A Study on Maximization of Dynamic Crushing Energy Absorption of Square Tubes with and without Stiffener*

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The axial impact crushing behavior of square tubes with and without stiffeners are studied by Finite Element method, and comparisons of mean axial crushing force between numerical solutions and theoretical prediction as well as qualitative comparisons of folding patterns between numerical simulations and experimental observations are made and discussed. Based on the numerical analyses, maximization problems of dynamic crushing energy absorption of square tubes with and without stiffeners, respectively, are solved using the crashworthiness maximization technique for tubular structures which combined the techniques of design-of-experiment, response surface approximation as well as usual mathematical programming. In addition, the energy absorbing capability of cylindrical tubes and square tubes with and without stiffeners are also compared.

Key Words: Structural Analysis, Optimum Design, Nonlinear Problem, Finite Element Method, Design-of-Experiment, Response Surface Approximation, Impact Crushing Behavior, Kinetic Energy Absorption

1. Introduction

The axial crushing behavior and energy absorption characteristics of square tubes used as simplified models of effective impact energy absorbers have received considerable attention. Many efforts have been made to predict the mean axial force and absorbed energy experimentally, theoretically and numerically⁽¹⁾⁻⁽⁴⁾. To stabilize crushing process and increase the energy absorbing capacity, effective methods such as filling tubes by crushable medium like foam or wood and arranging an appropriate number of beads have been investigated⁽⁵⁾⁻⁽⁷⁾. Stiffeners are placed axially or circumferentially on the outer or inner surface of tubes to improve the strength of light weight tubular structures⁽⁸⁾⁻⁽¹¹⁾. The impact crushing behavior of axially stiffened and

unstiffened square tubes was also studied as simple approximations to motor or railway coaches under impact^{(8),(9)}. On the other hand, the efforts toward optimizing highly nonlinear structural response have been made in recent years using the technique of design-of-experiment and response surface approximation⁽¹²⁾⁻⁽¹⁴⁾. The crashworthiness maximization system for tubular structures has been developed by applying the structural optimization techniques, and was applied to cylindrical tubes successfully⁽¹⁴⁾.

In this study, the axial impact crushing behavior of axially stiffened and unstiffened square tubes are studied by the explicit finite element method. The comparisons of the mean axial crushing force between numerical solutions, theoretical prediction and experiment results, and the qualitative comparisons of folding patterns between numerical simulations and experimental observations are made and discussed. Based on the numerical analyses, maximization problems of the dynamic crushing energy absorption of stiffened and unstiffened square tubes are solved respectively using the crashworthiness maximization technique for tubular structures which combined the

* Received 14th October, 1999. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 65, No. 632, A (1999), p. 887-893 (Received 19th June, 1998).

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techniques of design-of-experiment, response surface approximation as well as usual mathematical programming. In addition, the crashworthiness of cylindrical tubes and square tubes with and without stiffeners are also compared.

2. Impact Simulation

2.1 Model description

Figure 1 shows the design and analysis model of an unstiffened square tube and a stiffened square tube with four outer stiffeners. The aluminum square tube axially impacts to a rigid wall with an initial velocity $V_0=10$ m/s. Von Mises yielding condition and the bilinear constitutive law are assumed. The Young's modulus $E=70$ GPa, the Poisson's ratio $\nu=0.33$, the initial yielding stress $\sigma_0=100$ MPa, the plastic modulus $E_T=280$ MPa and the density $\rho=2.82 \times 10^3$ kg/m³ are assumed. By using the nonlinear dynamic finite element code, DYNA3D, five types of unstiffened and eight types of stiffened square tube models are analyzed for observing the crushing behavior of a square tube under an axial impact load. Table 1 shows the dimensions of mean width C , mean wall thickness H , initial length L of the square tube, and the depth T and width S of the stiffener. Each model has the same mass ($M_0=0.53$ kg) and is attached an added mass that is 500 times the tube mass at one end of the square tube in order to supply enough energy for crushing. The type and size of finite elements for each model are decided depending on the dimensions of the square tube and stiffener. Each stiffener is discretized into at least two elements along depth and

one shell element through thickness for a thin-walled tube or more than two solid elements for a thick tube.

2.2 Numerical results and discussions

2.2.1 Impact crushing deformation Figure 2 shows the typical folding patterns of unstiffened square tubes. Model UNS-1 and UNS-3 deformed into a symmetric pattern, i.e., two opposite sides deforming inwards with other two opposite sides deforming outwards. The thickest and shortest one, model UNS-2, developed one extensional wrinkle, i.e., four sides deforming outwards or inwards at the same time, with considerable circumferential extension and then buckling stopped because the energy is not great enough for further crushing. Model UNS-4 developed

Table 1 Dimension of square tube models with and without stiffener

Model	C (mm)	H (mm)	T (mm)	S (mm)	Length (mm)
UNS-1	60.0	2.0	--	--	392.7
UNS-2	40.0	8.0	--	--	149.6
UNS-3	60.0	5.0	--	--	157.1
UNS-4	37.5	3.0	--	--	418.9
UNS-5	30.0	5.0	--	--	314.2
STF-1	60.0	2.0	6.0	2.0	356.0
STF-2	60.0	2.0	30.0	2.0	261.8
STF-3	60.0	5.0	8.0	3.0	145.4
STF-4	30.0	5.0	10.0	5.0	254.7
STF-5	30.0	5.0	24.0	7.0	148.2
STF-6	60.0	2.0	12.0	2.0	327.2
STF-7	60.0	2.0	24.0	1.0	327.2
STF-8	60.0	2.0	24.0	2.0	280.5

Fig. 1 Analysis and design model

Fig. 2 Typical crushing deformations of square tubes

Fig. 3 Typical crushing deformations of stiffened square tubes

five symmetric wrinkles followed by column buckling when not much energy can be absorbed. It is considered that the symmetric crushing can introduce disturbances into the unchanged part of the square tube and can then produce column buckling. The relatively thick and long model, UNS-5, formed one slight extensional wrinkle followed by column buckling. The numerical simulation agrees with the experimental observation⁽¹⁾.

Simulations have also been performed for gaining insight into the influence of dimensions of the stiffener under the impact load. Figure 3 shows five types of crushing deformation of stiffened square tubes. The relatively thin stiffened square tube, model STF-1, developed three symmetric wrinkles as shown in Fig. 3(a). It is obvious that the stiffened tube has the same folding pattern but a longer hinge length as compared with the corresponding unstiffened square tube model UNS-1 (Fig. 2(a)), in which five symmetric wrinkles were developed. However, as in model STF-2 (Fig. 3(b)), the hinge length of the deformed stiffened square tube increases to a certain value then decreases as the depth of the stiffener increases.

In the case of a relatively thick square tube, attachment of stiffeners can change the folding pattern. As an example, model STF-3 (Fig. 3(c)) deformed into a mixed pattern including one symmetric wrinkle and one extensional wrinkle, rather than two symmetric wrinkles as in the case of the corresponding unstiffened tube model UNS-3 (Fig. 2(c)). Moreover, from the deformation of model STF-4 (Fig. 3(d)), it is observed that attachment of axial stiffeners to the model UNS-5 (Fig. 2(e)) can make the square tube develop an extensional wrinkle before buckling into column buckling. However, in the case of model STF-5 of larger additional stiffeners, no progressive wrinkles can be developed before column buckling occurs, as shown in Fig. 3(e).

Model STF-6 has the same folding pattern as model STF-1, while model STF-7 and model STF-8 deformed into the same folding pattern as model STF-2 did. Therefore, the dimension of additional

(a) UNS-1

(b) STF-1

Fig. 4 Deformation and impact force diagram

stiffeners strongly influence the folding patterns of the square tube subjected to an impact load. This observation also agrees well with experimental results⁽¹⁰⁾.

2.2.2 Axial impact force and absorbing energy

Figure 4 shows the axial impact forces of unstiffened square tube UNS-1 and its corresponding stiffened square tube STF-1, and the progressive process of wrinkles. It is observed that two force peaks appeared during the formation of one wrinkle, and that the axial impact force of stiffened square tube STF-1 is larger than that of the corresponding unstiffened

square tube UNS-1.

A comparison was made between the FE estimation and theoretical prediction or experimental results in the point of view of structural effectiveness η and solidity ratio ϕ in the progressive buckling models. The structural effectiveness η and solidity ratio ϕ are defined for the unstiffened model as

$$\eta = \frac{P_m}{A\sigma_0} = \frac{P_m}{4CH\sigma_0}, \quad \phi = \frac{A}{A_e} = \frac{4H}{C} \quad (1)$$

and for the stiffened model as

$$\eta = \frac{P_m}{A\sigma_0} = \frac{P_m}{4(CH+TS)\sigma_0}, \quad \phi = \frac{A}{A_e} = \frac{4(CH+TS)}{C^2} \quad (2)$$

where, A is the cross-sectional area of the thin-walled cross-section and A_e is the cross-sectional area which is enclosed by the cross-section. σ_0 is the yielding stress.

The relationship between the structural effectiveness η and solidity ratio ϕ are given for the unstiffened square tube according to the rigid plastic theory⁽¹⁾ as

$$\eta = 0.95\phi^{2/3}[1 + (0.33 V_0/CD)^{1/q}], \quad (3)$$

and also for the axially stiffened square tube according to experimental results under static loads and lower speed impacts⁽¹⁰⁾ as

$$\eta = 8\phi \quad (\phi < 0.05); \quad \eta = (2\phi)^{1/2} \quad (\phi > 0.05), \quad (4)$$

where $q=4$, $D=6500$ for aluminum. The comparison results are plotted in Fig. 5. It is obvious that the FEM analysis agrees very well with the theoretical prediction for the unstiffened square tube. Numerical estimates of the stiffened square tube are smaller than experimental results. It is considered that material property of tubes, impact velocity and boundary conditions can cause the difference between the numerical estimations and experimental results. However, both of them qualitatively agrees very well with each other.

Figure 6 shows the processes of energy absorbing for six square tubes with the same cross-sectional dimensions of the thin-walled square tube ($C=60$

Fig. 5 Structural effectiveness η vs solidity ratio ϕ

mm, $H=2$ mm) but different dimensions of stiffeners. It is observed that stiffened square tubes can absorb more energy than the corresponding unstiffened square tube, and the larger the cross-sectional areas of the stiffener, the larger the absorbed energy.

For the thicker square tube UNS-3 and STF-3, the comparison of the absorbing energy shows the same tendency as above. Stiffened tube STF-4 absorbed more energy than the unstiffened tube UNS-5, which indicates that the stiffeners have a reinforcement effect to the square tube in which column buckling occurs. However, stiffened square tube STF-5 with larger stiffeners can not absorb more energy. Hence, the dimensions of the stiffener must be decided corresponding to the dimension of the square tube in order to increase the energy absorbing capacity.

3. Maximization of the Absorbing Energy

The crashworthiness maximization technique for tubular structures suggested in a previous paper⁽¹⁴⁾ is applied to maximize the absorbing energy of a square tube. An orthogonal array in the design-of-experiment is employed to assign analysis points and DYNA3D is adopted to simulate the complicate crushing behavior of tubular structures. Based upon the numerical results of the structural behavior, the response surface approximation technique is applied to generate the approximated function of the absorbing energy in terms of the design variables that are evaluated to be significant at high levels for the response by means of analysis-of-variance. The approximated function of the absorbing energy is maximized under constraints of a constant mass and the mean axial force by the numerical optimization program DOT. This optimization process is repeated until the given convergence condition is satisfied.

Fig. 6 Comparison of energy absorbing progress of stiffened square tubes with different dimensions of stiffeners

3.1 Formulation of the maximization problem

The mean width C and the wall thickness H of the square tube as well as the depth T and width S of the stiffener are taken as design variables. The maximization problem of the crushing energy absorption of the square tubes can then be stated as

$$f_1 = U_1(C, H) \text{ or } f_2 = U_2(C, H, T, S) \rightarrow \max, \quad (5)$$

subject to the mean axial impact force P_m and the upper and lower bounds of the design variables as

$$g = P_m/P_{ma} - 1 \leq 0, \quad (6)$$

$$C_{\min} \leq C \leq C_{\max}, \quad H_{\min} \leq H \leq H_{\max}, \\ T_{\min} \leq T \leq T_{\max}, \quad S_{\min} \leq S \leq S_{\max}, \quad (7)$$

where U and P_{ma} denote the absorbing energy during crushing and the allowable limit of the mean axial impact crushing force, respectively. The mass of the unstiffened and stiffened square tube are kept constant as

$$M_0 = 4\rho CHL \text{ or } M_0 = 4\rho(CH + TS)L \quad (8)$$

3.2 Maximization of the crushing energy absorption of an unstiffened square tube

The design and analysis conditions are the same as described in Section 2.1. The constraints of the constant mass M_0 and the allowable limit of the mean axial impact force P_{ma} are taken as 0.53 kg and 68.6 kN. The design levels of the design variables in the first design cycle are shown in Table 2 as design cycle 1. An orthogonal array L_9 is employed to arrange the

Table 2 Design levels for square tube

Design variables	Design levels in the 1st cycle			Design levels in the 2nd cycle			Design levels in the 3rd cycle		
C (mm)	60.	70.	80.	30.	45.	60.	30.	37.5	45.
H (mm)	2.	3.	4.	3.	4.	5.	3.	4.	5.

Table 3 Numerical solutions for square tubes (1st design cycle)

Design Point	C (mm)	H (mm)	Pattern W: wrinkle	Force (kN)	Energy (kJ)
1	60	2	5W	15.42	4.57
2	60	3	3W	30.89	5.82
3	60	4	2.5W	54.15	8.37
4	70	2	4W	16.68	4.41
5	70	3	3W	34.76	5.76
6	70	4	2W	58.72	7.62
7	80	2	3.5W	17.08	3.88
8	80	3	2.5W	37.45	5.48
9	80	4	2W	63.16	6.53

Table 4 Optimization results for square tube by DOT

Cycle No.	C mm	H mm	U_1 kJ	g
1	60.0	4.00	8.08	-2.1×10^{-2}
2	47.2	4.31	10.90	-8.6×10^{-5}
3	42.6	4.35	11.40	-3.5×10^{-4}

design points⁽¹⁴⁾ and the results of the structural analysis are shown in Table 3 for 9 design points. Based on the structural analysis, the functions of the absorbing energy and the mean axial impact force are approximated in the form of orthogonal polynomials as

$$U_1(C, H) = 5.9478 - 0.0668(C - 70.0) \\ + 0.0013\{(C - 70.0)^2 - 66.6667\} \\ + 1.2800(H - 3.0) - 0.09\{(H - 3.0)^2 - 0.6667\} \\ - 0.0175(C - 70.0)(H - 3.0) \\ - 0.0004(C - 70.0)\{(H - 3.0)^2 - 0.6667\} \quad (9)$$

$$P_m(C, H) = 36.3778 + 0.2345(C - 70.0) \\ + 0.0025\{(C - 70.0)^2 - 66.6667\} \\ + 20.2633(H - 3.0) + 2.50\{(H - 3.0)^2 - 0.6667\} \\ + 0.0862(C - 70.0)(H - 3.0) \\ + 0.0022(C - 70.0)\{(H - 3.0)^2 - 0.6667\} \\ + 0.0002\{(C - 70.0)^2 - 66.6667\}(H - 3.0) \\ + 0.0002\{(C - 70.0)^2 - 66.6667\}\{(H - 3.0)^2 - 0.6667\} \quad (10)$$

The approximated sub-problem is then optimized by using DOT. The optimum results of the first design cycle are shown in Table 4 as design cycle 1. It is found that the smaller the C or the larger the H , the more energy is absorbed. Thus, the design space is then improved by reducing the width C and increasing the thickness H for the second design cycle, as shown in Table 2 as design cycle 2. From the numerical results in two design cycles for the preassigned design space, it is observed that the absorbing energy increases to a maximum point and then decreases suddenly and rapidly due to the column buckling with the ratio C/H decreasing, as shown in Fig. 7. To obtain more precise approximated response surface, three new analysis points are inserted for the third design cycle, as shown in Table 2 as design cycle 3. The optimization results by DOT are shown in Table 4 as design cycle 3.

The simulation of the crushing deformation of the optimized square tube is shown in Fig. 8. The numerical estimations of the absorbing energy and mean

Fig. 7 Absorbing energy - C/H

Fig. 8 Simulation of the optimized square tube

axial impact force of the optimized square tube are 11.5 kJ and 65.2 kN, respectively. It is observed from the optimization process that in the square model, the optimum tubes without stiffeners have the dimensions so as to attain the allowable limit of the axial impact force and then to have the most number of symmetric progressive wrinkles avoiding column buckling. It is obvious that the energy absorbing capability of the optimized square tube is about the same, as compared with the maximization result of a cylindrical tube⁽¹⁴⁾ under the same constraints of the mean axial impact force and constant tube mass. But as the energy absorber, it can be considered that the energy absorbing process of a cylinder is more stable than that of a square tube because the square tube shows the instability of transition from the progressive buckling to column buckling during crushing, according to the experiment observation⁽¹⁾.

3.3 Maximization of the crushing energy absorption of a stiffened square tube

To stabilize the crushing process, the square tube is axially stiffened and optimized for absorbing more crushing energy. The design and constraint conditions are the same as the case of unstiffened square tubes. Three design levels of equal interval are selected for each design variable as shown in Table 5 as design cycle 1. From the numerical analysis results at each design point (Table 6), it is observed that the number of wrinkles decreased and the absorbing energy and mean axial impact force increased as compared to the numerical results of the corresponding unstiffened square tubes (Table 3). Based on the numerical results, the variance analysis for the mean impact force are performed as shown in Table 7, where *S.S*, *D.F* and *V*, respectively, denote the Sum of Squares, Degrees of Freedom and mean square, and F_0 is the variance ratio, ** means significant at 1% level. It is obvious that the thickness of the square tube affects the axial impact force much greater than other design variables do. Approximate functions of the absorbing energy and mean axial impact force are constructed as

Table 5 Design levels of each design variable

Design Variable	Design cycle 1			Design cycle 2		
	Design level 1	Design level 2	Design level 3	Design level 1	Design level 2	Design level 3
<i>C</i> (mm)	60	80	100	30	45	60
<i>H</i> (mm)	2	3	4	3	4	5
<i>T</i> (mm)	6	8	10	8	10	12
<i>S</i> (mm)	2	3	4	3	5	7

Table 6 Numerical solutions in design cycle 1 for stiffened square tubes

Design Point	<i>C</i> (mm)	<i>H</i> (mm)	<i>T</i> (mm)	<i>S</i> (mm)	Pattern	Force (kN)	Energy (kJ)
1	60	2	6	2	3.5W	18.69	5.44
2	60	3	8	3	2.5W	37.90	6.75
3	60	4	10	4	2W	72.69	9.60
4	80	2	8	4	1.5W	24.94	5.00
5	80	3	10	2	1.5W	47.07	7.00
6	80	4	6	3	1.5W	65.86	7.03
7	100	2	10	3	1W	25.24	4.53
8	100	3	6	4	1W	45.82	5.35
9	100	4	8	2	1W	74.91	6.18

Table 7 Variance analysis for mean impact force

Design Variable	<i>S.S</i>	<i>D.F</i>	<i>V</i>	F_0
<i>C</i>	46.439	1	46.439	7.37
1	46.426	1	46.426	7.36
2	0.0133	0		
<i>H</i>	3508.3	1	3508.3	556.43**
1	3484.38	1	3484.38	552.64**
2	23.92	0		
<i>T</i>	35.6737	1	35.6737	5.66
1	35.6728	1	35.6728	5.66
2	0.0009	0		
<i>S</i>	39.1912	1	39.1912	6.22
1	1.2881	0		
2	37.9031	1	37.9031	6.01
Error	25.22	4	6.305	
Total	3654.83	8		

$$\begin{aligned}
 U_2(C, H, T, S) = & 6.3200 \\
 & -0.0478(C-80.0) - 0.0001\{(C-80.0)^2 \\
 & -266.6667\} \\
 & +1.3067(H-3.0) - 0.0700\{(H-3.0)^2 - 0.6667\} \\
 & +0.2758(T-8.0) + 0.1288\{(T-8.0)^2 - 2.6667\} \\
 & +0.2217(S-3.0) + 0.3250\{(S-3.0)^2 - 0.6667\}
 \end{aligned} \quad (11)$$

$$\begin{aligned}
 P_m(C, H, T, S) = & 45.9022 \\
 & +0.1391(C-80.0) - 0.0002\{(C-80.0)^2 \\
 & -266.6667\} \\
 & +24.0983(H-3.0) + 3.4583\{(H-3.0)^2 - 0.6667\} \\
 & +1.2192(T-8.0) - 0.0054\{(T-8.0)^2 - 2.6667\} \\
 & +0.4633(S-3.0) + 4.3533\{(S-3.0)^2 - 0.6667\}
 \end{aligned} \quad (12)$$

The optimization result in the first design cycle are shown in Table 8 as design cycle 1. It is observed that the smaller the width *C* or the larger the thickness *H*, the depth and width of the stiffener, the more

Table 8 Optimized results of stiffened square tube by DOT

Design Cycle	C mm	H mm	T mm	S mm	U_1 kJ	g
1	60.00	3.87	10.00	4.00	9.45	2.2×10^{-7}
2	48.72	3.64	11.49	4.46	11.3	-1.0×10^{-6}

Fig. 9 Simulation of optimized stiffened square tube

energy is absorbed. Thus, the design space is improved by reducing the width C and increasing the thickness H , depth and width of the stiffener for the second design cycle, as shown in Table 5 as design cycle 2. The dimensions of the optimum square tube are obtained by DOT as shown in Table 8 as design cycle 2. The simulation of the impact deformation of the optimized stiffened square tube is shown in Fig. 9. The numerical estimations of the absorbing energy and the mean axial impact force are 11.4 kJ and 71.4 kN, respectively. Because of the reinforcement of the stiffener, the relatively long tubes can absorb much energy even in the case of column buckling, so the reduction of the absorbed energy is not rapid. On the other hand, the errors between the numerical estimations and optimization results by DOT (Table 8) are as small as 4.1% and -1.3% for the mean axial impact force and the absorbing energy, respectively. Hence, the precision of the approximated response surface of the absorbing energy is adequate enough to stop the optimization process. From the design optimization of the stiffened square tube, it is concluded that by attaching stiffeners to a square tube, the stability of the crushing behavior can be improved without losing the maximum amount of absorbed energy.

4. Conclusions

The axial impact crushing behavior of axially stiffened and unstiffened square tubes were studied by the explicit finite element method. The comparisons of the mean axial crushing force between numerical estimations and theoretical and experimental predic-

tion, and the qualitative comparisons of folding patterns between numerical simulations and experimental observations were made and discussed. Based on the numerical analyses, maximization problems of the dynamic crushing energy absorption of stiffened and unstiffened square tubes are solved respectively by using the crashworthiness maximization technique for tubular structures. As compared to the design optimization of cylindrical and unstiffened square tubes, it is observed that stiffeners give the same influence to the energy absorbing capability as the thickness does. In addition, it is concluded that adding stiffeners to a square tube can reduce the danger of transition from the progressive folding pattern to column buckling during crushing.

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