Numerical Modeling and Analysis of Journal Bearing with coupled Elastohydrodynamic Lubrication and Flexible Multibody Dynamics

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This study deals with the modeling and analysis method for the elastohydrodynamic lubrication system such as journal bearing coupled with flexible multibody dynamics (or Multi-Flexible-Body Dynamics, MFBD) in order to analyze dynamic bearing lubrication characteristics such as pressure distribution and oil film thickness. In order to solve coupled fluid-structure interaction system, this study uses two main parts. The one is the MFBD solver and the other is elastohydrodynamic module. The elasto-hydrodynamic lubrication module developed in this study transmits the force and torque data to the MFBD solver which can solve general dynamic systems that include lots of rigid and flexible bodies, joints, forces, and contact elements. And then, the MFBD solver analyses the position, velocity, and accelerations of the flexible multibody system with the pressure distribution results of the elasto-hydrodynamic module. And the MFBD solver transmits the position and velocity information to the elasto-hydrodynamic solver continuously. Moreover, other functions such as mesh grid control and oil hole and groove effects are implemented. Finally, numerical examples for bearing lubrication systems are demonstrated.

Keywords:  
Elastohydrodynamics, Lubrication, Journal Bearing, Fluid-Structure Interaction, Multi-Flexible-Body Dynamics
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ELASTOHYDRODYNAMIC LUBRICATION AND FLEXIBLE MULTIBODY DYNAMICS

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ABSTRACT
This study deals with the modeling and analysis method for the elastohydrodynamic lubrication system such as journal bearing coupled with flexible multibody dynamics (or Multi-Flexible-Body Dynamics, MFBD) in order to analyze dynamic bearing lubrication characteristics such as pressure distribution and oil film thickness. In order to solve coupled fluid-structure interaction system, this study uses two main parts. The one is the MFBD solver and the other is elastohydrodynamic module. The elastohydrodynamic lubrication module developed in this study transmits the force and torque data to the MFBD solver which can solve general dynamic systems that include lots of rigid and flexible bodies, joints, forces, and contact elements. And then, the MFBD solver analyses the position, velocity, and accelerations of the flexible multibody system with the pressure distribution results of the elastohydrodynamic module. And the MFBD solver transmits the position and velocity information to the elastohydrodynamic solver continuously. Moreover, other functions such as mesh grid control and oil hole and groove effects are implemented. Finally, numerical examples for bearing lubrication systems are demonstrated.

1. INTRODUCTION
The journal bearings, which is the one of the widely used machine elements, transmit the power while reducing the friction and resisting the external loads. In particular, in the internal combustion engine which is frequently used for power generation, the various and lots of journal bearings are used between the piston, piston pin, connecting rod, crankshaft, and engine block. These journal bearings, which is under the alternating loads caused by the gas forces of the internal engines, guarantee the smooth operation of the engine and are tightly related to the durability of the engine system. Recently, in order to achieve the high-performance output and to reduce the engine weight, the importance of the bearing lubrication analysis is increasing (Taylor 1993, Oh and Goenka 1985, Labouff and Booker 1985).

The research on the lubrication characteristics and performance for journal bearing has been studied widely in the area of tribology (Nair, Sinhasan, and Singh 1987, Makino and Koga 2002). The lubrication study for the bearing is based on the Reynolds equations (Reynolds 1886) which is related to the thickness and pressure of fluid film generated by the relative motion of objects. In particular, the study on the trajectory by the relative motion of the journal bearing such as the engine bearing which is resisting the alternating loads was first tried by Ott (1948) and Hahn (1957). Dowson and Higginson (1959) had studied about the numerical solutions for the elastohydrodynamic problems. And Hamrock and Dowson (1976) had studied about the oil film thickness and the relations between contacts. In order to estimate the lubrication film characteristics such as the oil film thickness, pressure, power loss, and flow rate, the analysis using the elastohydrodynamics lubrication is needed. In particular, in order to calculate the relative displacements between bearing and journal, the theory for flexible multibody dynamics (or MFBD) is also needed (Peiskammer et al. 2002, Riener et al. 2001, Choi 2009).

Generally, elastohydrodynamic lubrication can be classified by two types, which are based on the relation of surface roughness and oil film thickness. One is the full-film lubrication. It has been widely used when the lubricant film is sufficiently thick so that there is no significant asperity contact. In this case, the pressure is only governed by Reynolds equation which is first established by

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Reynolds (1886). The other is the mixed lubrication. When the lubricant film is not enough to thick, the asperity contacts between two bodies can be occurred (Zhu and Cheng 1988, Greenwood and Tripp 1970-1971). As a result, the pressure by the fluid flow and the pressure by the asperity contact should be considered together. Therefore, in mixed lubrication region, the total pressure can be treated by the sum of the pressures induced by the fluid flow and the asperity contact.

In order to consider the full-film and mixed regions together, this paper uses the Greenwood and Tripp’s asperity contact model (1971) and Reynolds equation to obtain the hydrodynamic pressure. The oil hole and groove effects are considered by applying the pressure boundary conditions. Also the dynamic viscosity of oil is considered as the function of the pressure by using the Barus law (Dowson and Higginson 1977).

In the Section 2, the MFBD theory used in this study is introduced. The EHD governing equations are introduced in Section 3 and the analysis procedure for fluid-structure interactions is explained in Section 4. A numerical example is discussed in Section 5, and finally the conclusions are in Section 6.

2. Multi-Flexible-Body Dynamics (MFBD)

The MFBD formulation which is used in this study is described well in Choi (2009). In this section, the brief formulations for MBD and MFBD are introduced.

2.1 MBD Formulation

The coordinate systems for two contiguous rigid bodies in 3D space are shown in Fig. 1. Two rigid bodies are connected by a joint, and an external force \( F \) is acting on the rigid body \( j \). The X-Y-Z frame is the inertial or global reference frame and the \( x'-y'-z' \) is the body reference frame with respect to the X-Y-Z frame. The subscript \( i \) means the inboard body of body \( j \) in the spanning tree of a recursive formulation (Bae et al. 2001). And, in this section, the subscript \( j \) can be replaced with the subscript \( (i+1) \).

Velocities and virtual displacements of the origin of body reference frame \( x'-y'-z' \) with respect to the global reference frame X-Y-Z, respectively, defined as

\[
\begin{bmatrix}
\dot{r} \\
\omega
\end{bmatrix}
\quad \text{(1)}
\]

and

\[
\begin{bmatrix}
\delta \dot{r} \\
\delta \pi
\end{bmatrix}
\quad \text{(2)}
\]

Their corresponding quantities with respect to the body reference frame \( x'-y'-z' \) are, respectively, defined as

\[
\begin{bmatrix}
\dot{r}' \\
\omega'
\end{bmatrix} = \begin{bmatrix} A^T \dot{r'} \ A^T \omega \end{bmatrix}
\quad \text{(3)}
\]

and

\[
\begin{bmatrix}
\delta \dot{r}' \\
\delta \pi
\end{bmatrix} = \begin{bmatrix} A^T \delta \dot{r'} \ A^T \delta \pi \end{bmatrix}
\quad \text{(4)}
\]

where \( A \) is the orientation matrix of the \( x'-y'-z' \) frame with respect to the X-Y-Z frame.

The recursive velocity equations for a pair of contiguous bodies is obtained as

\[
Y_j = B_j^0 Y_i + B_j^0 q_i \quad \text{(5)}
\]

where \( Y \) is the combined velocity of the translation and rotation as defined in Eq. (3) and \( B_j^0 \) and \( B_j^1 \) are defined as follows:

\[
\begin{bmatrix}
A_j^0 & 0 \\
0 & A_j^0
\end{bmatrix}
\begin{bmatrix}
I \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{s}_0 + \delta s' \ A_j \dot{s}' \ A_j^T \\
0
\end{bmatrix}
\quad \text{(6)}
\]

and

\[
\begin{bmatrix}
A_j^0 & 0 \\
0 & A_j^0
\end{bmatrix}
\begin{bmatrix}
I \\
0
\end{bmatrix}
\begin{bmatrix}
d_j' \ A_j \ A_j^T \ H_j' \\
0
\end{bmatrix}
\quad \text{(7)}
\]

It is important to note that matrices \( B_j^1 \) and \( B_j^2 \) are only functions of \( q_i \). Similarly, the recursive virtual displacement relationship is obtained as follows:

\[
\delta Z_j = B_j^0 \delta Z_i + B_j^0 \delta q_i \quad \text{(8)}
\]

If the recursive formula in Eq. (5) is respectively applied to all joints along the spanning tree, the following relationship between the Cartesian and relative generalized

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velocities can be obtained:

\[ Y = Bq \]  

(9)

where \( B \) is the collection of coefficients of the \( \dot{q}_i \) and

\[ Y = \begin{bmatrix} Y_0^T, Y_1^T, Y_2^T, \ldots, Y_n^T \end{bmatrix}^T \]  

(10)

and

\[ \dot{q} = \begin{bmatrix} Y_0^T, \dot{q}_1^T, \dot{q}_2^T, \ldots, \dot{q}_{n+1,n}^T \end{bmatrix}^T \]  

(11)

where \( nc \) and \( nr \) denote the number of the Cartesian and relative generalized coordinates, respectively. The Cartesian velocity \( Y \in \mathbb{R}^{nc} \) with a given \( q \in \mathbb{R}^{nr} \) can be evaluated either by using Eq. (9) obtained from symbolic substitutions or by using Eq. (5) with recursive numerical substitution of \( Y_i \).

It is often necessary to transform a vector \( G \) in \( \mathbb{R}^{nc} \) into a new vector \( g = B^t G \) in \( \mathbb{R}^{nr} \). Such a transformation can be found in the generalized force computation in the joint space with a known force in the Cartesian space. The virtual work done by a Cartesian force \( Q \in \mathbb{R}^{nc} \) is obtained as follows:

\[ \delta W = \delta Z^T Q \]  

(12)

where \( \delta Z \) must be kinematically admissible for all joints in the system. Substitution of \( \delta Z = B \delta q \) into Eq. (12) yields

\[ \delta W = \delta q^t B^t Q = \delta q^t Q' \]  

(13)

where \( Q' = B^t Q \).

The equations of motion for a constrained mechanical system (García de Jalón et al. 1986) in the joint space (Wittenburg 1977) have been obtained by using the velocity transformation method as follows:

\[ F = B^t ( M \ddot{Y} + \Phi^e_\lambda \ddot{\lambda} - Q ) = 0 \]  

(14)

where \( \Phi \) and \( \lambda \), respectively, denote the cut joint constraint and the corresponding Lagrange multiplier. \( M \) is a mass matrix and \( Q \) is a force vector including the external forces in the Cartesian space.

### 2.2 MFBD Formulation

The equation of motion for the rigid body can be expanded from the Eq. (14) as follows:

\[ F' = B^t \left( M \ddot{Y} + \Phi^e_\lambda \ddot{\lambda} + \Phi^e \lambda^5 - Q' \right) = 0 \]  

(15)

where, the superscript \( r \) means the quantity for the rigid body. The superscripts \( rr \) means the quantities between rigid bodies and the superscript \( er \) means the quantities between a flexible nodal body and a rigid body. The constraints equations between rigid bodies are expressed as the function of rigid body generalized coordinates \( q^r \) as follows:

\[ \Phi^r = \Phi^r ( q^r, t ) \]  

(16)

Similarly, we can derive the equations of motion for the flexible body as follows:

\[ F' = M' \ddot{q}' + \Phi^e_\lambda \ddot{\lambda}' + \Phi^e \lambda^5 - Q' = 0 \]  

(17)

where, the superscript \( e \) means the quantities for the flexible nodal body and \( q' \) is the generalized coordinate for the flexible nodal bodies. The superscript \( ee \) means the quantities between flexible nodal bodies and the superscript \( er \) means the quantities between a flexible nodal body and a rigid body. The forces \( Q' \) between flexible nodal bodies can be expressed as the sum of the element forces and applied forces such as gravity or contact forces as follows:

\[ Q' = Q^\text{element} + Q^\text{applied} \]  

(18)

Also, for the flexible body joint constraints between a flexible nodal body and a virtual rigid body, we can express the \( \Phi^e \) as follows:

\[ \Phi^e = \Phi^e ( q^e, q^r, t ) \]  

(19)

Similarly, the constraint equations \( \Phi^r \) between flexible nodal bodies can be expressed as Eq. (20).

\[ \Phi^r = \Phi^r ( q^r, t ) \]  

(20)

Finally, we can make the whole system matrix for the MFBD problems as Eq. (21) and we can solve Eq. (21) using the sparse matrix solver for the incremental quantities. The equations of motion for the rigid body can be expanded from the Eq. (14) as follows:

\[
\begin{bmatrix}
\frac{\partial F'}{\partial q^r} & \frac{\partial F'}{\partial q^r}^T & 0 & 0 \\
\frac{\partial F'}{\partial q^e} & 0 & 0 & 0 \\
\frac{\partial F'}{\partial q^e} & 0 & 0 & 0 \\
\frac{\partial F'}{\partial q^e} & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\Delta q^r \\
\Delta \lambda^e \\
\Delta \lambda^r \\
\end{bmatrix} =
\begin{bmatrix}
F' \\
F' \\
F' \\
F' \\
\end{bmatrix}
\]  

(21)
3. Elastohydrodynamic (EHD)

3.1 Governing Equation of Hydrodynamics

Fig. 2 shows a schematic diagram for the relative motion and dimensions between a bearing and a journal.

If we consider the journal radius \( R \) and the clearance \( C_r \) in the journal bearing lubrication problems of the laminar flow, following assumptions can be used.

\[
\left( \frac{C_r}{R} \right)^2 \ll 1
\]

\[
\text{Re} \frac{C_r}{R} \ll 1 \quad \text{(Generally, \ Re} \frac{C_r}{R} = O(0.001))
\]

Under the above assumptions, the governing equation for the fluid flow becomes the Couette-Poiseuille flow equation (Sabersky et al. 1989, Gohar 2001). And then, if the mass or flow rate conservation law is applied, the Reynolds’ equation for the hydrodynamic problems can be expressed as follows:

\[
\frac{\partial}{\partial x} \left( \frac{1}{\text{Re}} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\text{Re}} \frac{\partial p}{\partial z} \right) = 12V + 6U \frac{\partial H}{\partial x} + 6W \frac{\partial H}{\partial z}
\]

\[
\Gamma = \frac{H}{\mu}
\]

Here, \( U, V, \) and \( W \) are \( x, y, \) and \( z \) relative velocities of the journal surface (at \( y = H \)) based on the bearing, respectively. \( H \) and \( \mu \) are the oil film thickness and the dynamic (or absolute) viscosity, respectively. In this study, the oil film thickness is defined as follows:

\[
h(\theta) = C_r - e_y \cos \theta - e_z \sin \theta
\]

Also, as shown in Eq. (23), the dynamic viscosity can be varied in space, it depends on the oil pressure. So, in order to consider the pressure-viscosity relations, this paper uses the Barus Law (Dowson and Higginson 1977) as Eq. (25).

\[
\mu = \mu_0 e^{\alpha p}
\]

Here, \( \mu_0 \) is a dynamic viscosity at the atmosphere state, \( \alpha \) is the pressure-viscosity coefficient which is related with the lubricant properties.

3.2 Asperity Contact Model

As shown in Fig. 3, when the oil film thickness is not enough to thick compared to the surface roughness, the contact pressure resulting from the asperities between bodies should be considered.

\[
p_a(H) = KEF_{5/2}(H/\sigma_r)
\]

\[
F_{5/2} \left( \frac{H}{\sigma_r} \right) = \begin{cases} 
4.4086 \times 10^{-5} \left( 4 - \frac{H}{\sigma_r} \right)^{6.004}, & \text{if} \quad \frac{H}{\sigma_r} < 4 \\
0, & \text{otherwise}
\end{cases}
\]

Here, \( K \) is the elastic factor and \( \sigma_r \) is the root mean square (rms) of the asperity summit heights and \( E' \) is the composite elastic modulus, which is defined from the material properties of contacting surfaces, as Eq. (27).

\[
\frac{1}{E'} = \frac{1}{E_1} + \frac{1}{E_2}
\]

where, the subscripts 1 and 2 mean the bearing and journal bodies, respectively. \( E \) is the Young’s modulus and \( \nu \) is the Poisson’s ratio.
4. Fluid-Structure Interactions

In this study, as shown in Fig. 4, EHD and MFBD solvers are used together to analyze the lubrication and flexible multibody dynamics characteristics of the journal bearing. First, the pressure distributions are evaluated from the hydrodynamic lubrication analysis. Then, the calculated pressure field and resulting forces and torques are transmitted into the MFBD solver. In the MFBD solver, the transmitted pressure, force and torque data are used as the external forces or torques acting on the journal and bearing bodies. Then, from the MFBD analysis, the positions and velocities of all the related bodies are calculated. From these position and velocity data, the oil film thickness is evaluated and transferred to the EHD solver. Like these procedures, in order to analyze the lubrication and dynamic characteristics of the journal bearing, EHD and MFBD solvers are used iteratively.

![Fluid-Structure Interactions](image)

Figure 4. FLUID-STRUCTURE INTERACTIONS BETWEEN EHD AND MFBD SOLVER.

Also, in order to support the general-purpose EHD solution, the groove and oil hole effects, which are treated as the pressure boundary condition in EHD solver, are implemented.

5. NUMERICAL EXAMPLES

In order to implement the EHD module with MFBD solver together, this study used the Recurdyn™ (2010) MFBD environment.

To validate the numerical results of this study, the experimental and numerical analysis results of Nakayama et al. (2003) are used. The detailed explanation about the numerical model is described well in Nakayama et al. (2003). Fig. 5 shows the numerical model and measurement points of the oil film thickness. In the numerical model, in order to check the effects on the external loads, various external loads such as 100N, 500N, 1000N, 1500N, and 2000N are applied. Also, 3570 rpm is used for the rotational speed of crank shaft.

Table 1 shows the simulation parameters used in the numerical model. And the Fig. 6 shows the numerical results and the results are compared with the results of EHL solutions of Nakayama et al. (2003). As shown in Fig. 6, the numerical results of current study also shows a good agreement with the EHL results at center position.

![Numerical Model](image)

Figure 5. NUMERICAL MODEL FOR JOURNAL BEARING BETWEEN THE CONNECTING ROD AND CRANK SHAFT.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh size (circum. × depth)</td>
<td>60×5</td>
</tr>
<tr>
<td>Journal Diameter</td>
<td>45 mm</td>
</tr>
<tr>
<td>Bearing Width</td>
<td>15.6 mm</td>
</tr>
<tr>
<td>Lubrication Gap</td>
<td>0.068 mm</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>3.5e-2 Pa·s</td>
</tr>
<tr>
<td>Pressure-Viscosity Coeff.</td>
<td>1.28e-8 Pa⁻¹</td>
</tr>
<tr>
<td>Roughness</td>
<td>5.0e-4 mm</td>
</tr>
<tr>
<td>Composite Elastic Modulus</td>
<td>206000 MPa</td>
</tr>
<tr>
<td>Elastic Factor</td>
<td>3.0e-3</td>
</tr>
</tbody>
</table>

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6. CONCLUSIONS

In this study, the elastohydrodynamic lubrication system coupled with flexible multibody dynamics (or MFBD) is developed in order to analyze the dynamic bearing lubrication characteristics such as the pressure distribution and oil film thickness. In order to solve coupled fluid-structure interaction system, this study uses two main parts. The one is the MFBD solver and the other is elastohydrodynamic module. The elastohydrodynamic lubrication module developed in this study transmits the force and torque data to the MFBD solver which can solve general dynamic systems. And then, the MFBD solver analyzes the positions and velocities of the flexible multibody system with the pressures, forces and torques results of the elastohydrodynamic module. And the MFBD solver transmits the position and velocity data, which can evaluate the oil film thickness, to the EHD solver. These kinds of procedures are used iteratively between MFBD and EHD solvers. Moreover, other functions such as mesh grid control and oil hole and groove effects are implemented. Finally, the numerical results are validated and compared with other experimental and numerical solutions by using the journal bearing example between the connecting rod and the crank shaft.

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