Approximate Solution of Interactions among Spatially Distributed Broken Fiber, Matrix and Interface in the Progress of Interfacial Debonding in Multifilamentary Unidirectional Composites*

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In order to describe the relationship of mesoscopic phenomena of interfacial debonding and breakage of fiber and matrix to the macroscopic stress-strain curve of unidirectional composites, an approximate nondimensional solution method was presented using a two-dimensional model. By applying this method to several examples in which the species and locations of broken components were varied, the following features were revealed: (a) fiber-breakage-induced debonding occurs at a lower strain than the matrix-breakage-induced one; (b) the overall debonding is hastened due to mechanical interactions under the existence of many broken fibers and matrices; (c) the progress of overall debonding is dependent on the species of the broken components and on the geometrical location of broken components and debonded interfaces; and (d) the stress-strain curve shows drops in stress due to the progress of debonding. As an extended application of the present method, a nondimensional Monte Carlo simulation method was presented to describe the behavior of the composite in which the number and location of broken components and debonded interface vary with increasing strain.

Key Words: Composite Material, Tensile Properties, Damage Mechanics, Interfacial Debonding, Geometrical Effect, Mechanical Interaction, Stress-Strain Curve, Shear Lag Analysis, Monte Carlo Simulation

1. Introduction

In unidirectional fiber-reinforced composites, mesoscopic-scale mechanical events such as breakage of the components (fiber and matrix) and debonding of the interface occur at many places between micro- and macroscales. The spatially distributed events, interacting mechanically with each other, increase in number with increasing applied stress (strain), resulting in overall fracture of composites. Therefore, to describe the overall mechanical response and related

properties, clarification of the elemental processes of such mesoscopic mechanical events from a more microscopic viewpoint and also the mechanical interactions and accumulation of events which are spatially distributed and increase in number with increasing stress, is necessary. Namely, to describe the overall property in the light of microscopic mechanical response, systemization of the "elemental process of mesoscopic events", "accumulation and interaction of mesoscopic events" and "related macroscopic properties" is needed. In the present study, interfacial debonding is the subject, and we attempted to solve the interactions among many spatially distributed broken components and interfaces and to correlate them with the overall debonding behavior of the composite.

To describe the progress of interfacial debonding, the mechanical interactions shall be solved as stated

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above. It is, however, difficult to obtain a rigid solution. Thus the development of simplified method to obtain approximate solutions is needed. One of the candidates for obtaining an approximate solution for the interactions is the shear lag analysis method⁽¹⁾⁻⁽⁵⁾, with which the complex stress distribution affected by many mesoscopic events can be solved to a first approximation from the viewpoint of stress transfer through interface. This method is rough; however, due to its roughness, it can be used to solve interactions.

Ordinary shear lag analysis has been developed with the approximation that only fibers carry applied stress and the matrix acts only as a stress-transfer medium. Due to this approximation, the application of this method has been limited to polymer matrix composites in which the elastic modulus of the matrix is two order of magnitude lower than that of the fiber, and to low-yield-stress metal matrix composites, since the longitudinal tensile stress of the matrix can be ignored in comparison with that of the fiber in these composites. This, in turn, means that the ordinary method cannot be applied to elastic fiber/elastic matrix composites such as ceramic matrix composites in which the elastic modulus of the matrix is comparable to that of the fiber. In order to overcome this disadvantage, we have recently proposed a modified method, with which the general situation that both fiber and matrix carry applied stress and both act as stress transfer media can be described^{(6),(7)}. The aim of the present work is to present a method for describing the debonding behavior in ceramic fiber/ceramic matrix composites by converting the simultaneous differential equations derived from the modified method to finite difference equations, and by combining these equations with the Monte Carlo simulation method which has been applied to describe the behavior of polymer and light metal matrix composites by Oh⁽²⁾, Kimpara et al.⁽³⁾, Goda et al.⁽⁴⁾ and us^{(8),(9)}. It is noted here that in order to derive a systematic method of description, nondimensional forms of the necessary parameters for calculation, such as Young's modulus, shear modulus, length of the sample and diameter of the fiber, were used in this work.

2. Simulation Method

2.1 Modeling and stress equilibrium equations

A two-dimensional model composite with width W , length L_c and thickness T shown in Fig. 1, was employed. The Young's modulus, shear modulus, cross-sectional area and width of the fiber are E_f , G_f , A_f and D_f and those of the matrix are E_m , G_m , A_m and D_m , respectively. Each component was numbered 1, 2, ..., i , ..., n from left to right. The width, cross-sectional

Fig. 1 Schematic representation of (top) model of composite for calculation and (bottom) the notation of the displacement $U_{i,j}$, component element (i, j) and interface element (i, j)

area, Young's modulus and shear modulus of the " i " th component are noted by D_i , $A_i (=TD_i)$, E_i and G_i , respectively. The distance x was taken from the top end, the top and bottom ends being defined by $X=0$ and $X=L_c$, respectively. Noting the displacement of the i -component at $X=X$ as $U_i(X)$ and the shear stress between i - and $i+1$ -components at $X=X$ as $\tau_i(X)$, the stress equilibrium equation in the shear lag analysis is expressed as

$$A_i E_i \frac{d^2 U_i(X)}{dX^2} + T \{ (\tau_i(X) - \tau_{i-1}(X)) \} = 0. \quad (1)$$

Each component ($i=1$ to N) was regarded to be composed of $k+1$ short component elements with length ΔX . The position at $X=0$ was assigned 0 and then 1, 2, 3, ..., j , ..., $k+1$ downward, in steps of ΔX . $(k+1)\Delta X$ is equal to L_c . $j=0$ and $k+1$ correspond to $X=0$ and L_c , respectively. Hereafter, the " i " component from $X=(j-1)\Delta X$ to $j\Delta X$ is called the (i, j) -component, and the interface from $X=(j-1/2)\Delta X$ to $(j+1/2)\Delta X$ between " i " and " $i+1$ " components is called the (i, j) -interface, as shown in Fig. 1. The displacement of the (i, j) -component at $X=j\Delta X$ is defined as $U_{i,j}$. Introducing an interfacial parameter $a_{i,j}$, which is equal to 1 (unity) when the (i, j) -interface is bonded and is 0 (zero) when it is debonded, the interfacial shear stress at the (i, j) -interface, $\tau_{i,j}$, is expressed as^{(6),(7)}

$$\tau_{i,j} = \frac{2a_{i,j}G_iG_{i+1}(U_{i+1,j} - U_{i,j})}{D_iG_{i+1} + D_{i+1}G_i} \quad (i=1 \text{ to } n-1, j=1 \text{ to } k). \quad (2)$$

Introducing a new parameter $\gamma_{i,j}^{(3),(4)}$, which is 1 and 0 when the (i, j) -component is unbroken and broken, respectively, and combining with Oh's approximation⁽²⁾, the first term $d^2U_i(X)/dX^2$ of the (i, j) component is expressed as

$$\frac{4\gamma_{i,j}(U_{i,j-1} - U_{i,j}) + \gamma_{i,j+1}(U_{i,j+1} - U_{i,j})}{(2 + \gamma_{i,j} + \gamma_{i,j+1})\Delta X^2}. \quad (3)$$

We define Young's and shear moduli of the composite as E_c and G_c , respectively, and the composite stress as σ_c . Hedgepeth⁽¹⁾ developed a mathematical technique to solve simultaneous equations in the shear lag analysis. According to his technique, parameters are expressed in nondimensional forms, providing convenient mathematical expressions for the problem. However, he treated the case where the elastic modulus between the fiber and the matrix differs considerably, and the matrix does not carry applied stress and the matrix acts only as a stress transfer medium, as in polymer matrix composites. As the present work treats the case where both fiber and matrix carry applied stress and both act as stress transfer media, his expression cannot be used directly. Also, as he took the fiber stress as the standard for nondimensionalization, the correspondence of the composite stress with progress of breakage of components and interfacial debonding cannot be expressed explicitly. In this work, his mathematical expression was modified to suit the present problem, and $U_{i,j}$, X , L_c , D_i , A_i , E_i , G_i and $\tau_{i,j}$ were converted to the nondimensional expressions of $u_{i,j}$, ξ , l_c , d_i , a_i , e_i , g_i and $\tau'_{i,j}$, respectively, as follows.

$$\begin{aligned} U_{i,j} &= \sigma_c D_f \{1/(E_c G_c)\}^{1/2} u_{i,j}; \quad X = D_f (E_c/G_c)^{1/2} \xi; \\ \Delta X &= D_f (E_c/G_c)^{1/2} \Delta \xi; \quad L_c = D_f (E_c/G_c)^{1/2} l_c; \\ D_i &= D_f d_i; \quad A_i = D_f T a_i = D_f T d_i; \quad E_i = E_c e_i; \\ G_i &= G_c g_i; \quad \tau_{i,j} = \sigma_c (G_c/E_c)\}^{1/2} \tau'_{i,j}. \end{aligned} \quad (4)$$

Substituting Eqs. (2)–(4) into Eq. (1), we have the equations for the nondimensional displacement $u_{i,j}$:

$$\begin{aligned} &\frac{4\gamma_{i,j}}{(2 + \gamma_{i,j} + \gamma_{i,j+1})\Delta \xi^2} u_{i,j-1} \\ &+ \frac{2a_{i-1,j}g_{i-1}g_i}{d_i e_i (d_{i-1}g_i + d_i g_{i-1})} u_{i-1,j} \\ &+ \left\{ -\frac{4(\gamma_{i,j} + \gamma_{i,j+1})}{(2 + \gamma_{i,j} + \gamma_{i,j+1})\Delta \xi^2} - \frac{2a_{i,j}g_i g_{i+1}}{d_i e_i (d_i g_{i+1} + d_{i+1} g_i)} \right. \\ &\quad \left. - \frac{2a_{i-1,j}g_{i-1}g_i}{d_i e_i (d_{i-1}g_i + d_i g_{i-1})} \right\} u_{i,j} \\ &+ \frac{4\gamma_{i,j+1}}{(2 + \gamma_{i,j} + \gamma_{i,j+1})\Delta \xi^2} u_{i,j+1} \\ &+ \frac{2a_{i,j}g_i g_{i+1}}{d_i e_i (d_i g_{i+1} + d_{i+1} g_i)} u_{i+1,j} = 0. \end{aligned} \quad (5)$$

2.2 Boundary conditions

We take the displacements of all components at $X=0$ as zero, and those at $X=L_c$ as U_c . The un-

known nondimensional parameters are $u_{i,j}$ ($i=1$ to n , $j=1$ to k) and u_c . As the load of the (i, j) -component is $\gamma_{i,j} E_i A_i (U_{i,j} - U_{i,j-1})/\Delta X$, the overall load of the composite, P , is expressed by Eq. (6) using the nondimensional forms of E_i , A_i , $U_{i,j}$, $U_{i,j-1}$ and X :

$$P = \sigma_c W T = \sum_{i=1}^N \gamma_{i,j} D_f T \sigma_c e_i d_i \frac{u_{i,j} - u_{i,j-1}}{\Delta \xi}. \quad (6)$$

As the load is the same at any cross section, we have

$$\sum_{j=1}^{k+1} \left\{ \sum_{i=1}^N \gamma_{i,j} e_i d_i \frac{u_{i,j} - u_{i,j-1}}{\Delta \xi} \right\} = \left\{ \sum_{i=1}^N d_i \right\} (k+1). \quad (7)$$

The unknown nondimensional displacements, $u_{i,j}$ ($i=1$ to n , $j=1$ to k) and u_c , can be solved by substituting $a_{i,j=1}$ and 0 for unbonded and debonded interfaces, respectively, and $\gamma_{i,j=1}$ and 0 for unbroken and broken components, respectively.

2.3 Simulation of debonding for a given geometry of broken fibers and matrices

2.3.1 Nondimensional expression of stress, strain and Young's modulus of composite The (i, j) -interface is debonded when $\tau_{i,j} = \tau_c$ (interfacial shear strength). From Eq. (4), the nondimensional shear stress at debonding of the (i, j) -interface $\tau'_{i,j}$ satisfies

$$\tau'_{i,j} = \frac{\tau_{i,j}}{\sigma_c} \left[\frac{E_c}{G_c} \right]^{1/2}. \quad (8)$$

Using Eq. (2) and the calculated values of $u_{i+1,j}$ and $u_{i,j}$, $\tau'_{i,j}$ is calculated by

$$\tau'_{i,j} = \frac{2a_{i,j}g_i g_{i+1}}{(d_i g_{i+1} + d_{i+1} g_i)} (u_{i+1,j} - u_{i,j}). \quad (9)$$

Debonding occurs at the (i, j) -interface at composite stress σ_c when $\tau'_{i,j}$ in Eq. (9) satisfies Eq. (8). This means that the stress of the composite σ_c upon debonding is proportional to τ_c . From Eq. (8), $1/\tau'_{i,j}$ can be defined as the nondimensional composite stress σ'_c at debonding of the (i, j) -interface:

$$\sigma'_c = \frac{1}{\tau'_{i,j}} = \frac{\sigma_c}{\tau_c} \left[\frac{G_c}{E_c} \right]^{1/2}. \quad (10)$$

When the components and interfaces are broken, the overall Young's modulus is reduced to $E_{c,eff}$ from E_c . $E_{c,eff}$ satisfies $U_c = (\sigma_c/E_{c,eff})L_c$. We define the nondimensional Young's modulus of composite E'_c as $E_{c,eff}/E_c$. Then, E'_c can be calculated by

$$E'_c = \frac{E_{c,eff}}{E_c} = \frac{l_c}{u_c}. \quad (11)$$

by using the nondimensional forms of $U_c (= U_{i,k+1})$ in Eq. (4) and L_c . In this way, the present nondimensionalization has the advantage that the reduction in Young's modulus of the composite with progress of debonding can be estimated simply from the calculation results of l_c/u_c in the nondimensional form.

The overall strain of composite ε_c is expressed as

$$\varepsilon_c = \frac{U_c}{L_c} = \frac{\sigma'_c}{l_c/u_c} \frac{\tau_c}{(E_c G_c)^{1/2}}. \quad (12)$$

We define the nondimensional strain of composite ε'_c as

$$\varepsilon_c' = \frac{\varepsilon_c (E_c G_c)^{1/2}}{\tau_c} \quad (13)$$

From Eqs.(11)-(13), we have

$$\varepsilon_c' = \frac{\sigma_c'}{l_c/u_c} = \frac{\sigma_c'}{E_c'} \quad (14)$$

In this way, the nondimensional strain ε_c' is expressed as the ratio of the nondimensional composite stress σ_c' to nondimensional Young's modulus E_c' as per Hooke's law.

2.3.2 Procedure for simulation of debonding

For the case where the locations of broken components are known, the debonding simulation was carried out according to the following procedure.

(1) $\tau_{i,j}'$ -values were calculated for all interface elements.

(2) The interface with the highest $\tau_{i,j}'$ is debonded first. Noting such an interface to be the (p, q) -interface, the debonding of this interface occurs at $\sigma_c' = 1/\tau_{p,q}'$. The nondimensional strain ε_c' upon debonding is given by Eq.(14).

(3) After setting $\alpha_{p,q} = 0$, $\tau_{i,j}'$ -values were calculated again for all interface elements. Applying procedure (2), the interface element to be debonded next was identified, and the pair values of $(\varepsilon_c', \sigma_c')$ were calculated.

(4) By repeating procedures (2) to (4), the interfaces to be debonded were identified in the order of debonding and the corresponding pair values of $(\varepsilon_c', \sigma_c')$ were determined one after another.

(5) When debonding occurs at one interface, say the (r, s) -interface, the stress distribution becomes different and the shear stress of each interface is changed. Some interfaces may have higher shear stresses after debonding of the (r, s) -interface than before, but some may have lower ones. The value of the maximum shear stress may also be changed. Depending on the situation, two cases arise: in one case, no more debonding occurs and in another case, more debonding occurs, under the same applied stress (strain). If we take the case where the (r, s) -interface had the highest shear stress $\tau_{r,s} (= \tau_c)$ before debonding and the (r', s') -interface comes to have the highest shear stress $\tau_{r',s'}$ after the debonding of the (r, s) -interface, the (r', s') -interface will be debonded following the debonding of the (r, s) -interface under the same applied stress (strain) if $\tau_{r,s} < \tau_{r',s'}$, but not if $\tau_{r,s} > \tau_{r',s'}$. Taking such a situation into consideration, the nondimensional stress-strain curve was determined as follows. As the curve becomes different, depending on whether the tensile test is carried under displacement or stress control, the procedure for both controls will be mentioned.

Figure 2(a) shows the schematic drawing of the calculated pair values of $(\varepsilon_c', \sigma_c')$ at first (closed circle

Fig. 2 Schematic representation of (a) the calculated pairs of $(\varepsilon_c', \sigma_c')$ for the 1st to 5th debondings indicated by A to E, respectively, for (b) displacement and (c) stress controls. F and G refer to the stress levels after the 2nd and 4th debondings following the 1st and 3rd debondings, respectively, under displacement control. H and I refer to the strain levels after the 2nd and 4th debondings following the 1st and 3rd debondings, respectively, under stress control. The stress-strain curves vary along AFCGE and AHGIE for displacement and stress controls, respectively

indicated by A on line 1), second (B on line 2)... and fifth (E on line 5) debondings. One to 5 shows the order of debonding and the corresponding lines are drawn by connecting $(0, 0)$ and the calculated $(\varepsilon_c', \sigma_c')$. The thick lines in (b) and (c) show the stress-strain curves under displacement and stress controls, respectively. Taking the displacement control as an example, the first debonding occurs at A. The 2nd debonding occurs at B in the calculation. However, the strain at B is lower than that at A. In practice, as the 2nd debonding never occurs unless the 1st one occurs, the 2nd debonding occurs following the 1st debonding under the same strain as that for the 1st. After the 2nd debonding, as Young's modulus is reduced to the slope of the 3rd line, the stress is reduced to F. At this stage, no more debonding occurs. With increasing applied strain, composite stress increases on the 3rd line, and the 3rd debonding occurs at C. The 4th debonding occurs, following the 3rd one at the same strain, similarly to the 1st and 2nd

debondings, and the stress drops to G . After that, the stress of the composite increases with increasing strain on the 5th line and the 5th debonding occurs at E . In this way, the stress-strain curve is judged to vary along 0AFCEG, as shown in Fig. 2(b). In the case of stress control, the stress-strain curve is judged to vary along 0AHCEI by replacing displacement for stress in the procedure stated above.

2.4 Monte Carlo simulation for the case where more breakages of fibers and matrices occur with increasing applied stress (strain)

When the (i, j) -component is not broken, the tensile strain $\varepsilon_{i,j}$ is given by

$$\varepsilon_{i,j} = \frac{U_{i,j} - U_{i,j-1}}{\Delta X} = \frac{u_{i,j} - u_{i,j-1}}{\Delta \xi} \frac{\sigma'_c \tau_c}{(E_c G_c)^{1/2}}. \quad (15)$$

Noting the nondimensional strain $\varepsilon'_{i,j}$ as

$$\varepsilon'_{i,j} = \frac{\varepsilon_{i,j} (E_c G_c)^{1/2}}{\tau_c} \quad (16)$$

and combining with Eq. (15), we have

$$\varepsilon'_{i,j} = \frac{u_{i,j} - u_{i,j-1}}{\Delta \xi} \sigma'_c. \quad (17)$$

In this expression, $(u_{i,j} - u_{i,j-1})/\Delta \xi$ is equal to $\{(U_{i,j} - U_{i,j-1})/\Delta x\}/(\sigma_c/E_c)$; namely, it corresponds to the strain concentration factor with respect to the strain σ_c/E_c of the composite without breakage of components and interfaces.

Assuming that the distribution of fracture strain (ε_f) of the components obeys the Weibull distribution, the cumulative function F of the nondimensional fracture strain $\varepsilon'_f (= \varepsilon_f (E_c G_c)^{1/2} / \tau_c)$ is given by

$$F = 1 - \exp\left\{-\left[\frac{L}{L_0}\right]\left[\frac{\varepsilon'_f}{\varepsilon_0}\right]^m\right\}, \quad (18)$$

where L is the length, L_0 is the standard length, and m and ε_0 are the shape and scale parameters, respectively. In the present simulation, we took $L_0 = L_c$ and $L = \Delta X$ as examples. Therefore, L_0/L is equal to $k+1$ in this work.

The nondimensional fracture strain of each component element $\varepsilon'_{f,i,j}$ -value was given by generating a random value as in the former Monte Carlo simulation studies^{(2)-(4),(8),(9)} using Eq. (18), in which m and ε_0 values were given for fiber and matrix.

In this simulation, breakages of both the component and the interface occur. The order of breakage was determined as follows. Among the many component elements which have not been broken, the component element whose nondimensional composite stress at which $\varepsilon'_{i,j} (= \{u_{i,j} - u_{i,j-1}\}/\Delta \xi) \sigma'_c$ reaches $\varepsilon'_{f,i,j}$ is the minimum, is broken. Thus $\varepsilon'_{f,i,j}/\{(u_{i,j} - u_{i,j-1})/\Delta \xi\}$, which is equal to σ'_c from Eq. (17), was calculated for all unbroken component elements, and then the element which has the minimum value was determined. On the other hand, for interfacial debonding, the interface element which has the maximum $\tau'_{i,j}$ (correspond-

ing to the minimum σ'_c from Eq. (9)) was determined. Then comparing the minimum composite stress for breakage of the component with that for interfacial debonding, the event corresponding to the lower minimum composite stress was identified to occur. Under the repetition of such a procedure up to the overall fracture, the order, species of event (breakage of component and debonding of interface), location, and corresponding pair values of $(\varepsilon'_c, \sigma'_c)$ were determined one after another. From the values of $(\varepsilon'_c, \sigma'_c)$ in the order of occurrence, the stress-strain curve was determined under the same concept as in Section 2.3.2. The overall fracture occurs when all components from one to n are broken and, in addition, the broken ends are connected directly in the same cross section or the broken ends are connected through debonded interfaces from right to left of the composite.

3. Results and Discussion

As an example, the following nondimensional values were used for simulation: $N=9$, $k=12$, $d_f = d_m=1$, $e_c=1$, $e_f=1.71$, $e_m=0.43$, $g_c=1$, $g_f=2.67$, $g_m=0.67$, $l_c=16.4$, $\Delta \xi=1.26$, which corresponded to the dimensional case of $E_f/E_m = G_f/G_m = 4$, $D_f = D_m$, $L_c = 26D_f$ and $\Delta X = 2D_f$.

First, in order to know the influence of geometrical location on the progress of debonding under the existence of two cut component elements, T1 to T4 in Fig. 3 were taken as examples. T1 shows the progress of debonding for the case where two broken component elements are separated (nearly isolated). T2 to T4 show the influence of the longitudinal distance between the cut elements of neighboring fiber and matrix on the debonding behavior. Next, in order to know the influence of the geometrical location of many broken ends on the stress-strain curve, six cut elements were distributed as shown by T5 and T6 in Fig. 3. Monte Carlo simulation was carried out for the followings cases: case (a) $\{m=5$ (fiber) and 10 (matrix), $\varepsilon'_0=1$ (fiber) and 2 (matrix) $\}$ and case (b) $\{m=10$ (fiber) and 5 (matrix), $\varepsilon'_0=2$ (fiber) and 1 (matrix) $\}$. Case (a) corresponds to the case where the fiber is broken at lower strain than the matrix and case (b), to the reverse case. The simulation results obtained under the displacement control will be shown below.

3.1 Influence of geometry on the progress of interfacial debonding under the existence of two cut component elements

Figure 4 shows the simulated progress of interfacial debonding for T3 as an example. Figures 1, 2, ..., within the composite show the order of occurrence. The debonding occurs first at $\varepsilon'_c=1.71$, followed by further debondings indicated by 2 to 6 at the same

Fig. 3 Schematic representation of the geometrical locations of broken components for T1 to T6, for which the debonding behavior was simulated.

strain, and then stops. When the strain is raised to $\epsilon'_c=1.73$, the debondings indicated by 7 and 8 occur and stop. In this way, the debonding progresses by repeated growing and stopping. The debonding near the sample ends occurs in the late stage since the shear stress is low due to the constraint. This is a common feature among T1 to T6.

Figure 5 shows the simulated progress of interfacial debonding at $\epsilon'_c=1.6, 2.0, 2.5$ and 5.5 for T1 to T4. Figure 6 shows the number of debonded interface-elements, N_d , which is proportional to the total debonded length, as a function of nondimensional composite strain ϵ'_c also for T1 to T4. In T1, in which the cut elements are apart each other, first the cut-fiber-induced debondings occur and reach to the sample end. Then the cut-matrix-induced debondings occur at the strain indicated by A in Fig. 6 and grow. This means that the debonding in the composite with high fiber volume fraction as in the present examples tends to grow at lower applied stress when the fiber with high elastic modulus is broken than when the matrix is broken.

When the longitudinal distance between the cut elements of the neighboring components is large (T2), first the cut-fiber-induced debonding grows to some

Fig. 4 Simulated progress of interfacial debonding for T3

extent, then the cut-matrix-induced one grows, and the two are connected. When the longitudinal distance is small (T3), the mechanical interaction hastens the debonding. When cut elements exist in the same cross section (T4), the debonding is hastened much more. In T2 to T4, the area surrounded by the cut components and debonded interfaces (Region (R) in Fig. 5) appears in the final stage of debonding. As stress is not transferred to such a region, the shear stress becomes zero, and therefore, no more debonding occurs in this region. As a result, when components are broken individually, the debonding reaches the end of sample, while when the neighboring components are broken, the debonding does not necessarily occur in all possible interfaces and the region where debonding does not occur may appear.

While the cut-matrix-induced debonding first occurs at $\epsilon'_c=5$ when the cut element is isolated (T1 in Fig. 6), the debonding is saturated at $\epsilon'_c=4$ when the neighboring components are cut (T2 to T4). In this way, the mechanical interactions among broken elements act to hasten the overall debonding. The present method is useful to obtain an approximate

Fig. 5 Simulated progress of interfacial debonding at $\epsilon'_c = 1.6, 2.0, 2.5$ and 5.5 for T1 to T4 in which two components are broken at different geometrical locations (Fig. 3). The progress varies due to the difference in the geometrical location of the broken components. R refers to the region where no debonding occurs.

Fig. 6 Number of debonded interface-elements, N_d , as a function of nondimensional composite strain ϵ'_c for T1 to T4

Fig. 7 Simulated progress of interfacial debonding at $\epsilon'_c = 1.6, 2.2, 3.2$ and 5.0 for T5 and T6

Fig. 8 Number of debonded interface-elements, N_d , as a function of nondimensional composite strain ϵ'_c for T5 and T6

solution for the interactions-hastened overall debonding.

3.2 Progress of interfacial debonding under the existence of many cut component elements

Figure 7 shows the simulated progress of interfacial debonding for T5 and T6; Fig. 8 the number of debonded interface-elements, N_d , as a function of ϵ'_c ; Fig. 9 the simulated stress-strain curves; and Fig. 10 the reduction of nondimensional Young's modulus E'_c ($=E_{c,eff}/E_c$: ratio of effective Young's modulus to original one) of composite calculated by Eq. (11) as a function of ϵ'_c . The main features are summarized as follows.

(1) The debonding in T5 and T6 progresses at lower strains in comparison with those in T1 to T4, indicating that breakage of many components hastens overall debonding.

(2) With progress of debonding at a given strain, the stress of the composite is reduced. The reduction

Fig. 9 Simulated stress-strain curves for T5 and T6

Fig. 10 Reduction of nondimensional Young's modulus E_c' of composite as a function of composite strain ϵ_c' for T5 and T6

is, however, small for the debonding of one interface element, so that it is not clearly found in the stress-strain curve. The large reduction of stress in the curve in Fig. 9 indicates that debonding progressed at many interface elements under constant strain. In Fig. 2, an example that shows that the second debonding follows the first one at the same strain is given. The stress-strain curve in Fig. 9, on the other hand, corresponds to the case where the repetition of the debonding of new interface elements occurs one after another. The overall debonding stops when the shear stresses of all interfaces become lower than the shear strength at a given strain. Then, with increasing strain, the composite stress increases. The composite deforms with repetition of such progress and stoppage of debonding, resulting in the serrated stress-strain curve.

(3) The nondimensional Young's modulus E_c' before the occurrence of debonding was as high as 0.91 for T5 and 0.87 for T6 in spite of the existence of many cut component elements. Thus, the reductions of Young's modulus due to the introduction of cut component elements are 9% and 13% for T5 and T6,

Fig. 11 Simulated stress-strain curves for cases (a) and (b), together with the increase in number of broken fiber, matrix and interface elements for case (a). In case (b), the fiber, matrix and interface elements are broken one after another at the same strain at which initial breakage of fiber occurs, causing the overall fracture of the composite

respectively. E_c' decreases with progress of debonding, reaching about 0.44 for both T5 and T6. The reductions of Young's modulus after completion of the debonding are, therefore, 47% and 43% for T5 and T6, respectively. In this way, the influence of debonding on the effective Young's modulus of the composite becomes large as debonding progresses. The present method makes it possible to estimate the influence of the breakage of components and the debonding of interface separately, which is one of the features of the present approach.

(4) The comparison of T5 with T6 evidently shows that the progress of debonding, the resulting reduction of Young's modulus and the shape of stress-strain curve become different, depending on the spatial distribution of cut elements of components. The mechanical interaction is very sensitive to the geometrical locations of cut elements; the debonding occurs at lower strain when the cut elements exist in the restricted cross section (T6) than when they exist separately.

3.3 Behavior in the case where the breakage of components and debonding of interfaces increase with increasing strain

Figure 11 shows the simulated stress-strain curves for case (a) where the fracture strain of the matrix is lower than that of the fiber and case (b) where the former is higher than the latter, together with the increase in the number of broken fiber, matrix and interface elements for case (a). Figures 12 and 13 show the snapshots of the simulated

Fig. 12 Snapshots of simulated progress of breakage of fiber, matrix and interface for case (a)

Fig. 13 Snapshots of simulated progress of breakage of fiber, matrix and interface for case (b)

progress of breakage of fiber, matrix and interface for cases (a) and (b), respectively, together with the fracture morphology. A and B refer to the regions with and without interfacial debonding, respectively. In case (a), the matrix elements are broken first, followed by fiber breakage and debonding. In the final stage, the composite is broken when the broken matrix, fiber and interface are connected from the left to the right side, as shown by the bold line (abc...gh) in Fig. 12. In case (b), when the initial breakage of fiber element occurs, the breakage of matrix, fiber and interface are induced one after another, resulting in overall fracture at the same strain. The overall fracture occurs along abc...mn as shown by the bold line in Fig. 13.

Figure 14 shows fracture morphology for cases (a) and (b). It is noted that interfacial debonding occurred in region A but not in region B. Also, it is noted that in the fracture surface opposite to that containing region A, there are regions with (A) and without (B) debonding. Until now, the interface has been judged to be strong if there is no debonded interface in fracture surface and to be weak if there is debonded one. Also, the coexistence of debonded and unbonded regions has been attributed to the spatial scatter of interfacial strength. The present result demonstrates that, even if the interface is weak, a region without debonding can arise in the fracture surface, depending on the spatial distribution of bro-

Fig. 14 Fracture morphology for cases (a) and (b). A and B refer to the regions with and without interfacial debonding, respectively.

ken components. This point must be taken into consideration in the estimation of the interfacial strength from a fracture surface.

5. Conclusions

A nondimensional calculation method based on the modified shear lag analysis was presented to describe the relationship of the progress of mesoscopic-scale interfacial debonding and breakage of fiber and matrix to the macroscopic overall stress-strain curve of a two-dimensional composite. By applying the present method to the progress of debonding, the following features were revealed: the debonding in the composite with a high fiber volume fraction tends to grow at lower applied stress when the fiber is broken than when the matrix is broken; the mechanical interactions among the components hasten the progress of debonding; the progress of debonding varies, depending on the geometrical position of the broken components and the interface; and the intermittent progress of debonding results in serrated stress-strain curves. The present method was also combined with a Monte Carlo simulation method as an example for extended application, and the non-dimensional description method of the stress-strain behavior of composites, in which the number and spatial distribution of breakage of fiber, matrix and interface vary with increasing strain, was proposed.

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