Polynomial Evaluation of Stress Intensity Factors of CNS Specimen with an Interface Crack Subject to Mixed-Mode Loading*

Kenji MACHIDA**

Three-dimensional finite element analysis has been carried out on the compact normal and shear (CNS) specimens under mixed-mode loading. The complex stress-intensity factor K associated with an elastic interface crack is discussed by the virtual crack extension method. The effect of Young's modulus and Poisson's ratio on stress intensity factors is discussed under various kinds of mixed-mode loading. The distribution of stress intensity factors along the crack front is investigated. A polynomial fitting is proposed to evaluate the stress-intensity factors at the midsection of CNS specimen with an interface crack subject to mixed-mode loading. It is possible to evaluate the stress-intensity factors of CNS specimen with high accuracy by the present polynomial evaluation.

Key Words: Finite Element Method, Stress Intensity Factor, Fracture Mechanics, Interface Crack, Mixed Mode, Virtual Crack, Extension Method, Polynomial Fitting

1. Introduction

The characteristic oscillating stress singularity field near the interface-crack of dissimilar material was pointed out by Williams⁽¹⁾, and solutions to specific problems were proposed by England⁽²⁾, Erdogan^{(3),(4)}, and Rice and Sih⁽⁵⁾. Recently, the strength evaluation of electronic packages, bonded dissimilar materials, adhesive joints and composites have become very important. Many useful investigations have been carried out from the numerical and theoretical viewpoint⁽⁶⁾⁻⁽³¹⁾. The stress singularity field at the interface-crack tip is in the mixed-mode state even under the mode I loading. The compact normal and shear (CNS) specimen proposed by Richard and Benitz⁽³²⁾ enables us to carry out the experiments under the various kinds of mixed-mode loading.

The evaluation of stress-intensity factors of interface crack has been carried out by the conservation M-integral⁽⁷⁾, new method⁽¹⁵⁾ combined the idea of M-

integral with the virtual crack extension method (VCEM)⁽³³⁾, modified crack-closure integral^{(8),(34),(35)}, and stress extrapolation⁽¹⁶⁾. The high accuracy of virtual extension method has been reported in the previous papers^{(30),(31)}.

In this study, the three-dimensional finite element analysis was carried out by using the compact normal and shear (CNS) specimen with various kinds of Young's modulus, Poisson's ratio and load application angle. The stress-intensity factors at the central part of specimen are nearly uniform. A simple evaluation method for stress-intensity factors is needed in the fracture toughness test. Therefore, the polynomial evaluation of stress-intensity factors was carried out on CNS specimen with various kinds of Young's modulus ratio, Poisson's ratio and load application angles. It is predicted that the stress-intensity factors would vary with a decrease in thickness of specimen. Then, the thickness effect on stress-intensity factors was discussed. The stress-intensity factors of CNS specimen can be simply evaluated by the present polynomial approximation.

2. Stress-Intensity Factors of Interface Crack

2.1 Complex stress-intensity factor

Figure 1 shows the region surrounding a traction-free interface crack tip. Erdogan⁽³⁾ has introduced the

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** Faculty of Science and Technology, Science University of Tokyo, 2641 Yamazaki, Noda-shi, Chiba 278-8510, Japan. E-mail: mac @ kikujiro.me.noda.sut.ac.jp

Fig. 1 Region surrounding a traction-free interface crack

complex stress-intensity factor, which uniquely characterizes the singular field as:

$$(\sigma_{yy} + i\sigma_{xy})_{\theta=0} = K(r/l_k)^{i\epsilon}/\sqrt{2\pi r} \quad (1)$$

$$\epsilon = (1/2\pi) \ln[(\chi_1/\mu_1 + 1/\mu_2)/(\chi_2/\mu_2 + 1/\mu_1)] \quad (2)$$

$$\chi_j = \begin{cases} 3-4\nu_j & (\text{plane strain}) \\ (3-\nu_j)/(1+\nu_j) & (\text{plane stress}) \end{cases} \quad (j=1, 2) \quad (3)$$

where $(\sigma_{yy} + i\sigma_{xy})$ is the complex stress, K is a complex stress-intensity factor, l_k is the typical length like a crack length used to set the dimension of stress-intensity factor of interface crack in correspondence to that of homogeneous material, μ_1 and μ_2 are shear moduli of materials 1 and 2, r is the distance from the crack tip, ϵ is a bi-material constant, and ν_j is the Poisson's ratio.

2.2 Virtual crack extension method

The strain energy release rate in the virtual crack extension method proposed by Parks⁽³³⁾ is expressed as follows:

$$G = -\left(\frac{\partial U}{\partial A}\right) = -\frac{1}{2} \sum_{i=1}^{N_F} \mathbf{u}_i^T \frac{\Delta k_i}{\Delta A} \mathbf{u}_i \quad (4)$$

where U is the potential energy, ΔA is the increment of crack area, Δk_i is the variation in the stiffness matrix of elements $(1 - N_F)$ within an interior contour which surrounds the crack tip with ΔA and $\mathbf{u}_i$ is the vector of nodal point displacement.

According to Yau-Wang-Matos⁽⁷⁾⁽¹⁵⁾, the stress-intensity factor solution for the mixed-mode fracture problem can be obtained as follows. Consider two independent equilibrium states with field variables denoted by superscripts 1 and 2 within an interior contour $\mathcal{Q}$, which surrounds the crack tip. The stress-intensity factors of equilibrium state 2 are known. Denoting the superposed equilibrium state by a superscript 1+2, the principle of superposition for the mixed-mode stress-intensity factors holds as

$$\begin{aligned} K_I^{(1+2)} &= K_I^{(1)} + K_I^{(2)} \\ K_{II}^{(1+2)} &= K_{II}^{(1)} + K_{II}^{(2)} \end{aligned} \quad (5)$$

The relation between G and K_I , K_{II} is expressed as follows:

$$G = \beta(K_I^2 + K_{II}^2) \quad (6)$$

$$\beta = (c_1 + c_2)/16 \cosh^2(\pi\epsilon) \quad (7)$$

$$c_1 = (\chi_1 + 1)/\mu_1, \quad c_2 = (\chi_2 + 1)/\mu_2$$

The following equation is derived from Eqs. (5), (6)

Fig. 2 Configuration of compact normal and shear specimen

and (7).

$$2\beta[K_I^{(1)}K_I^{(2)} + K_{II}^{(1)}K_{II}^{(2)}] = G^{(1+2)} - G^{(1)} - G^{(2)} \quad (8)$$

In Eq. (8), $K_I^{(2)}$, $K_{II}^{(2)}$ and $G^{(2)}$ are known. $G^{(1)}$ and $G^{(1+2)}$ are derived from substituting $u^{(1)}$ and $u^{(1+2)}$ into Eq. (4) as follows:

$$G^{(j)} = -\frac{1}{2} \sum_{i=1}^{N_F} (\mathbf{u}_i^{(j)})^T \frac{\Delta k_i}{\Delta A} \mathbf{u}_i^{(j)} \quad (j=1, 1+2) \quad (9)$$

The stress-intensity factor solution for the mixed-mode fracture problem was evaluated by the superposition, using $u_i^{(2)}$ and $u_i^{(1+2)}$ which were obtained from the asymptotic solution for two-dimensional bi-material body derived by Sun and Jih⁽⁸⁾, and $u_i^{(1)}$ assumed to be zero.

First, K_I and K_{II} values are evaluated by setting $l_k = 2a$ where a is a crack length. The K_I and K_{II} values were transformed into the final stress-intensity factors (K_I' and K_{II}') at $l' = 12 \mu\text{m}$ considering the compatibility with the stress-intensity factors obtained by the modified crack-closure integral^{(30),(31)}. The transformation of stress-intensity factors is simply conducted by the following equation.

$$\begin{aligned} \begin{Bmatrix} K_I' \\ K_{II}' \end{Bmatrix} &= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix} \\ \theta &= \epsilon \ln\left(\frac{l'}{l_k}\right) \end{aligned} \quad (10)$$

3. Model for Analysis

Figure 2 shows the configuration of compact normal and shear specimen. The materials 1 and 2 denote the upper and lower parts of specimen, respectively. The width, height and thickness are 60, 100 and 18 mm, respectively and a crack length is 30 mm. Figure 3 shows the loading device for mixed-mode

Fig. 3 Loading device for mixed-mode loading

Fig. 4 2-D mesh pattern and loading condition

loading proposed by Richard and Benitz⁽³²⁾. This device enables us to carry out the experiments under seven kinds of mixed-mode loading. Figure 4 shows the two-dimensional mesh pattern and loading condition of the compact normal and shear specimen. Here, l_1 and l_2 are 36 and 72 mm, respectively. The three-dimensional mesh pattern was generated by developing the two-dimensional mesh pattern to the thickness direction. The three-dimensional finite element analysis was carried out on a half portion of the specimen due to the symmetry with respect to the midsection of the specimen. The ratio of layer division of specimen is 1:2:3 from the free surface. The numbers of elements and joints are 1440 and 7801, respectively. The material 2 is an epoxy resin whose Young's modulus is 2.94 GPa and constant. The Young's

modulus of material 1 is changed as $E_1/E_2=1.0, 2.0, 3.0, 6.7, 23.3, 40.0, 70.0$ and 100.0. Moreover, the effect of absolute value of Young's modulus and Poisson's ratio were investigated. To investigate the thickness effect, two models of 3 and 5 mm thickness were analyzed. The same loading condition as Kishimoto et al.⁽³⁶⁾ was employed for the mixed-mode loading. The forces and moments meet the following equilibrium equations.

$$\begin{aligned} P \sin \alpha &= P_2 \\ P \cos \alpha &= P_1 + P_2 \\ P_1 l_1 + P_2 l_2 &= P_3 l_3 \end{aligned} \quad (11)$$

P_1 , P_2 and P_3 were applied to the centers of each bolt hole in order to make the uniform loading. Moreover, two loading points on the left and low side were constrained in x_1 and x_2 directions. P is the load applied to the bolt hole on the periphery of loading device shown in Fig.3 and α is a load application angle.

4. Discussion and Results

4.1 Effect of Young's modulus, Poisson's ratio and load application angle

To simplify the comparison between the stress-intensity factors, the normalized stress-intensity factors (F_I , F_{II}) were evaluated by the following equation from the stress-intensity factors (K_I , K_{II}):

$$F_M = \frac{K_M}{\frac{P}{WB} \sqrt{\pi a}} \quad (M=I, II) \quad (12)$$

where W is the width of specimen and B the thickness.

Figure 5 shows the distribution of stress-intensity factors along the crack front in the case of $E_1/E_2=2.0$ and $\alpha=45$ degrees. Here Z is the distance from the midsection of the specimen. The F_I value is nearly uniform from the midsection of the specimen to $2Z/B \approx 0.4$ and decreases gradually toward the free surface. The effect of Poisson's ratio is greater at the central part of specimen than near the free surface. The Poisson's ratio largely influences the F_I value at the central part of specimen. The F_I value increases with an increase in ν_2 , while ν_1 does not influence it. The F_{II} value is nearly uniform from the midsection of the specimen to $2Z/B \approx 0.6$ and increases gradually toward the free surface. The F_{II} value increases with an increase in ν_1 , while it decreases with an increase in ν_2 . Figure 6 shows the distribution of stress-intensity factors along the crack front in the case of $E_1/E_2=70$ and $\alpha=45$ degrees. The variation of F_I and F_{II} values is very similar to that of $E_1/E_2=2.0$. The stress-intensity factors are hardly influenced by ν_1 . The F_I value increases and F_{II} value decreases with an increase in ν_2 .

Fig. 5 Distribution of stress intensity factors along the crack front ($E_1/E_2=2.0$ and $\alpha=45$ degrees)

Fig. 6 Distribution of stress intensity factors along the crack front ($E_1/E_2=70$ and $\alpha=45$ degrees)

(a) F_I value

(b) F_{II} value

Fig. 7 Variation of stress intensity factors with a load application angle α ($\nu_1=0.3$ and $\nu_2=0.38$)

Figure 7 shows the variation of stress-intensity factors with a load application angle α in the case of $\nu_1=0.3$ and $\nu_2=0.38$. The F_I value is maximum at $\alpha=0$ degree and decreases with an increase in α , and becomes nearly zero at $\alpha=90$ degrees. As the ratio of Young's modulus increases, the F_I value decreases. The F_{II} value is minimum at $\alpha=0$ degree and increases with an increase in α . As the ratio of Young's modulus increases, the F_{II} value increases and the peak moves to the left. The F_I and F_{II} values can be approximated by the 3rd polynomial of α (radian).

Figure 8 shows the variation of stress-intensity factors with the ratio of Young's modulus in the case

Fig. 8 Variation of stress intensity factors with the ratio of Young's modulus ($\nu_1=0.3$ and $\nu_2=0.38$)

of $\nu_1=0.3$ and $\nu_2=0.38$. In a small E_1/E_2 region, the F_I value decreases rapidly, while the F_{II} value increases. As the E_1/E_2 value exceeds 40, both values become nearly constant. The F_I and F_{II} values can be approximated by the polynomial of $\sqrt{E_2/E_1}$.

Figure 9 shows the variation of stress-intensity factors with Young's modulus in the case of $E_1/E_2=6.7$ and $\alpha=45$ degrees. When the ratio of Young's modulus is constant, both F_I and F_{II} values are nearly uniform independent of the absolute value of Young's modulus. The stress-intensity factors of interface crack do not depend on the absolute value of Young's

Fig. 9 Variation of stress intensity factors with Young's modulus ($E_1/E_2=6.7$ and $\alpha=45$ degrees)

modulus, but depend on the ratio of Young's modulus.

Figure 10 shows the variation of stress-intensity factors with Poisson's ratio, ν_1 in the case of $\alpha=45$ degrees and $\nu_2=0.38$. As the ν_1 value increases, the F_I value decreases, while the F_{II} value increases. Both values vary linearly with ν_1 and considerably with a decrease in the ratio of Young's modulus, and are nearly uniform at $E_1/E_2=100$. Even changing ν_2 with constant ν_1 , the stress-intensity factors vary linearly. Accordingly, it is considered that the F_I and F_{II} values include $(a_1\nu_1+b_1)(a_2\nu_2+b_2)$ as a factor. Here, a_i and b_i are constants.

4.2 Polynomial approximation

Considering the above-mentioned dependency of Young's modulus, Poisson's ratio and load application angle, the polynomial approximation was carried out as follows:

$$F = A_3\alpha^3 + A_2\alpha^2 + A_1\alpha + A_0 \quad (13)$$

$$\left. \begin{aligned} A_3 &= B_{33}x^3 + B_{32}x^2 + B_{31}x + B_{30} \\ A_2 &= B_{23}x^3 + B_{22}x^2 + B_{21}x + B_{20} \\ A_1 &= B_{13}x^3 + B_{12}x^2 + B_{11}x + B_{10} \\ A_0 &= B_{03}x^3 + B_{02}x^2 + B_{01}x + B_{00} \end{aligned} \right\} \quad (14)$$

$$B_{ij} = C_{3ij}\nu_1\nu_2 + C_{2ij}\nu_1 + C_{1ij}\nu_2 + C_{0ij} \quad (15)$$

where $x = \sqrt{E_2/E_1}$ and the unit of α is radian.

Tables 1 and 2 show the coefficients of polynomials for the evaluation of F_I and F_{II} values. The stress-intensity factors of CNS specimen can be evaluated by Eqs.(13), (14) and (15) using these coefficients. Tables 3 and 4 show several examples of stress-intensity factors at the midsection of specimen obtained by the virtual crack extension method and present polynomial approximation. A small difference between the stress-intensity factors obtained by both methods can be seen at $E_1/E_2=1.0$ and $\alpha=60-90^\circ$. However, both stress-intensity factors are in good agreement in other place. Thus, the

(a) F_I value (b) F_{II} valueFig. 10 Variation of stress intensity factors with Poisson's ratio, ν_I ($\nu_2=0.38$ and $\alpha=45$ degrees)

stress-intensity factors of CNS specimen under the mixed-mode loading can be evaluated with good accuracy by the present polynomial approximation.

4.3 Thickness effect

To investigate the thickness effect, two models of 3 and 5 mm thickness were analyzed as shown in Table 5. The values of upper, middle and lower part of each column denote the results of present polynomial approximation, 5 and 3 mm thick specimens obtained by the virtual crack extension method, respectively. As the thickness decreases, the F_I value decreases slightly, while the F_{II} value increases. The

Table 1 Coefficients of polynomials for evaluation of F_I value

	C_0	C_1	C_2	C_3
B_{00}	1.5129	2.9970	2.3351×10^{-1}	-6.4143×10^{-1}
B_{01}	-2.1392×10^{-1}	2.1190×10^{-1}	-1.7946×10^{-1}	8.2353×10^{-1}
B_{02}	9.2994	-16.259	-8.5646	19.650
B_{03}	-12.684	20.125	15.108	-29.865
B_{04}	4.9856	-7.5145	-6.7803	12.754
B_{10}	-1.0276×10^{-1}	2.5551×10^{-1}	-1.5486×10^{-1}	4.4882×10^{-1}
B_{11}	4.2221×10^{-2}	1.8290×10^{-3}	-7.0949×10^{-2}	-5.7336×10^{-2}
B_{12}	2.4866×10^{-1}	-2.8386×10^{-1}	-1.7045×10^{-2}	-6.3360×10^{-1}
B_{13}	-1.7367×10^{-1}	2.0399×10^{-1}	7.4085×10^{-2}	2.6044×10^{-1}
B_{20}	-1.0236×10^{-2}	-2.1428×10^{-2}	1.4773×10^{-4}	-3.8438×10^{-3}
B_{21}	-3.6387×10^{-3}	-9.8012×10^{-3}	3.7027×10^{-3}	2.1996×10^{-2}
B_{22}	-3.2713×10^{-2}	9.4532×10^{-2}	2.0679×10^{-2}	-1.1680×10^{-1}
B_{23}	2.5790×10^{-2}	-6.2004×10^{-2}	-2.2130×10^{-2}	8.1145×10^{-2}
B_{30}	3.6442×10^{-4}	9.6850×10^{-5}	2.2218×10^{-4}	-6.1008×10^{-4}
B_{31}	1.5511×10^{-4}	-2.8010×10^{-4}	-7.7832×10^{-5}	5.0834×10^{-4}
B_{32}	-9.7801×10^{-5}	-1.1195×10^{-4}	7.5094×10^{-5}	4.3886×10^{-4}
B_{33}	0.0	0.0	0.0	0.0

Table 2 Coefficients of polynomials for evaluation of F_{II} value

	C_0	C_1	C_2	C_3
B_{00}	2.3772	-3.5006	2.3010	-6.1260
B_{01}	3.4891×10^{-1}	-6.0912×10^{-2}	-3.4443×10^{-1}	-3.7090×10^{-1}
B_{02}	-7.3558	1.5442	7.4847	7.7522
B_{03}	6.2257	-2.2693	-10.816	3.4311
B_{04}	-1.6623	1.0785	4.4884	-4.7297
B_{10}	1.8332×10^{-1}	8.4268×10^{-2}	5.3498×10^{-2}	-1.5652×10^{-1}
B_{11}	2.6347×10^{-2}	-8.6986×10^{-2}	-7.0181×10^{-2}	2.2270×10^{-1}
B_{12}	-5.2390×10^{-2}	-1.3562×10^{-1}	-7.4552×10^{-4}	2.4414×10^{-1}
B_{13}	1.5698×10^{-2}	1.2086×10^{-1}	3.3277×10^{-2}	-2.4187×10^{-1}
B_{20}	-1.8710×10^{-2}	2.4666×10^{-2}	-1.6404×10^{-2}	4.5079×10^{-2}
B_{21}	6.8861×10^{-4}	4.5728×10^{-4}	-6.2385×10^{-3}	6.5455×10^{-3}
B_{22}	3.8538×10^{-2}	-5.2435×10^{-3}	-1.3428×10^{-2}	-8.7625×10^{-2}
B_{23}	-2.1259×10^{-2}	3.2180×10^{-3}	1.3360×10^{-2}	3.6407×10^{-2}
B_{30}	1.1904×10^{-4}	-6.2567×10^{-4}	2.8631×10^{-4}	-7.9025×10^{-4}
B_{31}	3.7306×10^{-6}	-8.8540×10^{-5}	1.0075×10^{-4}	5.6821×10^{-5}
B_{32}	-8.5377×10^{-4}	7.1789×10^{-4}	5.5305×10^{-4}	4.4672×10^{-4}
B_{33}	4.8897×10^{-4}	-4.6789×10^{-4}	-4.7733×10^{-4}	1.6876×10^{-4}

maximum difference of F_I values between 18 and 3 mm thick specimens is about 0.07 and not large. The maximum difference of F_{II} values between 18 and 3 mm thick specimens is about 0.25 and about 3 times as large as that of F_I value. The experiment revealed that the elliptical law was applicable to the fracture of interface crack^{(18),(37)-(39)}. Usually, the interface is strong to the shear and weak to the delamination. Therefore, the K_{IIC} value becomes 2 to 3 times as large as the K_{IC} value. Considering the large deviation in the experiment and above reason, the present polynomials are applicable to the evaluation of stress-intensity factors of specimen up to 3 mm thick. Further investigation is necessary to evaluate the stress-intensity factors of small and thin specimens.

5. Conclusion

The three-dimensional finite element analysis

Table 3 F_I values obtained by virtual crack extension method and present approximation
(Upper part: VCEM Lower part: Approximation)

(a) $\nu_1=0.3, \nu_2=0.34$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	2.975	2.948	2.893	2.762	2.617	2.585	2.564	2.556	2.549
		2.973	2.945	2.891	2.759	2.615	2.583	2.562	2.553	2.546
	15	2.912	2.862	2.795	2.644	2.484	2.448	2.426	2.416	2.409
		2.919	2.868	2.801	2.649	2.489	2.453	2.430	2.421	2.413
	30	2.651	2.581	2.506	2.346	2.181	2.145	2.122	2.112	2.105
		2.655	2.583	2.507	2.347	2.182	2.146	2.123	2.113	2.105
	45	2.208	2.124	2.046	1.888	1.729	1.695	1.673	1.664	1.657
		2.216	2.125	2.047	1.888	1.729	1.695	1.673	1.664	1.657
	60	1.616	1.523	1.448	1.302	1.160	1.130	1.111	1.103	1.097
		1.636	1.529	1.455	1.308	1.163	1.133	1.114	1.107	1.101
	75	0.914	0.818	0.750	0.627	0.512	0.488	0.473	0.467	0.462
		0.949	0.831	0.765	0.640	0.519	0.495	0.480	0.475	0.471
	90	0.149	0.058	0.002	-0.091	-0.171	-0.187	-0.197	-0.201	-0.205
		0.190	0.065	0.014	-0.080	-0.170	-0.186	-0.195	-0.198	-0.199

(b) $\nu_1=0.3, \nu_2=0.38$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	3.024	3.001	2.955	2.843	2.719	2.691	2.673	2.665	2.659
		3.022	2.999	2.954	2.841	2.717	2.689	2.671	2.663	2.657
	15	2.962	2.926	2.873	2.749	2.615	2.586	2.566	2.558	2.552
		2.968	2.932	2.878	2.754	2.620	2.590	2.571	2.563	2.556
	30	2.698	2.652	2.595	2.468	2.334	2.304	2.285	2.277	2.271
		2.701	2.652	2.594	2.467	2.333	2.303	2.284	2.276	2.270
	45	2.251	2.197	2.140	2.019	1.893	1.866	1.848	1.841	1.835
		2.256	2.195	2.138	2.015	1.889	1.862	1.844	1.837	1.832
	60	1.650	1.593	1.539	1.432	1.324	1.301	1.286	1.280	1.275
		1.668	1.595	1.543	1.433	1.322	1.299	1.285	1.279	1.275
	75	0.938	0.880	0.834	0.747	0.664	0.647	0.636	0.631	0.627
		0.972	0.889	0.845	0.756	0.667	0.649	0.638	0.635	0.633
	90	0.161	0.107	0.072	0.012	-0.041	-0.051	-0.058	-0.061	-0.063
		0.202	0.111	0.080	0.018	-0.044	-0.056	-0.061	-0.063	-0.063

(c) $\nu_1=0.34, \nu_2=0.38$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	2.990	2.986	2.947	2.842	2.719	2.691	2.673	2.665	2.659
		2.988	2.984	2.945	2.839	2.717	2.689	2.671	2.664	2.658
	15	2.944	2.922	2.873	2.752	2.617	2.586	2.567	2.559	2.552
		2.951	2.928	2.878	2.756	2.621	2.591	2.571	2.562	2.556
	30	2.697	2.659	2.604	2.475	2.336	2.306	2.286	2.278	2.271
		2.701	2.660	2.603	2.473	2.335	2.304	2.284	2.276	2.269
	45	2.267	2.215	2.157	2.030	1.897	1.868	1.849	1.842	1.836
		2.272	2.213	2.154	2.025	1.892	1.863	1.844	1.837	1.831
	60	1.682	1.621	1.563	1.446	1.328	1.303	1.287	1.280	1.275
		1.699	1.624	1.566	1.446	1.326	1.300	1.285	1.279	1.275
	75	0.983	0.915	0.863	0.764	0.669	0.649	0.637	0.632	0.629
		1.015	0.926	0.874	0.771	0.670	0.650	0.639	0.635	0.632
	90	0.217	0.148	0.104	0.030	-0.034	-0.048	-0.057	-0.060	-0.063
		0.255	0.154	0.112	0.033	-0.042	-0.055	-0.061	-0.062	-0.063

(d) $\nu_1=0.38, \nu_2=0.38$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	3.058	3.017	2.964	2.846	2.719	2.691	2.673	2.665	2.659
		3.056	3.015	2.962	2.843	2.717	2.689	2.671	2.663	2.657
	15	2.979	2.930	2.872	2.747	2.614	2.585	2.566	2.558	2.552
		2.986	2.936	2.878	2.751	2.619	2.590	2.571	2.563	2.556
	30	2.697	2.643	2.585	2.460	2.331	2.303	2.285	2.277	2.271
		2.702	2.644	2.586	2.460	2.330	2.302	2.284	2.276	2.271
	45	2.231	2.177	2.122	2.007	1.891	1.864	1.848	1.841	1.836
		2.240	2.176	2.121	2.005	1.886	1.860	1.844	1.838	1.833
	60	1.614	1.562	1.514	1.416	1.319	1.298	1.285	1.280	1.275
		1.637	1.567	1.520	1.420	1.319	1.298	1.284	1.279	1.276
	75	0.887	0.841	0.803	0.730	0.659	0.644	0.635	0.631	0.628
		0.928	0.852	0.817	0.741	0.663	0.647	0.638	0.635	0.633
	90	0.100	0.063	0.038	-0.007	-0.046	-0.054	-0.059	-0.061	-0.062
		0.149	0.069	0.048	0.003	-0.047	-0.057	-0.061	-0.063	-0.063

Table 4 F_{II} values obtained by virtual crack extension method and present approximation
(Upper part: VCEM Lower part: Approximation)

(a) $\nu_1=0.3, \nu_2=0.34$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	-0.221	0.292	0.547	0.905	1.160	1.206	1.234	1.246	1.254
		-0.227	0.284	0.540	0.895	1.150	1.197	1.225	1.236	1.244
	15	0.200	0.717	0.974	1.336	1.595	1.642	1.670	1.682	1.691
		0.198	0.713	0.972	1.332	1.592	1.639	1.668	1.679	1.687
	30	0.609	1.093	1.334	1.676	1.921	1.966	1.993	2.004	2.012
		0.604	1.086	1.329	1.668	1.913	1.958	1.985	1.996	2.004
	45	0.976	1.394	1.604	1.902	2.117	2.156	2.179	2.189	2.196
		0.970	1.386	1.597	1.891	2.106	2.146	2.169	2.178	2.185
	60	1.276	1.600	1.764	1.998	2.168	2.199	2.217	2.225	2.231
		1.273	1.596	1.760	1.992	2.162	2.194	2.212	2.220	2.225
	75	1.490	1.698	1.805	1.958	2.071	2.092	2.105	2.110	2.114
		1.490	1.699	1.806	1.959	2.074	2.095	2.107	2.112	2.115
	90	1.602	1.680	1.722	1.785	1.834	1.843	1.848	1.851	1.852
		1.600	1.679	1.719	1.781	1.831	1.840	1.846	1.847	1.848

(b) $\nu_1=0.3, \nu_2=0.38$

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	-0.236	0.207	0.426	0.733	0.952	0.991	1.015	1.024	1.032
		-0.232	0.209	0.429	0.733	0.951	0.991	1.015	1.024	1.031
	15	0.190	0.638	0.861	1.174	1.398	1.438	1.462	1.472	1.480
		0.197	0.644	0.868	1.179	1.403	1.403	1.469	1.478	1.485
	30	0.604	1.026	1.237	1.535	1.749	1.787	1.811	1.820	1.827
		0.608	1.028	1.241	1.536	1.749	1.749	1.812	1.821	1.828
	45	0.976	1.344	1.529	1.791	1.980	2.015	2.035	2.044	2.050
		0.978	1.344	1.529	1.789	1.977	1.977	2.033	2.041	2.047
	60	1.283	1.571	1.717	1.926	2.077	2.105	2.122	2.128	2.134
		1.285	1.572	1.719	1.925	2.077	2.077	2.122	2.128	2.133
	75	1.501	1.691	1.788	1.929	2.033	2.052	2.063	2.068	2.072
		1.505	1.695	1.793	1.933	2.038	2.038	2.069	2.073	2.076
	90	1.618	1.695	1.737	1.801	1.849	1.859	1.864	1.867	1.869
		1.617	1.695	1.735	1.798	1.848	1.848	1.863	1.865	1.866

(c) $\nu_1=0.34, \nu_2=0.38$

		E ₁ /E ₂								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	-0.355	0.123	0.362	0.698	0.941	0.984	1.011	1.022	1.030
		-0.356	0.122	0.363	0.698	0.942	0.986	1.013	1.023	1.031
	15	0.072	0.555	0.797	1.140	1.387	1.432	1.459	1.470	1.478
		0.074	0.558	0.803	1.145	1.394	1.440	1.467	1.478	1.486
	30	0.495	0.949	1.178	1.503	1.739	1.781	1.808	1.818	1.826
		0.495	0.950	1.181	1.505	1.741	1.784	1.810	1.821	1.828
	45	0.884	1.279	1.479	1.765	1.972	2.010	2.033	2.042	2.050
		0.882	1.277	1.479	1.762	1.971	2.009	2.032	2.041	2.048
	60	1.213	1.522	1.679	1.906	2.071	2.102	2.120	2.128	2.134
		1.213	1.522	1.680	1.906	2.073	2.103	2.122	2.129	2.134
	75	1.459	1.661	1.765	1.917	2.029	2.050	2.063	2.068	2.072
		1.461	1.665	1.769	1.921	2.036	2.057	2.069	2.074	2.078
90	1.606	1.687	1.731	1.797	1.849	1.859	1.865	1.867	1.869	
	1.605	1.686	1.728	1.795	1.848	1.858	1.864	1.866	1.867	

Table 5 Thickness effect on stress intensity factors in case of $\nu_1=0.3$ and $\nu_2=0.38$
(Upper part : Approximation, Middle part : $B=5$ mm, Lower part : $B=3$ mm)

(a) F_I value

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	2.988	2.984	2.945	2.839	2.717	2.689	2.671	2.664	2.658
		2.968	2.971	2.937	2.843	2.731	2.706	2.689	2.683	2.677
		2.930	2.939	2.912	2.832	2.736	2.715	2.701	2.695	2.690
	15	2.951	2.928	2.878	2.756	2.621	2.591	2.571	2.562	2.556
		2.918	2.899	2.853	2.739	2.612	2.584	2.565	2.558	2.552
		2.881	2.866	2.826	2.725	2.613	2.588	2.572	2.565	2.560
	30	2.701	2.660	2.603	2.473	2.335	2.304	2.284	2.276	2.269
		2.669	2.639	2.574	2.449	2.315	2.285	2.266	2.259	2.252
		2.635	2.598	2.547	2.433	2.311	2.285	2.268	2.261	2.255
	45	2.272	2.213	2.154	2.025	1.892	1.863	1.844	1.837	1.831
		2.239	2.181	2.120	1.992	1.860	1.831	1.813	1.805	1.799
		2.209	2.153	2.095	1.975	1.852	1.826	1.809	1.802	1.797
	60	1.699	1.624	1.566	1.446	1.326	1.300	1.285	1.279	1.275
		1.656	1.584	1.522	1.399	1.278	1.253	1.236	1.229	1.224
		1.634	1.561	1.500	1.382	1.267	1.243	1.228	1.221	1.216
	75	1.015	0.926	0.874	0.771	0.670	0.650	0.639	0.635	0.632
		0.961	0.879	0.820	0.711	0.610	0.589	0.575	0.570	0.565
		0.947	0.863	0.803	0.695	0.596	0.575	0.562	0.557	0.553
	90	0.255	0.154	0.112	0.033	-0.042	-0.055	-0.061	-0.062	-0.063
		0.200	0.115	0.062	-0.025	-0.100	-0.115	-0.125	-0.129	-0.132
		0.196	0.106	0.052	-0.039	-0.116	-0.132	-0.141	-0.145	-0.148

(b) F_{II} value

		E_1/E_2								
		1.0	2.0	3.0	6.667	23.33	40.0	70.0	100.0	150.0
α (deg)	0	-0.356	0.122	0.363	0.698	0.942	0.986	1.013	1.023	1.031
		-0.324	0.192	0.452	0.825	1.102	1.153	1.184	1.197	1.207
		-0.320	0.200	0.464	0.848	1.139	1.194	1.228	1.242	1.253
	15	0.074	0.558	0.803	1.145	1.394	1.440	1.467	1.478	1.486
		0.118	0.638	0.901	1.281	1.562	1.615	1.647	1.660	1.670
		0.129	0.655	0.923	1.314	1.611	1.667	1.701	1.715	1.726
	30	0.495	0.950	1.181	1.505	1.741	1.784	1.810	1.821	1.828
		0.551	1.041	1.290	1.649	1.917	1.967	1.997	2.010	2.019
		0.570	1.065	1.319	1.690	1.972	2.026	2.059	2.072	2.082
	45	0.882	1.277	1.479	1.762	1.971	2.009	2.032	2.041	2.048
		0.947	1.373	1.590	1.905	2.141	2.185	2.212	2.223	2.231
		0.971	1.403	1.625	1.951	2.200	2.247	2.276	2.288	2.297
	60	1.213	1.522	1.680	1.906	2.073	2.103	2.122	2.129	2.134
		1.278	1.612	1.783	2.032	2.219	2.254	2.276	2.284	2.291
		1.307	1.646	1.821	2.079	2.277	2.315	2.338	2.347	2.355
	75	1.461	1.665	1.769	1.921	2.036	2.057	2.069	2.074	2.078
		1.523	1.740	1.853	2.020	2.146	2.170	2.184	2.190	2.195
		1.554	1.776	1.893	2.065	2.199	2.225	2.241	2.247	2.252
	90	1.605	1.686	1.728	1.795	1.848	1.858	1.864	1.866	1.867
		1.663	1.750	1.798	1.870	1.927	1.938	1.945	1.947	1.949
		1.694	1.785	1.835	1.911	1.972	1.984	1.991	1.994	1.996

was carried out by using the compact normal and shear (CNS) specimen with various kinds of Young's modulus, Poisson's ratio and load application angle. The stress-intensity factors of interface crack were evaluated with high accuracy by the virtual extension method. The polynomial approximation was proposed to evaluate the stress-intensity factors at the midsection of specimen. The stress-intensity factors of compact normal and shear specimen under the mixed-mode loading can be simply evaluated by the present polynomials.

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