Photoelastic Analysis from a Single Image Using Elliptically Polarized Tricolored Light*

Satoru YONEYAMA** and Masahisa TAKASHI**

In this paper, the authors describe a new method for determining the fringe order and the principal direction of birefringence from a single image. Using an elliptically polarized tricolored light and color image processing, the fringe order and the principal direction of birefringence are obtained from data of a color image obtained by a single shot. The theory of the proposed method and the entire experimental and analytical system are described. Then, a successful application of the method to 2-D photoelastic analysis is demonstrated. It is emphasized that this method can be applied to time-dependent phenomena, since multiple exposures are not necessary for sufficient data acquisition in the completion of stress analysis.

Key Words: Photoelasticity, Image Processing, Elliptically Polarized Light, Tricolored Light, Experimental Stress Analysis

1. Introduction

It is well known that photoelasticity is one of the most effective experimental techniques for stress analysis. Recently, several image processing techniques and automated polariscope systems for the analysis of photoelastic fringe patterns have been proposed and developed. Half-fringe photoelasticity⁽¹⁾, the fringe center extraction technique⁽²⁾, phase-shifting methods⁽³⁾⁻⁽⁵⁾ and Fourier transform methods^{(6),(7)} have been fairly successful. White light photoelastic techniques, such as spectral-content analysis⁽⁸⁾⁻⁽¹⁰⁾, were also proposed, in which photoelastic fringe patterns could be analyzed in detail using spectral scanners and a computer. Ajovalasit et al.^{(11),(12)} and Ramesh and Deshmukh⁽¹³⁾ proposed different types of white light photoelastic method, using the database search technique based on primary color analysis.

In order to analyze the stress state of time-dependent phenomena, such as the case of a moving load, as well as impact or viscoelastic problems by the

photoelastic technique, it is necessary to determine the retardation (fringe order) and the principal direction of birefringence (principal stress direction) from data of a photoelastic image obtained by a single shot. However, currently available techniques require multiple images at different polarization angles or multiple measurements. Thus, these techniques cannot be applied to the time-dependent phenomena since reproducibility of experiments cannot be expected.

In this context, the authors proposed a tricolored photoelastic/photoviscoelastic technique^{(14),(15)} for the determination of fringe order and the principal direction of birefringence from a single color image with plane polarized light consisting of three different wavelengths. Then, an apparatus including a tricolored light source that can simultaneously emit three monochromatic lights of primary colors was developed⁽¹⁶⁾, and the applications of the method to photoviscoelastic analysis were demonstrated^{(17),(18)}. However, in the tricolored photoelastic technique proposed, isochromatics superposed on isoclinic lines could not be separated; thus no information about isochromatics could be obtained⁽¹⁶⁾. Some difficulties in fringe order analysis still exist; thus the subsequent calculation of the principal direction of birefringence involves additional errors⁽¹⁶⁾.

In order to eliminate these difficulties in the

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** Department of Mechanical Engineering, Aoyama Gakuin University, 6-16-1 Chitosedai, Setagaya, Tokyo 157-8572, Japan. E-mail: yoneyama@me.aoyama.ac.jp

photoelastic analysis from a single image, a new technique that utilizes an elliptically polarized tricolored light and the color image processing technique is proposed. In the process of fringe-order determination, a database search approach based on primary color analysis^{(11)–(13)} is employed. After determining the fringe order, the distributions of the principal direction of birefringence can be calculated by solving nonlinear equations.

In this paper, the theory of the tricolored photoelastic technique that uses an elliptically polarized light, which is a new method for determining the fringe order and the principal direction of birefringence from a single color image, is described. Then, the successful application of the method to photoelastic analysis is demonstrated.

2. Theory

2.1 Equation of emerging light intensity

A transparent specimen observed under a dark-field circular polariscope and a plane polariscope with monochromatic light is considered. When the specimen is stressed, the intensities of light emerging from the circular polariscope and the plane polariscope are expressed respectively as

$$I = I_0 \sin^2 N\pi \text{ (for a circular polariscope),} \quad (1)$$

$$I = I_0 \sin^2 N\pi \sin^2 2(\varphi - \theta) \text{ (for a plane polariscope),} \quad (2)$$

where I_0 is the maximum value of the emerging light intensity; φ , the principal direction of birefringence; and θ , the polarization angle. N denotes the fringe order given by⁽¹⁹⁾

$$N = \frac{\delta}{\lambda} = \frac{Cd}{\lambda} (\sigma_1 - \sigma_2) \quad (3)$$

where δ is the retardation; λ , the wavelength of the light used; C , the stress-optic coefficient; d , the thickness of a specimen; and σ_1 and σ_2 are principal stresses. Equations (1) and (2) show that in a circular polariscope, the parameter of the principal direction of birefringence does not exist in the light intensity equation; on the other hand, isochromatics superposed on isoclinics vanish in a plane polariscope. In order to analyze the fringe order and the principal direction of birefringence on a full field from a single image, the emerging light intensity should always include information of isochromatics and isoclinics. Thus, it is expected that the other arrangement of optical elements in a polariscope, i.e., the use of an elliptically polarized light, makes it possible to obtain the emerging light intensity which includes information of isochromatics and isoclinics on the full field.

The derivation of the equation of the emerging light intensity with the angles of the optical element

Fig. 1 Arrangement of optical elements

axes as unknown parameters is complicated. Here, the arrangement of optical elements shown in Fig. 1 is considered. The axes of the polarizer and the analyzer make angles of $\pi/12$ rad and $-5\pi/12$ rad with respected to the ox axis and the fast axes of the quarter-wave plates subtend angles of 0 rad and $\pi/2$ rad with respected to the ox axis.

Introducing matrix calculation such as the Mueller calculus⁽²⁰⁾, the intensity of light emerging from the polariscope can be obtained. Using the matching wavelength λ_0 for the quarter-wave plate, the light intensity equation for the polariscope shown in Fig. 1 is expressed as

$$I = \frac{I_0}{8} (5 - 3 \cos 4\varphi) \sin^2 \frac{\delta\pi}{\lambda_0}. \quad (4)$$

On the other hand, when monochromatic light of arbitrary wavelength λ is used, the emerging light intensity is

$$I = \frac{I_0}{16} (9 - 5 \cos 4\varphi + 2 \cos 2\varepsilon \sin^2 2\varphi + 4\sqrt{3} \sin \varepsilon \sin 4\varphi) \sin^2 \frac{\delta\pi}{\lambda}, \quad (5)$$

where ε represents the retardation error of a quarter-wave plate given by

$$\varepsilon = \frac{\pi}{2} \left(\frac{\lambda_0}{\lambda} - 1 \right). \quad (6)$$

The light intensity I in other combinations of optical axis inclination in a polariscope can be easily obtained by a similar manner.

The analysis of Eqs. (4) and (5) shows that light intensity attenuation occurs where the principal direction of birefringence $\varphi = 0$ rad. On the other hand, there is no attenuation where $\varphi = \pi/4$ rad. In these cases, Eq. (4) becomes

$$I = \frac{I_0}{4} \sin^2 \frac{\delta\pi}{\lambda_0} \text{ for } \varphi = 0, \quad (7)$$

$$I = I_0 \sin^2 \frac{\delta\pi}{\lambda_0} \text{ for } \varphi = \pi/4. \quad (8)$$

The above relations and Eqs. (4) and (5) indicate that information of both isochromatics and isoclinics remains on the full field of a specimen. Thus, the analysis of isochromatics and isoclinics can be performed from a single image when an elliptically polarized light is used.

The use of light containing three wavelengths λ_r ,

λ_0 (= matching wavelength λ_0 for the quarter-wave plate) and λ_b , which correspond to red, green and blue lights emitted from the tricolored light source^{(16),(17)}, is considered. A photoelastic fringe pattern is recorded by a color camera with independent sensors for primary colors and stored in a computer as color image data. The digitized color image data consist of three monochromatic images corresponding to the three wavelengths, and each monochromatic image has its own distribution of intensity levels. Thus, the intensities, I_r (red), I_g (green) and I_b (blue), of light emerging from filters in the camera can be expressed as follows.

$$I_r = \frac{S_r}{16} (9 - 5 \cos 4\varphi + 2 \cos 2\varepsilon_r \sin^2 2\varphi + 4\sqrt{3} \sin \varepsilon_r \sin 4\varphi) \sin^2 N_r \pi, \quad (9)$$

$$I_g = \frac{S_g}{8} (5 - 3 \cos 4\varphi) \sin^2 N_g \pi, \quad (10)$$

$$I_b = \frac{S_b}{16} (9 - 5 \cos 4\varphi + 2 \cos 2\varepsilon_b \sin^2 2\varphi + 4\sqrt{3} \sin \varepsilon_b \sin 4\varphi) \sin^2 N_b \pi, \quad (11)$$

where S_i ($i=r, g, b$) is the product of the light source emittance, the transmittance of the overall optical system adopted and the spectral response of a camera; ε_i ($i=r, g, b$) is the quarter-wave plate error; and N_i ($i=r, g, b$) is the fringe order for each wavelength.

2.2 Determination of fringe order

In order to determine the fringe order, the database search approach^{(11)–(13)} is applied. When light involving multiwavelengths such as white light or a tricolored light is used in photoelasticity, isochromatics are colored because the distribution of isochromatic fringe is dependent on the wavelength. The observed color variation implies that red, green and blue components of the emerging light vary with retardation. In fact, red, green and blue components of the light intensities are unique to a certain fringe order. Thus, by comparing the light intensity values at each data point with the corresponding values in the calibration table that relates fringe order to light intensity values, the fringe order can be determined.

In the white light photoelastic technique proposed by Ajovalasit et al.^{(11),(12)}, the fringe order can be determined by minimizing an error function defined as

$$E_j = (R_j - R_m)^2 + (G_j - G_m)^2 + (B_j - B_m)^2, \quad (12)$$

where R_m , G_m and B_m are light intensity values digitized at each point of the model; and R_j , G_j , and B_j are also digitized values listed in the calibration table. The fringe order at a certain point can be determined by finding index j which minimizes the error function E_j .

In the proposed method, however, the error function Eq. (12) cannot be applied because the light intensities are attenuated due to ellipticity of the light used.

Thus, to ignore the effect of the principal direction of birefringence φ , the gray levels must be normalized before comparing the values of the model with those in the calibration table. The analysis of fringe order can be performed by means of a newly defined error function E'_j as

$$E'_j = \left(\frac{R_j}{R_j + G_j + B_j} - \frac{R_m}{R_m + G_m + B_m} \right)^2 + \left(\frac{G_j}{R_j + G_j + B_j} - \frac{G_m}{R_m + G_m + B_m} \right)^2 + \left(\frac{B_j}{R_j + G_j + B_j} - \frac{B_m}{R_m + G_m + B_m} \right)^2 \quad (13)$$

In the same manner with the method proposed by Ajovalasit et al.^{(11),(12)}, the fringe order can be determined in the following manner by searching index j which minimizes the error function E'_j .

It is realized that if the variation of the retardation in the calibration beam is linear and the first pixel of the calibration table is zero fringe order, the fringe order N_j corresponding to a pixel index as j is⁽¹¹⁾

$$N_j = N_m \frac{j}{j_m}, \quad (14)$$

where N_m is the maximum fringe order in the calibration table and j_m is the number of values listed in the calibration table.

2.3 Determination of the principal direction of birefringence

After determining the fringe order, the principal direction of birefringence can be calculated. From Eqs. (9), (10) and (11), the following equation is derived.

$$I_r + I_g + I_b - \frac{S_m}{16} (9 - 5 \cos 4\varphi + 2 \cos 2\varepsilon_r \sin^2 2\varphi + 4\sqrt{3} \sin \varepsilon_r \sin 4\varphi) \sin^2 N_r \pi - \frac{S_g}{8} (5 - 3 \cos 4\varphi) \sin^2 N_g \pi - \frac{S_b}{16} (9 - 5 \cos 4\varphi + 2 \cos 2\varepsilon_b \sin^2 2\varphi + 4\sqrt{3} \sin \varepsilon_b \sin 4\varphi) \sin^2 N_b \pi = 0. \quad (15)$$

Here, the term $S_i \sin^2 N_i \pi$ can be determined from the values in the calibration table constructed where the principal direction of birefringence is $\pi/4$ rad. Equation (15) is a nonlinear equation with respect to the principal direction of birefringence φ . Thus, the principal direction of birefringence φ can be obtained by solving Eq. (15) numerically.

3. System Configuration

In this study, a tricolored light source, previously developed by the authors^{(16),(17)}, which simultaneously emits three monochromatic lights of wavelength $\lambda = 436$ nm, 546 nm and 619 nm, is used to obtain photoelastic image data represented by Eqs. (9), (10) and (11). The arrangement of the light source is

Fig. 2 Construction of the tricolored light source

Fig. 3 Spectral distribution of the light source emittance

shown in Fig. 2. Light emitted from a mercury lamp passes through a dichroic mirror, and is separated into blue and green lights. Red light is extracted from a halogen lamp. These three lights pass through interference filters. Then, the three monochromatic lights are compounded by use of dichroic mirrors again. Figure 3 shows the spectral distribution of the tricolored light source. The full-width at half maximum of each spectrum of tricolored lights are about 3 - 5 nm. Light intensities of the three wavelengths are adjusted by using neutral-density (ND) filters, taking the spectral sensitivity of the camera into account. The entire experimental setup, including the tricolored light source, is shown in Fig. 4. This setup includes a set of polarizers and a set of quarter-wave plates matched to 546 nm wavelength, a color hi-vision camera, the tricolored light source, a digitizer (1 920 × 1 035 × 24 bits), a computer and a color monitor. Light emitted from the tricolor light source passes through

Fig. 4 Entire experimental setup

a plane polarizer, a first quarter-wave plate, a birefringent specimen, a second quarter-wave plate and an analyzer. Then, a color photoelastic fringe pattern is recorded by the color hi-vision camera as 24 bits image data of 1 024 × 1 024 pixels and 256 gray levels of brightness for each color.

4. Photoelastic Analysis and Results

4.1 Calibration

In current white light photoelastic techniques such as primary color analysis with the database search method⁽¹¹⁾⁻⁽¹³⁾ and spectral-content analysis⁽⁸⁾⁻⁽¹⁰⁾, calibration tests should be performed where the principal direction of birefringence is $\pi/8$ rad, to minimize the influence of quarter-wave plate error, as proposed by Baud⁽²¹⁾. In the proposed method, however, the calibration test has to be carried out where the principal direction of birefringence is $\pi/4$ rad to reconstruct the term $S_i \sin^2 N_i \pi$ in Eq. (15).

In order to analyze a stress-frozen test specimen, the calibration test for creating a table relating digitized light intensity levels to fringe order was performed using a stress-frozen specimen prepared in the same manner. A beam specimen 170 mm in height, 50 mm in width and 6 mm in thickness (Fig. 5(a)) was subjected to tension-bending load and stresses were frozen in an oven. Then, light intensity levels at each pixel along the transverse line on the calibration specimen were acquired using the apparatus shown in Fig. 4 and were stored in an ascending order array with index j . Figure 5(b) shows an example of a part of the fringe pattern observed in the calibration beam specimen.

The digitized values of the light intensities along the transverse section of the calibration beam are

Fig. 5 (a) Calibration beam under tension bending and (b) an example of fringe pattern

shown in Fig. 6(a). Figure 6(b) shows the corresponding fringe order for green light (546 nm) obtained by the method proposed with the values obtained manually by searching extreme values of the gray level. The linearity of the fringe order is recognized. Fringe orders obtained by the method proposed show good agreement with those obtained manually. The average deviation of both values is estimated to be about 0.008 fringe orders whereas a maximum deviation of 0.045 fringe orders is estimated at fringe order two.

In the current database search approach in white light photoelasticity, the maximum measurable fringe order is limited to three or four orders due to the attenuation of the light intensity⁽¹¹⁾. As shown in Fig. 6(b), however, fringe orders up to about 6.5 can be analyzed using tricolored light in this study⁽²²⁾.

4.2 Photoelastic analysis

In order to evaluate the effectiveness of the method, a plate with a center hole under tension load was analyzed. A plate model made of epoxy resin, which is 170 mm in height, 74 mm in width and 6 mm in thickness and has a hole of 8 mm radius, was adopted in this study. A tensile axial load of 43.5 N was applied to the plate. Then, stresses were frozen in the usual manner in an oven heated to 393 K. Photoelastic fringe patterns were recorded by the apparatus shown in Fig. 4 as 24 bits image data.

A color photoelastic fringe pattern of the plate with a hole under tension in elliptically polarized light is shown in Fig. 7. Note that the color image is expressed by black and white here. Attenuation of isochromatic fringes due to the influence of the elliptically polarized light is observed, as predicted in

Fig. 6 (a) Digitized light intensity levels along the transverse section on the calibration beam and (b) corresponding fringe order

Fig. 7 Color photoelastic fringe pattern of the plate with a hole (Color image, originally)

Fig. 8 Contour map of the fringe order obtained by (a) the method proposed and (b) the theory of elasticity

Fig. 9 Contour maps of the principal direction of birefringence obtained by (a) the method proposed, (b) the phase shifting method and (c) the theory of elasticity

Eqs. (9), (10) and (11).

The analysis of the fringe order is performed on the full field of the image data. Figure 8(a) shows the distributions of the fringe order obtained by the method proposed. Also, the fringe order obtained by the theory⁽²⁴⁾ is shown in Fig. 8(b). Here, the values are expressed by a contour map of 12 steps. By comparing the result of the proposed method with the fringe pattern in Fig. 7 and the theoretical value in Fig. 8(b), it is recognized that the almost all fringe orders on the field are evaluated accurately.

The distribution of the principal direction of birefringence is calculated using the bisection method, as shown in Fig. 9(a). Figures 9(b), (c) show the principal direction of birefringence obtained by the

Fig. 10 Principal direction of birefringence along the A-A' line

Fig. 11 Principal direction of birefringence along the B-B' line

phase-shifting method⁽³⁾ and the theory, respectively. It is found that the accuracy of the calculated results using the method proposed is poorer than that of the results obtained by other methods. Figures 10 and 11 show the principal direction of birefringence along the lines A-A' and B-B' in Fig. 9(a), respectively. The results of the principal direction of birefringence were affected by dark isochromatics. From this study, it is found that the determination of the fringe order and the principal direction of birefringence from a single image be performed by the proposed method. In static photoelastic experiments, accuracy can be improved by combining images obtained under different loads. In time-dependent problems, however, some improvement should be introduced for more precise analysis.

Using the proposed method, time-dependent problems can be analyzed since multiple exposures are not necessary for the acquisition of sufficient data in the completion of stress analysis.

5. Conclusions

A new method for determining the fringe order and the principal direction of birefringence from a single image using an elliptically polarized tricolored light and color image processing is described. Different from the method proposed previously by the authors^{(16),(17)} previously, the newly proposed method can determine the fringe order and the principal direction of birefringence on the full field of the image data.

As a result of this study, time-dependent problems in which good reproducibility of experiments is not expected can be analyzed by the proposed method since multiple exposures are not necessary for acquisition of sufficient data in the completion of stress analysis.

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References

- (1) Voloshin, A.S. and Burger, C.P., Half-Fringe Photoelasticity: A New Approach to Whole-field Stress Analysis, *Exp. Mech.*, Vol. 23 (1983), p. 304-313.
- (2) Seguchi, Y., Tomita, Y. and Watanabe, M., Computer-Aided Fringe-Pattern Analyzer—A Case of Photoelastic Fringe, *Exp. Mech.*, Vol. 19 (1979), p. 362-370.
- (3) Takashi, M., Mawatari, S., Toyoda, Y. and Kunio, T., A New Computer Aided System for Photoelastic Stress Analysis with Structure-Driven Type Image Processing, *Applied Stress Analysis*, Hyde, T.H. and Ollerton, E. Eds., (1990), p. 516-525, Elsevier Applied Science.
- (4) Patterson, E.A. and Wang, Z.F., Towards Full Field Automated Photoelastic Analysis of Complex Components, *Strain*, Vol. 27 (1991), p. 49-56.
- (5) Umezaki, E., Watanabe, H. and Shimamoto, A., Automatic Whole-Field Measurement of Photoelastic Fringe Orders Using Generalized Phase-Shift Method, *Post Conf. Proc. 1996 VIII Int. Cong. Exp. Mech.*, (1996), p. 154-159.
- (6) Quan, C., Bryanston-Cross, P.J. and Judge, T.R., Photoelasticity Stress Analysis Using Carrier Fringe and FFT Technique, *Optics and Lasers in Engineering*, Vol. 18 (1993), p. 79-108.
- (7) Morimoto, Y., Morimoto, Y. Jr., and Hayashi, T., Separation of Isochromatics and Isoclinics Using Fourier Transform, *Exp. Tech.*, Vol. 18 (1994), p. 13-17.
- (8) Redner, A.S., Photoelastic Measurements by Means of Computer-Assisted Spectral-Content Analysis, *Exp. Mech.*, Vol. 25, No. 2 (1985), p. 148-153.
- (9) Sanford, R.J., On the Range and Accuracy of Spectrally Scanned White Light Photoelasticity, *Proc. 1986 SEM Conf. Exp. Mech.*, (1986), p. 901-908.
- (10) Haake, S.J. and Patterson, E.A., Photoelastic Analysis of Frozen Stressed Specimens Using Spectral-contents Analysis, *Exp. Mech.*, Vol. 32 (1992), p. 266-272.
- (11) Ajovalasit, A., Barone, S. and Perucci, G., Towards RGB Photoelasticity: Full-Field Automated Photoelasticity in White Light, *Exp. Mech.*, Vol. 35 (1995), p. 193-200.
- (12) Ajovalasit, A., Barone, S. and Perucci, G., Automated Photoelasticity in White Light: Influence of Quarter-Wave Plates, *J. Strain Anal. Eng. Des.*, Vol. 30 (1995), p. 29-34.
- (13) Ramesh, K. and Deshmukh, S.S., Three Fringe Photoelasticity—Use of Colour Image Processing Hardware to Automate Ordering of Isochromatics, *Strain*, Vol. 32 (1996), p. 79-86.
- (14) Gotoh, J., Yoneyama, S., Mawatari, S. and Takashi, M., Photoelastic Analysis from a Single Piece of Image with Linearly Polarized RGB Lights, *Post Conf. Proc. 1996 VIII Int. Cong. Exp. Mech.*, (1996), p. 160-166.
- (15) Yoneyama, S., Takashi, M. and Gotoh, J., Time- and Temperature-Dependent Stress-Strain Analysis Using White Light Photoviscoelasticity, *Proc. SPIE*, Vol. 2921 (1997), p. 486-491.
- (16) Yoneyama, S., Gotoh, J. and Takashi, M., Photoelastic Analysis from Single Image with the Three Primary Colors, *Proc. Int. Conf. Advanced Tech. Exp. Mech.*, (1997), p. 429-434.
- (17) Yoneyama, S., Gotoh, J. and Takashi, M., Determination of Time-Dependent Fringe Order and Principal Birefringence Direction Using Tricolor Photoviscoelasticity, *Trans. Jpn. Soc. Mech. Eng.*, Vol. 64, No. 620, A (1998), p. 1007-1013.
- (18) Yoneyama, S., Gotoh, J. and Takashi, M., Tricolor Photoviscoelastic Technique and Its Application to Moving Contact, *Exp. Mech.*, Vol. 38 (1998), p. 211-217.
- (19) Burger, C.P., Photoelasticity, *Handbook on Experimental Mechanics*, 2nd Ed., Kobayashi, A. S., Ed., (1993), p. 165-266, Prentice-Hall.
- (20) Theocaris, P.S. and Gdoutos, E.E., *Matrix Theory of Photoelasticity*, (1979), McGraw-Hill.
- (21) Baud, R.V., Contribution to Study of Effect of Elliptical Polarization upon Energy Transmission, *J. Opt. Soc. Am.*, Vol. 21 (1931), p. 119-123.
- (22) Yoneyama, S., Shimizu, M. and Takashi, M., Higher Retardation Analysis in Automated White Light Photoelasticity, *Allison, I. M., Ed., Experimental Mechanics (Proc. 11th Int. Conf. Exp. Mech.)*, p. 527-532, A.A. Balkema.
- (23) Dally, J.W. and Riley, W.F., *Experimental Stress Analysis*, 2nd Ed., (1978), McGraw-Hill.
- (24) Timoshenko, S.P. and Goodier, J.N., *Theory of Elasticity*, 3rd Ed., (1970), McGraw-Hill.