Study of Stress-Analyzing System by Speckle Photography*

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The point-by-point measurement of in-plane displacement was conducted by the pointwise filtering approach of speckle photography. Young's fringes analysis is based on the 2 D fast Fourier transform (FFT) and bounded Newton-Raphson algorithm. Many Young's fringes patterns were taken by a charge coupled devices (CCD) and analyzed by the image-processing system developed in my laboratory. The displacement obtained using speckle photography is not as smooth as that obtained by the finite-element analysis. Therefore, the displacement data were smoothed by 2 D FFT and the least-squares method. Then, stress-intensity factors of asymptotic solution derived by Sun and Jih were evaluated using the displacement data obtained using speckle photography by applying the least-squares method. From the nodal displacement calculated using the stress-intensity factor, the stress-strain analysis was conducted by using the latter algorithm of the finite element method (FEM). Finally, the stress-strain relationship was discussed in terms of comparison between the speckle photography and finite-element analysis.

Key Words: Experimental Stress Analysis, Stress Intensity Factor, Finite Element Analysis, Speckle Photography, 2 D Fast Fourier Transform, Newton-Raphson Algorithm, Young's Fringes, Image Processing

1. Introduction

A large amount of displacement data must be taken to investigate the singular field near the crack tip. The Young's fringes analysis procedure is usually time-consuming and tedious. Therefore, image processing is indispensable to carry out fringe analysis with high speed and accuracy. Numerous algorithms for automatic Young's fringes analysis have been proposed. In earlier works on fringe pattern analysis, Kaufmann et al.⁽¹⁾ used a linear photodiode array and Ineichen et al.⁽²⁾ used a television camera. Meynart^{(3),(4)} and Robinson⁽⁵⁾ digitized the 2 D fringe pattern by a television camera, and proposed an algorithm for analyzing the fringe spacing and orientation. In addition, several algorithms have been developed by Isacson and Kaufmann⁽⁶⁾, Huntley^{(7),(8)}, Chen and Taylor⁽⁹⁾ and Deng and Yamaguchi⁽¹⁰⁾. Robinson has presented a simple algorithm.

We presented an image-processing system^{(11),(12)} for the numerical processing of the diffraction pattern of Young's fringes from a double-exposure speckle photograph. The Young's fringes pattern is in the form of a circular halo modulated with straight uniformly spaced fringes. The fringe spacing and orientation are determined using only one Young's fringes pattern without any other diffraction halo patterns. This system is based on the 2 D FFT and iteration using the bounded Newton-Raphson algorithm of the fringe pattern with the resolution of $256 \times 256 \times 8$ bits. Therefore, it enables us to automatically analyze the specklegrams of a noisy and poor fringe pattern. The measurement of deformation using laser speckle photography can be carried out with high accuracy, reliability and speed. To date, the stress-intensity factors of homogeneous and dissimilar materials have been evaluated with high accuracy by measuring the displacement along the crack-line on the specklegram⁽¹³⁾⁻⁽¹⁵⁾.

By setting up the mesh pattern similar to that in the finite-element analysis, the displacement of each nodal point was measured using speckle photography. However, the displacement data scatter widely and

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are lost at some points. It is difficult to evaluate the stress-strain accurately using the raw displacement data. Hence, the displacement data were smoothed by the 2D FFT filtering and least-squares methods. Then, stress-intensity factors of the asymptotic solution derived by Sun and Jih⁽¹⁶⁾ were evaluated using the displacement data obtained from speckle photography by the least-squares method. The displacement of each node was calculated from the asymptotic solution using stress-intensity factors obtained by the previous process. From the nodal displacement, the stress-strain analysis was conducted by using the latter algorithm of FEM. Finally, the stress-strain relationship was discussed in terms of comparison between the speckle photography and finite-element analysis.

2. Theoretical Background

2.1 Light intensity of Young's fringes pattern

The light intensity of the diffraction pattern is given by the following equation⁽¹⁰⁾:

$$I_d(x, y) = I_0(x, y) + I_h(x, y) \times \left[1 + \gamma \cos\left(2\pi \frac{xu_x + yu_y}{\lambda L}\right) \right] \quad (1)$$

where $I_0(x, y)$ is the zeroth-order spot located at the central part of Young's fringes, $I_h(x, y)$ is the diffraction halo, x, y are coordinates of a point at the observation plane, γ is the fringe contrast which depends on speckle decorrelation, u_x, u_y are the speckle displacement components in the illuminated area of the specklegram, λ is the wavelength of the laser, and L is the focal length of the Fourier transform lens.

An example of an analyzed image and a typical diffraction spectrum of double-exposure specklegram is given in Fig. 1. The intensity distribution in Fig. 1 (b) corresponds to the case of $\gamma=1$ in the second term of Eq.(1), and the maxima of the Young's fringes are bounded on the diffraction halo.

The second term of Eq.(1) is rewritten as follows:

$$I(x, y) = \gamma I_h(x, y) \left[1 + \cos\left(2\pi \frac{xu_x + yu_y}{\lambda L}\right) \right] + (1 - \gamma) I_h(x, y) \quad (2)$$

The term in brackets in Eq.(2) for an $N \times N$ image is given by the following equation:

$$G(m, n) = 1 + \cos\left(2\pi \frac{k_m m + k_n n + C}{N}\right) \quad (3)$$

where k_m and k_n are the number of fringes along the x and y axes on the transform planes, respectively, and are proportional to the displacement components u_x and u_y , m and n are the number of column and row, respectively, C is a constant related to the position of the halo, and N is the number of pixels in

a row and column. The fringe spacings in the x and y directions are expressed as follows:

$$S_x = \frac{W_x M_s}{k_x M_c}, \quad S_y = \frac{W_y M_s}{k_y M_c} \quad (4)$$

where W_x and W_y are the number of pixels in the x and y directions, respectively. M_s and M_c are the recording magnifications of the specklegram and CCD camera, respectively. From S_x and S_y , the displacement components of each point of interest, u_x, u_y are obtained using the following equation.

$$u_x = \frac{\lambda L}{S_x}, \quad u_y = \frac{\lambda L}{S_y} \quad (5)$$

2.2 Smoothing of displacement data

The displacement data obtained using speckle photography show so little scattering that the error must be removed. Thus, the smoothing process was carried out on the displacement components (u, v) in the x and y directions. Two smoothing methods were employed. One was the masking method using the 2D FFT. Another was the line-smoothing method using the least-squares method. Figure 2 shows an example of power spectra obtained by the 2D FFT of 8×8

Fig. 1 An example of an analyzed image and a typical diffraction spectrum of a double-exposure specklegram

displacement data. In conventional image processing, the smoothing is carried out by removing the high-frequency component in the FFT. However, even the main data are destroyed in the case of small pixel data. Therefore, the frequency components were omitted except on the main axes, as shown in Fig. 2. This method enables us to remove the large error without destroying the main data.

Since an 8×8 -filtering mask was too big for smoothing the displacement data near the crack tip, a 4×4 -filtering mask was employed as shown in Fig. 3. A 2 D FFT of 4×4 displacement data was carried out, and then a 2 D inverse FFT was carried out by setting the real and imaginary parts of the low power spectrum at zero. This filtering process was performed by moving the mask step by step from the left to the right, and then was performed downward in turn. This process was carried out by moving the mask over all data one to four times. The protruding data were removed without destroying the main frame by this process. A small error of data on the mesh line in Fig. 3 was removed by the polynomial approximation of the least-squares method. In addition, the lost data can be complemented using the least-squares method during image processing.

2.3 Stress analysis

The stress-intensity factor was evaluated from the smoothed displacement data by using the asymptotic solution derived by Sun and Jih⁽¹⁶⁾. Then, the displacement at each node was estimated from the stress-intensity factor using the asymptotic solution. Finally, the stress-strain values were calculated by using the latter algorithm of FEM.

The displacement of asymptotic solution is evaluated by the following equations⁽¹⁶⁾:

Fig. 2 Power spectra and omitted points

$$u_j = \frac{K_I \sqrt{2\pi r}}{4\pi\mu_j \cosh(\pi\epsilon)} \left[\kappa_j \omega_j h_{11} - \frac{1}{\omega_j} h_{12} + \omega_j h_{13} \right] + \frac{K_{II} \sqrt{2\pi r}}{4\pi\mu_j \cosh(\pi\epsilon)} \left[\kappa_j \omega_j h_{21} - \frac{1}{\omega_j} h_{22} + \omega_j h_{23} \right] \quad (6)$$

$$\nu_j = \frac{K_I \sqrt{2\pi r}}{4\pi\mu_j \cosh(\pi\epsilon)} \left[\kappa_j \omega_j h_{21} - \frac{1}{\omega_j} h_{22} - \omega_j h_{23} \right] + \frac{K_{II} \sqrt{2\pi r}}{4\pi\mu_j \cosh(\pi\epsilon)} \left[-\kappa_j \omega_j h_{11} + \frac{1}{\omega_j} h_{12} + \omega_j h_{13} \right] \quad (7)$$

$$\epsilon = \frac{1}{2\pi} \ln \left[\left(\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2} \right) / \left(\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1} \right) \right] \quad (8)$$

$$\Theta = \epsilon \ln \left(\frac{r}{2a} \right) + \frac{\theta}{2} \quad (9)$$

$$\kappa_j = \begin{cases} 3 - 4\nu_j & (\text{plane strain}) \\ (3 - \nu_j)/(1 + \nu_j) & (\text{plane stress}) \end{cases} \quad (10)$$

μ = shear modulus

ν = Poisson's ratio

$$\omega_1 = e^{-\epsilon(\pi - \theta)} \quad (11)$$

$$\omega_2 = e^{\epsilon(\pi + \theta)} \quad (12)$$

$$h_{11} = \frac{1}{1 + 4\epsilon^2} [\cos(\theta - \Theta) - 2\epsilon \sin(\theta - \Theta)] \quad (13)$$

$$h_{12} = \frac{1}{1 + 4\epsilon^2} [\cos \Theta + 2\epsilon \sin \Theta] \quad (14)$$

$$h_{13} = \sin \theta \sin \Theta \quad (15)$$

$$h_{21} = \frac{1}{1 + 4\epsilon^2} [\sin(\theta - \Theta) + 2\epsilon \cos(\theta - \Theta)] \quad (16)$$

$$h_{22} = \frac{1}{1 + 4\epsilon^2} [-\sin \Theta + 2\epsilon \cos \Theta] \quad (17)$$

$$h_{23} = \sin \theta \cos \Theta \quad (18)$$

where subscript j refers to the material of the upper or lower part of the interface, a is crack length, r is the distance from the crack tip, and θ is an angle in the counterclockwise direction.

$$\{\epsilon\} = [B]\{\delta\}^e \quad (19)$$

$$\{\sigma\} = [D]\{\epsilon\} \quad (20)$$

$$[D] = \frac{E}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1 - \nu)/2 \end{bmatrix} \quad (\text{plane stress})$$

Fig. 3 Smoothing method by 2 D FFT mask

(21)

$$[D] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \times \begin{bmatrix} 1 & \nu/(1-\nu) & 0 \\ \nu/(1-\nu) & 1 & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2(1-\nu)} \end{bmatrix} \quad (22)$$

(plane strain)

Here, $[B]$ is the strain matrix and $\{\delta\}^e$ is the nodal displacement. The nodal displacement, strain and stress are derived from Eqs. (6), (7), (19) and (20), respectively.

3. Experiment

The edge-cracked specimen of 15 mm thickness made of acrylic resin is shown in Fig. 4. Young's modulus and Poisson's ratio are 3.06 GPa and 0.38, respectively. First, the upper and lower parts of the specimen as shown in Fig. 4 were machined separately and then two pieces were bonded. A crack was introduced by introducing a Teflon sheet of 0.02 mm thickness between the two pieces during bonding. Then, the surface roughness was controlled by polishing with #500 emery paper. Mode I loading was applied to the specimen by pulling two left-hand holes up and down.

Figure 5 shows the schematic of the optical arrangement. A 50 mW He-Ne gas laser was used as the light source. A camera was used to record the double-exposed specklegram of the specimen being deformed between exposures. The experimental set-up requires no special vibration isolation. Furthermore, the illumination direction and specimen size are

Fig. 4 Specimen configuration

arbitrary.

The displacement at any point on the specimen was obtained by pointwise filtering of the specklegram using a thin laser beam. A CCD camera was used to record the resulting fringe patterns. For a full field analysis, the specklegram was mounted on an X-Y stage driven by stepping motors controlled by microcomputer. The displacement data were taken in the area of 10 mm (H) $\times$ 12.8 mm (W) around the crack tip. The positions of scan points in the y direction were 0.05, 0.1, 0.25, 0.4, 0.65, 0.9, 1.25, 1.6, 2.05, 2.5, 3.05, 3.6, and 6.4 mm from the crack tip. The positions of scan points in the x direction were 0.1, 0.4, 0.9, 1.6, 2.5, 3.6, and 6.4 mm from the crack tip. Displacement analysis was carried out using an engineering workstation.

4. Model for Analysis

Figure 6 shows the model for the 3D finite-element analysis. One-half portion of the specimen was modeled by the symmetry with respect to the midsection of the specimen by using the isoparametric elements of 20 nodes. The layer division ratio was 1:2:3, and the number of elements and nodes was 1440, and 7801, respectively. The stress-intensity factor was evaluated by the following three approaches: virtual crack extension method (VCEM)⁽¹⁷⁾ modified crack-closure integral (MCCI)⁽¹⁸⁾ and displacement extrapolation⁽¹⁵⁾. The stress-intensity factor at the free surface was evaluated by assuming the plane strain or plane stress condition.

5. Results and Discussion

5.1 Smoothing process

The most important thing in the smoothing pro-

Fig. 5 Schematic of optical arrangement

Fig. 6 3D FEM Model

cess is to remove the large error without destroying the original data. The smoothing process should not destroy the displacement data near the crack tip. Figure 7 shows the 3D representation of the displacement component in the y direction. The x - y plane is the free surface of the specimen, and the frame shows the measurement region $10\text{ mm}(H) \times 12.8\text{ mm}(W)$. Let the crack tip and the positive direction of displacement be the origin of the coordinates and the upper direction of the crack line, respectively. A considerable scattering of raw data in the experiment is seen in Fig. 7(a). Therefore, it is not easy to evaluate the stress and strain from this data with high accuracy. Figure 7(b) shows the distribution of displacement after smoothing by the 2D FFT and least-squares method. As the smoothed distribution without any large error can be seen in this figure, the smoothing by the two methods is very useful. Figure 7(b) agrees well with Fig. 7(c) obtained by the finite-element analysis.

5.2 Distribution of stress-intensity factor along the crack front

Figure 8 shows the distribution of stress-intensity factor along the crack front obtained by the finite-element analysis. Here, Z is the distance from the midsection and B is the thickness of the specimen. The stress-intensity factor was evaluated by assuming the plane strain condition except for the free surface. SIFV, SIFM, Plane strain and Plane stress in the legend denote the stress-intensity factors obtained by VCEM, MCCI, plane strain and plane stress analyses of displacement extrapolation, respectively. The stress-intensity factors obtained by the VCEM, MCCI and displacement extrapolation are nearly the same and uniform from the midsection to a 3 mm

(a) Before smoothing

(b) After smoothing

(c) FEM

Fig. 7 3D representation of displacement in y direction

Fig. 8 Distribution of stress intensity factor along the crack front

Fig. 9 Contour map of σ_y (FEM, $2Z/B=0.25$)

depth. They decrease gradually toward the free surface. The stress-intensity factors near the free surface evaluated by the VCCM and MCCI are located between those obtained by the displacement extrapolation assuming the plane strain and plane stress conditions. It is considered that the stress condition at the free surface is somewhere between the plane strain and plane stress conditions.

5.3 Evaluation of stress-intensity factor

The stress-intensity factor at the free surface was $347 \text{ kN/mm}^{3/2}$ in the case of the displacement extrapolation of finite-element analysis, assuming the plane strain condition. That of the asymptotic solution obtained from the displacement data of speckle photography was $343 \text{ kN/mm}^{3/2}$. The latter was about 1% lower than the former, but both values were in good agreement. The stress-intensity factor can be estimated with high accuracy provided that the dis-

(a) Plane strain

(b) Plane stress

Fig. 10 Contour map of σ_y (EXP, $2Z/B=1.0$)

placement is measured with good accuracy by the experiment.

5.4 Stress distribution

Figure 9 shows the contour map of σ_y (normal stress in the y direction) within $2.5 \text{ mm} (W) \times 2.5 \text{ mm} (H)$ at $2Z/B=0.25$ obtained by the finite-element analysis. The distribution was nearly the same from the midsection to a 3 mm depth, as shown in the distribution of stress-intensity factor along the crack front. Figure 10 shows the contour map of σ_y at the free surface obtained by speckle photography. The stress-state obtained by assuming the plane strain condition is higher than that obtained by assuming the plane stress condition. The stress-intensity factor at the free surface obtained by assuming the plane strain condition coincides with that at the midsection of the specimen, as shown in Fig. 8. This was also demonstrated

Fig. 11 Contour map of σ_y (FEM, $2Z/B=0.917$)

Fig. 12 Contour map of weighted σ_y (EXP, $2Z/B=1.0$)

experimentally from the result that the distribution of Fig. 9 is nearly the same as that of Fig. 10 (a). Figure 11 shows the contour map of σ_y at a 0.625 mm depth obtained by the finite-element analysis. The stress-state is similar to that of the plane stress condition, but slightly higher. Then, the weighted average was calculated by using the ratio of plane strain to plane stress of 1 : 3. The contour map of weighted σ_y shown in Fig. 12 agrees very well with Fig. 11 obtained by the finite-element analysis. It is considered that the stress-state at the free surface is slightly higher than the plane stress condition.

Figure 13 shows the variation of σ_y along the x -axis at the center of elements that are close to the extension of the crack line. The σ_y value of plane strain is the largest. The σ_y value of weighted average agrees with that of finite-element analysis 1.2 mm inside the crack tip. That of plane stress is slightly

Fig. 13 Variation of σ_y along the x -axis

low. At a distance of 1.2 mm outside the crack tip, the σ_y value of finite-element analysis is slightly lower than the other values. The stress state at the free surface is close to that of plane stress, but is slightly higher.

6. Conclusion

1. The 2 D FFT filtering and least-squares method enable us to remove the large error without destroying the main data and to smooth the displacement data.
2. The stress-intensity factor was evaluated with high accuracy using speckle photography.
3. Stress and strain closely corresponding to those of the finite-element analysis can be obtained by using the asymptotic solution derived by Sun and Sih.

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