path independence of separated dynamic J integrals in the transonic and supersonic regimes

wave (Mach wave). During a certain time increment dt , the shear wave and dilatational wave emanated at time t travel the distances of $C_s \cdot dt$ and $C_d \cdot dt$, respectively while the crack propagates $C \cdot dt$. Based on the geometrical relations shown in Fig. 17, the angles of the shear and dilatational shock waves can be

Fig. 19 Variations of the separated dynamic J integrals in the transonic and supersonic regimes

evaluated by

$$\Psi_a = \tan^{-1} \{ (C^2/C_a^2 - 1)^{-1/2} \}; \quad \alpha = s, d. \quad (30)$$

These theoretical shock wave angles will be compared with the computational ones.

The separated dynamic J integrals in the regimes of transonic and supersonic interfacial crack propagation are plotted against the integral path number in Figs. 18(a) and (b). In both regimes, all components of the separated dynamic J integrals are excellently path-independent.

Figures 19(a) and (b) show the variations of the separated dynamic J integrals in the transonic regime and in the supersonic regime, respectively. In the transonic regime (see Fig. 19(a)), the energy flow rate from the material-1 side ($J_1^{0(1)}$) reaches a very small negative value after a steady-state condition prevails, while the energy flow rate from the material-2 side ($J_1^{0(2)}$) reaches a very small positive value. Both $J_2^{0(1)}$ and $J_2^{0(2)}$ integrals become very small negative values.

In the supersonic regime, as seen in Fig. 19(b), all components of the separated dynamic J integrals are zero after a steady-state condition prevails.

The variations of the dynamic J integrals in the transonic and supersonic regimes are shown in Fig. 20. The energy flow rate into the propagating crack tip drastically drops immediately after the onset of

Fig. 20 Variations of the dynamic J integral in the transonic and supersonic regimes

Fig. 21 Strain energy density distribution in the transonic regime

Fig. 22 Strain energy density distribution in the supersonic regime

dynamic crack propagation, and then remains zero during transonic and supersonic crack propagation.

The strain energy density distributions for the transonic regime at $a=0.2732W$ ($t=1.130 \mu s$) and $a=0.3464W$ ($t=2.261 \mu s$) are shown in Figs. 21(a) and (b), respectively. In these figures, the theoretically predicted angle of the shear shock wave ($\Psi_s=56.4^\circ$) is indicated. It is seen that the highly dense contours of the strain energy density agree excellently with the theoretical shock wave angle.

The shock waves emanated from the tip of a dynamically propagating interfacial crack have been observed experimentally by Rosakis and coworkers^{(1),(2)}. The present results may be the first computational visualization of distinct shock waves

Fig. 23 Crack opening profiles during interfacial crack propagation

emanated from a transonically propagating interfacial crack tip. The present moving finite element method made it possible to obtain the distinct and accurate shock waves.

Figures 22(a) and (b) show the distributions of the strain energy density for the supersonic regime at $a=0.2732W$ ($t=0.678 \mu s$) and $a=0.3464W$ ($t=1.356 \mu s$), respectively. For this crack velocity, theoretical angles of the shear shock wave and the dilatational shock wave are $\Psi_s=30.0^\circ$ and $\Psi_d=66.6^\circ$, respectively. As seen in the figures, numerical shear shock wave appears along the theoretically predicted direction $\Psi_s=30.0^\circ$. However, the dilatational shock wave is not apparent in the numerical results. This is reasonable due to the fact that the shear waves generally have much energy compared with the dilatational waves.

7. Discussion

The crack opening profiles during interfacial crack propagation are summarized in Figs. 23(a)–(d) for various crack velocity regimes. The

Fig. 24 Comparison of crack opening profiles for various crack-velocity regimes

coordinates of the deformed crack surfaces are normalized by the constrained displacement $\bar{u}_0$ (see Fig. 8). In all cases, the first crack opening profiles are of the initial cracks under the static loading.

In the subsonic regime I, the profiles of the dynamically propagating crack are almost the same with the initial static one. The well-known feature of $\sqrt{r}$ in the crack opening displacement near the crack tip can be seen in Fig. 23(a). In the subsonic regime II, the profiles become sharper than those of the subsonic I, and lose the $\sqrt{r}$ feature in the material-1 side. The crack profiles in the transonic regime become further sharper. In the supersonic regime, the crack rapidly changes its profile from the static one with the $\sqrt{r}$ feature to a linear feature.

In Fig. 24, the crack opening profiles at $a=0.3464$ W for various crack-velocity regimes are compared. In the material 1, the $\sqrt{r}$ feature of the crack opening displacement vanishes in the subsonic regime II. In the material 2, the $\sqrt{r}$ feature can be observed until the transonic regime while it vanishes in the supersonic regime. This can be attributed that the crack velocity of the transonic regime is still in a subsonic state in terms of the elastic wave velocities in the material 2 ($C=0.750C_k^{(2)}=0.693C_s^{(2)}=0.379C_d^{(2)}$), while the supersonic regime becomes a transonic state in terms of the material 2 ($C=1.25C_k^{(2)}=1.155C_s^{(2)}=0.632C_d^{(2)}$).

Similarly with the static interfacial crack problems, the analytical crack-tip solutions^{(3),(4)} for a dynamically propagating interfacial crack, predict the existence of a micro-contact zone near the crack-tip as well as an oscillatory singularity. Thus, the contact of crack surfaces may be classified into the micro-contact due to the oscillatory singularity and the macro-contact under a mechanical compression.

The present numerical results reveal no macro-contact zones for all regimes of interfacial crack propagation, even for the supersonic regime. This can be attributed to the tension-dominated nature of the

present problems. So far in experiments^{(1),(2)} the macro-contact zones have been observed⁽¹⁾ only under the shear-dominated loading.

Now, we consider the theoretical limit of the velocity of interfacial crack propagation, from the energy point of view.

If a crack propagates without a macro-crack contact zone under a tension-dominated loading condition, crack propagation greater than the transonic regime is not possible due to zero energy flow rate as shown in Fig. 20.

So far, several experimental observations of the transonic interfacial crack propagation have been made by Rosakis and coworkers⁽¹⁾, and Singh and Shukla⁽²¹⁾. The maximum interfacial crack velocities that have been observed in these experiments were about $1.4C_s^{(1)}$ for the bimaterial of Polymethyl-methacrylate (PMMA) and steel, and about $1.3C_s^{(1)}$ for the bimaterial of Homalite-100 and aluminum. However, these transonic interfacial crack velocities were only observed under shear-dominated loading conditions. Moreover, in each case a macro-contact zone at the vicinity of the transonically propagating crack was observed.

So far, to the authors' knowledge, no transonic interfacial crack propagation has been observed under remote tension-dominated loading conditions. These experimental evidences support the aforementioned theoretical limit of the velocity of interfacial crack propagation without a macro-contact zone under tension-dominated loading.

Therefore, the discrepancy between the terminal velocities of interfacial crack propagation under shear-dominated and tension-dominated conditions can be attributed to the occurrence of macro-contact zone. Thus, we may arrive a hypothesis that a macro-contact zone provides energy flow to a transonically propagating crack tip. However, further studies are needed to clarify the mechanism of transonic interfacial crack propagation with a macro-contact zone under a shear-dominated condition. The presently developed moving finite element method can be a powerful numerical tool for this type of problem, if appropriate numerical schemes for the contact of crack surfaces are incorporated.

8. Conclusions

In this paper, the concept of separated dynamic J integrals was presented for dynamic interfacial fracture mechanics. The separated dynamic J integrals have the physical meaning of the energy flow rates (or equivalently energy release rates) from the adjacent material sides to the crack tip. In addition to the separated dynamic J integral, various forms of the

separated energy release rates were also presented. Since the dynamic J integral⁽¹⁰⁾ reduces to the static J integral⁽¹¹⁾ under static conditions, the concepts of the separated J integral and the separated energy release rates can be readily developed for static interfacial fracture mechanics as well. The separated dynamic J integral and the separated energy release rate should be very useful to identify the fracture mechanics effects of individual material in a nonhomogeneous material system.

Furthermore, in this paper, the moving isoparametric element method for dynamic interfacial fracture analysis was developed. This made it possible to accurately analyze supersonic as well as transonic interfacial crack propagation. Using the moving finite element method, numerical simulations were carried out for subsonic, transonic and supersonic interfacial crack propagation in the bimaterial plates. So far, to the authors' knowledge, these were the first simulations of transonic and supersonic crack propagation with the moving finite element procedure.

The numerical results successfully revealed the shock waves (Mach waves) emanated from transonically and supersonically propagating crack tips. These are also the first computational visualization of the distinct and accurate shock waves that agree with the theoretically predicted characteristics.

The moving finite element method demonstrated also accurate calculation of crack opening profiles in all crack-velocity regime. No macro-contact zones behind the propagating crack tips were found even in the supersonic regime under the tension-dominated loading.

The dynamic J integral and the separated dynamic J integrals were accurately evaluated by the moving finite element method. These integrals showed excellent path independence for all crack-velocity regimes even for supersonic crack propagation. It was found in the subsonic I and II regimes that the energy flow rate to the propagating interfacial crack tip from the compliant material is much larger than that from the stiff material.

From the energy flow rate to the propagating interfacial crack tip, the theoretical limit of interfacial crack velocity without a macro-contact zone under the tension-dominated loading was found to be the shear wave velocity of the compliant material. The transonic interfacial crack propagation without a macro-contact zone was found to be impossible unless input energy is directly supplied to the propagating crack tip.

Acknowledgments

This study was supported by the Grant-in-Aid for

Scientific Research given to "Basic Studies on Dynamic Interfacial Fracture Mechanics" (No. 08455063) from the Ministry of Education, Science and Culture in Japan.

References

- (1) Lambros, J. and Rosakis, A.J., Shear Dominated Transonic Interfacial Crack Growth in a Bimaterial-I. Experimental Observation, *J. Mech. Phys. Solids*, Vol. 43, No. 2 (1995), p. 169-188.
- (2) Rosakis, A.J., Samudrala, O. and Singh, R., Dynamic Shear-Dominated, Supersonic Crack Growth in Bimaterial and Layered Systems and Relationship to Earthquake Rupture, *Proc. Int. Conf. Adv. Tech. in Exp. Mech.*, (1997), p. 11-18.
- (3) Yang, W., Suo, Z. and Shih, C.F., Mechanics of Dynamic Debonding, *Proc. R. Soc. Lon. Vol. A433* (1991) p. 679-697.
- (4) Liu, C., Huang, Y. and Rosakis, A.J., Shear Dominated Transonic Interfacial Crack Growth in a Bimaterial-II. Asymptotic Fields and Favorable Velocity Regimes, *Journal of the Mechanics and Physics of Solids*, Vol. 43, No. 2 (1995) p. 189-206.
- (5) Nakamura, T., Kushner, A. and Lo, C.Y., Interlaminar Dynamic Crack Propagation, *International Journal of Solids and Structures*, Vol. 32, No. 17/18 (1995), p. 2657-2675.
- (6) Xu, X.P. and Needleman, A., Numerical Simulations of Dynamic Crack Growth along an Interface, *International Journal of Fracture*, Vol. 74 (1996), p. 289-324.
- (7) Kanninen, M.F., *A Critical Appraisal of Solution Techniques in Dynamic Fracture Mechanics*, *Numerical Methods in Fracture Mechanics* (D.R.J. Owen and A.R. Luxmoore, eds.), (1978), p. 612-634, Pineridge Press.
- (8) Nishioka, T., Computational Dynamic Fracture Mechanics, *International Journal of Fracture*, Vol. 86, No. 1/2 (1997), p. 127-159.
- (9) Nishioka, T., The State of the Art in Computational Dynamic Fracture Mechanics, *JSME International Journal, Series A*, Vol. 37, No. 4 (1994), p. 313-333.
- (10) Nishioka, T. and Atluri, S.N., Path-Independent Integrals, Energy Release Rates, and General Solutions of Near-Tip Fields in Mixed-Mode Dynamic Fracture Mechanics, *Eng. Fract. Mech.*, Vol. 18 (1983), p. 1-22.
- (11) Rice, J.R., A Path Independent Integral and Approximate Analysis of Strain Concentration by Notches and Cracks, *J. Appl. Mech.*, Vol. 35 (1968), p. 379-386.
- (12) Irwin, G.R., Analysis of Stresses and Strains Near the End of a Crack in an Elastic Solid, *Journal of Applied Mechanics*, Vol. 24 (1957), p. 361-364.
- (13) Nishioka, T. and Atluri, S.N., Computational Methods in Dynamic Fracture, *Computational Methods in the Mechanics of Fracture*, (S.N.

- Atluri, Ed.), (1986), Chapter 10, p. 336-383, Elsevier Science Publishers.
- (14) Nishioka, T. and Atluri, S.N., A Path-Independent Integral and Moving Isoparametric Elements for Dynamic Crack Propagation, *AIAA Journal*, Vol. 22, No. 3(1984), p. 409-414.
 - (15) Nishioka, T. and Takemoto, Y., Moving Finite Element Method Aided by Computerized Symbolic Manipulation and Its Application to Dynamic Fracture Simulation, *JSME International Journal*, Series I, Vol. 32, No. 3(1989), p. 403-410.
 - (16) Nishioka, T., Recent Developments in Computational Dynamic Fracture Mechanics, *Dynamic Fracture Mechanics*, (M.H. Aliabadi, Ed.), (1995), Chapter 1, p. 1-60. Computational Mechanics Publications.
 - (17) Nishioka, T. and Atluri, S.N., Numerical Modeling of Dynamic Crack Propagation in Finite Bodies, By Moving Singular Elements-Part II. Results, *Journal of Applied Mechanics*, Vol. 47, No. 3(1980), p. 577-582.
 - (18) Nilsson, F., Dynamic Stress-Intensity Factors for Finite Strip Problems, *International Journal of Fracture*, Vol. 8, No. 4(1972)p. 403-411.
 - (19) Freund, L.B., *Dynamic Fracture Mechanics*, (1990), Cambridge University Press.
 - (20) Fry'ba, L. *Vibration of Solids and Structures under Moving Load*, (1972), Noordhoff.
 - (21) Singh, R.P. and Shukla, A., Subsonic and Inter-sonic Crack Growth along a Bimaterial Interface, *Journal of Applied Mechanics*, Vol. 63 (1996), p. 919-924.
-