ABSTRACT

This presentation gives a brief review of past and present challenges in the multibody modeling and the dynamic analysis of wheeled vehicles. The emphasis of the presentation is on the weakest link of a vehicle model—the tires. The evolution of tire models, like any other component in a multibody model, has experienced major advances during the past three decades. Early tire models were comprised of simple analytical expressions or they were purely empirically based curve fitting procedures. Recent models are more complex and have moved towards representing the structure of a tire in more detail. In this presentation we investigate the possibility of employing existing high-resolution finite-element tire models in multibody dynamics.

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Keynote Lecture

CHA水资源 IN MULTIBODY DYNAMIC ANALYSIS OF VEHICLES

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This presentation gives a brief review of past and present challenges in the multibody modeling and the dynamic analysis of wheeled vehicles. The emphasis of the presentation is on the weakest link of a vehicle model—the tires. The evolution of tire models, like any other component in a multibody model, has experienced major advances during the past three decades. Early tire models were comprised of simple analytical expressions or they were purely empirically based curve fitting procedures. Recent models are more complex and have moved towards representing the structure of a tire in more detail. In this presentation we investigate the possibility of employing existing high-resolution finite-element tire models in multibody dynamics.

1. INTRODUCTION
It would not be an exaggeration to claim that one of the main reasons for the inception of the discipline that we call multibody dynamics was our interest in analyzing the performance of wheeled vehicles. Almost four decades ago, vehicle models consisted of highly simplified one-dimensional mass-spring components. But today, due to the evolution of multibody dynamics, a vehicle model can contain deformable bodies, joints with bushing, controls for handling and stability, or other sophisticated features. Despite all the advances in multibody dynamics, the weakest component in a vehicle model today is the same as it was three decades ago—the modeling of tires and the accurate prediction of ground-tire interaction forces and moments.

Research and development in multibody dynamics began with the assumption of rigidity for the bodies. Tires are not rigid bodies and the interaction between a tire and the ground is complex. Today, deformable bodies, described as finite-element models, can be included in a multibody model as long as the number of degrees-of-freedom is not too large. Present day finite-element models for tires contain large numbers of degrees-of-freedom not suitable for multibody dynamics.

In this presentation we examine several possible strategies to incorporate high-resolution finite-element tire models in multibody dynamics. Some of the problems associated with adopting such finite-element models, besides the issue of large numbers of degrees-of-freedom, are discussed and possible solutions are proposed.

The presentation begins with a brief review of some of the past and present challenges in the evolution of multibody dynamics as a discipline. The review, and therefore the corresponding references, is mainly based on the experience of the first author. This review is not meant to represent the complete history of the development of multibody dynamics. Following the general review, a short discussion on some of the earlier and present tire models is provided. Then, the presentation concentrates on its main objective: Can high-resolution tire models be adopted for multibody dynamics?

2. CHALLENGES
The discipline of multibody dynamics is comprised of numerous subjects: kinematics and dynamics, solid mechanics and structural analysis, controls, numerical methods, development of algorithms and programming, etc. However, in its early days, multibody dynamics meant solving kinematic and dynamic problems using a computer.

Some of the early formulations of the equations of motion for multibody dynamics, going back to 1970’s, employed the classical Lagrangian approach [Schiehlen (1978)]. Another approach that appeared in literature used Newton-Euler equations [Orlandea (1977)]. This approach eventually gave rise to general-purpose multibody software packages such as ADAMS and DADS. These formulations were initially called Cartesian Coordinates and later on Body Coordinates. In these formulations the main assumption was that the bodies were non-deformable, and the joints and compliance elements were ideal.

The methodologies used in ADAMS and DADS assumed a free rigid body to have six degrees-of-freedom (DoF). Kinematic joints provided algebraic constraints reducing the number of DoF, which resulted into mixed differential-algebraic (DAE) equations. The numerical integration of these equations gave rise to issues such as computational inefficiency and constraint violation. By this time the topic of multibody dynamics had advanced enough that graduate level courses were introduced in the mechanical engineering curricula at several universities.
and textbooks were published covering the subject [Wittenberg (1977); Nikravesh (1988); Haug (1989)]. In another formulation called Natural Coordinates, a body was presented by two points, two vectors, and six constraints [Garcia de Jalon (1994)]. This representation of a rigid body eliminated the need for rotational coordinates and the associated transformation between reference frames. Another representation, known as the Point Coordinates, considered a body as a collection of four points and six constraints [Nikravesh (1994)]. Although these formulations increased the number of DAE’s, they provided simplicity in the corresponding equations of motion.

In order to improve on the shortcomings of the numerical solution of DAE’s, new methodologies were introduced. The so-called coordinate partitioning method selected a sub-set of the absolute coordinates (and velocities), equal to the number of DoF, for integration [Wehage (1981)]. To some extent this method resolved the computational efficiency and the constraint violation issues, but introduced other numerical difficulties. Later on another method known as the Joint Coordinates transformed the equations of motion from the body (or absolute) coordinates to joint (or relative) coordinates [Jerkovsky (1978); Kim (1986); Nikravesh (1989)]. This method was able to either partially or completely eliminate the drawbacks of the Body Coordinate formulation.

In addition to the preceding methodologies for eliminating the problem of constraint violation, other techniques based on feedback control were developed to control the growth of the violation during numerical integration. Examples of such methods are: constraint stabilization; penalty method; and augmented Lagrangian method [Baumgarte (1972); Garcia de Jalon (1994)]. Furthermore, new numerical integration algorithms were introduced that considered lower index DAE’s, resulting in better control over the propagation of constraint violations [Petzold (1982, 1986)].

By mid 1980’s the assumption of non-deformable bodies were relaxed and formulations representing deformable bodies that could undergo gross motion began to appear [Shabana (1989)]. These formulations incorporated the finite-element methodology into the multibody formulations. Similarly, in modeling of kinematic joints, the concept of an ideal joint was relaxed by introducing compliance and clearance in a joint [Flores (2006)].

Unilateral contact or impact may occur in multibody systems that undergo intermittent motion. A variety of methodologies have been presented for modeling impact. Some methods employed piece-wise analysis formulations, based on the classical concept of coefficient of restitution, to determine the change in the velocities immediately after impact. Other methods suggested continuous contact force models for impact analysis [Brach (1989); Lankarani (1990); Pfeiffer (1996); Pereira (1996)]. Some of these ideas have been extended from a simple contact to the crash analysis of vehicles. Despite the development of these sophisticated models, one issue still remains illusive and challenging—the determination of the exact time of contact during the numerical integration of equations of motion.

Inclusion of various components in a multibody model has introduced other challenges. For example, a vehicle model comprised of mechanical and electro-mechanical components, such as ABS, exhibits a wide spectrum of frequencies. When a single numerical integrator is used to determine the response of the complete system, the integrator must adjust the time-steps based on the highest frequency component. Computationally, this procedure cannot be very efficient. Development of multi-rate integration schemes, where two or more integrators operate simultaneously on different parts of the system, but with different time steps, is another challenge for today’s researchers.

In multi-physics or inter-disciplinary applications, multibody dynamics may be one of several disciplines needed for analyzing a system. This would require different computational packages to communicate with each other. This view can be extended to perform an analysis through distributed computing with packages that reside at different sites.

The present state of multibody dynamics allows us to model and analyze a vehicle in great detail. Whether the objective of the analysis is ride comfort or cornering, the accuracy of the results mostly depends on the tire model. A highly simplified tire model that is computationally fast may not produce reliable results. On the other hand, a sophisticated tire model that could produce acceptable results, most likely would be too time consuming. Therefore finding a balance between the two conflicting requirements remains a challenge.

In the following sections, a brief review of some of the existing tire models is provided. Then, several formulations and strategies are discussed that might assist us in our progress towards developing more accurate and efficient tire models for vehicle dynamics.

3. TIRE MODELS

Studies of static and dynamic properties of tires date back almost a century. Tires have been studied for their cornering characteristics and ride comfort, and more recently for their noise emission. Many tire models have been developed with the focus on these three particular characteristics. They differ in their approaches, level of complexity, and as a result, in computational effort.

Tire models can be categorized into four groups based on their complexity. The first group includes analytically based models that are aimed at computational efficiency. These models rely heavily on empirical data for the tire-road slip properties, thereby enabling accurate slip characteristics for a particular condition. On the other hand, the representation of the actual physical structure of a tire is very simplified. Examples of analytically based models are HSRI [Dugoff (1970)], the U-of-A tire models [Gim (1990)], and the Magic Formula [Bakker (1993)]. While analytical models are perfectly suited for steady-state
(combined) slip simulation, their lack in representing the physical characteristics of a tire prevents them for use in applications that require higher frequency responses, such as rolling over obstacles. Also, in situations where the tire may come to a full stop and reverse its direction, it can become difficult for some of these models to properly simulate.

The second group includes models that are based on mechanical components. The complexity of these components can vary significantly, depending on the application. Most of the models consist of an elastic string, beam, or ring, which represents the tread, attached to an elastic foundation that is the sidewall. For example, Eichler (1997) created a flexible I-ring belt model, where the belt is represented by mass points that are connected to each other and the rim through a collection of springs and dampers. Other models represent the belt as a rigid ring that is elastically suspended with respect to the rim for all six degrees-of-freedom. Furthermore they might include springs that connect the ring to the contact patch for a better representation of the total tire stiffness. For example, the rigid ring models [Bruni (1997); Allison (1997)], or the more recently developed MF-SWIFT model [Pacejka (2006)]. While the tire models belonging to this group can produce results quickly and with less computational cost, they need an extensive amount of experiments to obtain the properties of the mechanical components. The other drawback of these tire models is their accuracy when it comes to higher frequency response. This makes these tires only valid for a limited range of applications.

The third group consists of hybrid models that are basically a bridge between the mechanical component-based models and the finite-element tire models. Hybrid models are more physically based systems; i.e., their mechanical structure is closer to a real tire and, hence, includes more degrees-of-freedom, but avoids the high fidelity of finite-element models. Rolling over short wavelength obstacles, such as a cleat, can be captured with these models. An example of such tire models is the FTire [Gipser (1999)]. The model can be separated into two parts: the flexible belt that describes tire’s structural stiffness, damping, and inertia properties; and the tread model that represents the tire-road contact and computes the contact pressure distribution and friction forces. The belt consists of several segments that are mutually connected through springs. Friction functions are used to distinguish between sticking and sliding friction. Additionally a thermal and wear model is included. Another hybrid based tire model can be found in the RMOD-K system [Oertel (1999)]. Here the belt structure is modeled with finite elements and is connected to the rim via a membrane. The membrane itself is a simplified sidewall model that considers inflation pressure. Road contact is realized through an additional sensor layer that envelops the belt structure. At the sensor points, the normal and frictional forces are calculated. Temperature and contact pressure dependent friction models are included that allow both adhesion and sliding of the sensor points.

The fourth and last group is comprised of finite-element (FE) based models with the highest grade of complexity. Unlike the previous models, the finite-element tire models are capable of modeling the tire in much greater detail. Furthermore, geometric and material non-linearities can be captured. The first numerical simulations with finite-element based tire models date back as early as the 70’s [Padovan (1977)]. Today the tire industry uses FE models during development processes with focus on steady-state rolling and vibration analysis [Becker (1997); Faria (1992); Nackenhorst (1993)]. The use of explicit finite-element programs, such as LS-DYNA3D, allows even performing transient dynamic analysis with full scale FE tire models [Kao (1997)], but at an extreme computational expense. Acoustic analysis has become another application for these high-resolution models [Brinkmeier (2007)].

An overview of these models reveals that each group is equally important in the performance evaluation of a tire. While empirically based models are better suited to investigate the effect of changing performance parameters on out-of-plane cornering forces, slip angle changes, self-aligning torque, lateral deformations, and stiffness, the more complex and physically based models are better qualified to examine the effects of the change of physical parameters, such as material properties and cross section contour. Looking from a different perspective, it is obvious that the analytical and mechanical based models are best suited for multibody simulation, due to their computational efficiency. However, due to the advancements in vehicle dynamics, more accurate models, such as the hybrid models, find their way into multibody simulation. An apparent question remains whether finite-element based models could be adopted for multibody dynamic simulations as well. Although these models serve a valuable purpose in the design phase of a tire, they are not suitable for multibody simulation purposes in their current state. Their high-resolution leads to thousands of degrees-of-freedom, which make a multibody simulation almost impossible, even on today’s high performance computers using several parallel processors. Only a reduction in their number of degrees-of-freedom would make a multibody simulation feasible. Whether such a reduction is possible without restricting the model in its applications is the issue that this presentation will attempt to address.

4. A FE-BASED WHEEL-TIRE MODEL

For many years tire manufacturers have developed high-resolution FE models for tires that contain detailed structural and material characteristics. These models are used to study different designs in various static and dynamic loading conditions. Such models can also exhibit the nonlinear behavior of a tire under different conditions. These models normally contain degrees-of-freedom in the order of 10^6 or greater. Due to the large number of degrees-of-freedom, these models are not suitable for multibody dynamic simulation of vehicles.
FE wheel-tire model provides mass and stiffness matrices in the following form:

\[
M_{FE} = \begin{bmatrix}
M^r & 0 & 0 \\
0 & M^{rot} & 0 \\
0 & 0 & M
\end{bmatrix}
\]

\[
K_{FE} = \begin{bmatrix}
K^r & K^{rot} & \bar{K}^r \\
K^{rot} & K^{rot} & K^{rot} \\
\bar{K}^r & K^{rot} & K
\end{bmatrix}
\]  

(1)

Even, for example, for simulating a single tire rolling on a track, the computation time can become prohibitive.

In order to investigate whether a high-resolution FE tire model could be employed for use in multibody dynamics, we need to look at the equations of motion for a wheel-tire model. We assume that a finite-element model for the tire is available. We first present the equations of motion for a wheel-tire model in several standard forms. Then we attempt to transform the equations to other forms that are more suitable for our purpose.

In this document, matrix notation is used in order to keep the attention on concepts without losing sight of details. The following nomenclature is used:

Vectors and arrays:
- **Bold-face, lower-case characters**
- **Bold-face, upper-case characters**

Matrices:
- A vector or matrix described in body-fixed frame
- Transforms a 3-vector to a skew-symmetric matrix
- Stacks vertically 3-vectors or 3×3 skew-symmetric matrices
- Repeats a 3×3 matrix to form a block-diagonal matrix

The following examples should clarify the notation. Assume that \( \mathbf{b} \) is a 3-vector and \( \mathbf{b}' \) is a 3×3 skew-symmetric matrix for \( i = 1, \ldots, n \). Let \( \mathbf{I} \) be a 3×3 identity matrix and \( \mathbf{A} \) be a 3×3 rotation matrix. Then the following stacked arrays and matrices can be constructed:

\[
\mathbf{b} = \begin{bmatrix} \mathbf{b} \\ \mathbf{b}' \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{b} \\ \mathbf{b}' \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} 1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 1 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \cdots & 0 \\ \vdots & \ddots \end{bmatrix}
\]

4.1 *Finite-Element Model*

In a multibody simulation scenario, whether we have a complete vehicle model or a single tire on a track, our concern is the accurate prediction of the tire-ground forces and moments, and their transfer to the wheel through the tire. If the number of degrees-of-freedom of a model is lowered, but the tire-ground forces and moments, and their transmission, are accurately predicted, the model would be acceptable for our purposes.

In most high-resolution wheel-tire models, the wheel is considered to be non-deformable (rigid body) and the tire deformable. A reference frame is attached to the wheel with its origin at the mass center of the assembly, as shown in Figure 1. In this figure only some typical nodes on the periphery are shown. We assume that the high-resolution track, the computation time can become prohibitive. 

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4.2.1 Absolute Nodal Accelerations

The equations of motion for a wheel-tire can be expressed as

\[
\begin{bmatrix}
M^r & 0 & 0 \\
0 & M^{in} & 0 \\
0 & 0 & M
\end{bmatrix}
\begin{bmatrix}
r \\
\omega \\
d
\end{bmatrix}
= \begin{bmatrix}
f^r-f^\delta \\
n^\delta-\omega M^{in} \omega - n^r \\
f-f^\delta
\end{bmatrix}
\]

(2)

In this equation \( \dot{r} \) and \( \dot{\omega} \) denote the translational and rotational accelerations of the wheel, \( \dot{d} \) is the array of absolute accelerations of the nodes, \( \dot{\omega} \) is the angular velocity of the wheel, \( f^r \), \( n^\delta \), and \( f \) denote external forces and moments that may act on the wheel and the nodes.

In any tire-ground contact scenario, some of the nodes would be in contact with the ground. In a dynamic simulation, as a tire rolls, some nodes from the contact patch leave the ground and some other nodes enter the contact area. Contact problems in finite-element methods are incorporated either via the penalty approach or the method of Lagrange multipliers. Although both methods provide adequate results, the former method can introduce significant numerical stiffness in the tire-wheel system that is undesirable in a multibody simulation. On the other hand, the method of Lagrange multipliers is an accepted tool in multibody dynamics. Modeling the transition of some nodes into and out of the contact patch requires the boundary conditions, or the constraints, in the equations of motion to be adjusted accordingly. In order to handle this issue, we partition the nodes into two groups: those that are in contact with the ground, designated as \( b \) or boundary nodes; and those that are free, designated as \( f \) nodes. The number of \( b \) nodes depends on the simulation scenario. The bounded nodes need to cover the largest possible size for the contact patch when the tire is under a possible maximum load. Note that a \( b \) node is only a candidate to become in contact with the ground. Three possible contact scenarios are illustrated in Figure 2.

![Figure 2. THREE POSSIBLE SCENARIOS FOR THE BOUNDARY NODES AND THE GROUND.](image)

With the partitioning of the nodes into \( b \) and \( f \) sets, the mass and stiffness sub-matrices are accordingly partitioned:

\[
M = \begin{bmatrix}
M^b & 0 \\
0 & M^f
\end{bmatrix}, \quad K = \begin{bmatrix}
K^b & K^{bf} \\
K^{fb} & K^f
\end{bmatrix}
\]

(3)

Then, the equations of motion are expressed as:

\[
\begin{bmatrix}
M^b & 0 & 0 & 0 & 0 \\
0 & M^{in} & 0 & 0 & 0 \\
0 & 0 & M^{in} & 0 & 0 \\
0 & 0 & 0 & D & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\omega \\
n^\delta-\omega M^{in} \omega - n^r \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
f^r-f^\delta \\
n^\delta-\omega M^{in} \omega - n^r \\
f-f^\delta
\end{bmatrix}
\]

(4)

The constraints on the boundary nodes are included in these equations of motion through the use of Lagrange multipliers, where \( D \) is the Jacobian matrix of the constraints and \( \gamma \) is the array of quadratic velocity terms in the acceleration constraints. Note that not all of the \( b \) nodes are constrained—only those that are in contact with the ground. In most cases, for simple contact with the ground, the Jacobian is an identity matrix and \( \gamma \) is a zero array.

In order to simplify the notation in the upcoming equations, the nodal mass matrices are moved from the coefficient matrix to the force array as:

\[
\begin{bmatrix}
M^b & 0 \\
0 & M^{in}
\end{bmatrix}
\begin{bmatrix}
r \\
\omega \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
M^{in} (f^r-f^\delta) \\
M^{in} (f-f^\delta)
\end{bmatrix}
\]

(5)

There is no need to replace \( -D^T \) with \( -M^{in} D^T \) because the Lagrange multipliers will adjust their values to compensate for the difference.

4.2.2 Nodal Deflections

The deflection of a typical node is denoted as \( \delta \), and its time derivatives are expressed as

\[
\dot{\delta} = \dot{\delta} \omega + \dot{\delta}_{rel}
\]

\[
\ddot{\delta} = \ddot{\delta} \omega + \ddot{\delta}_{rel} + \dddot{\delta} \omega + 2\dot{\omega} \delta_{rel}
\]

In this notation \( \delta \) denotes the total time derivative of \( \delta \), where \( \dot{\delta}_{rel} \) represents the time derivative of \( \delta \) relative to (or within) the rotating frame.
The absolute position of a typical node can be expressed in terms of its deflection as
\[
d' = r + s' + \delta' \quad \text{(a)}
\]
\[
d = r + b' \quad \text{(b)}
\]
where \(s'\) and \(b'\) represent the nodal position vectors with respect to the wheel mass center in the undeformed and deformed states respectively. The time derivative of Eq. (6) is written as
\[
\dot{d}' = r - \dot{s}' \omega + \dot{\delta}' \quad \text{(a)}
\]
\[
= r - b' \omega + \delta_{rel} \quad \text{(b)}
\]

The second time derivative of Eq. (6) is expressed as
\[
\ddot{d}' = \dot{r} - \dot{s}' \omega + \ddot{\delta}' + \omega \dot{\delta}' \quad \text{(a)}
\]
\[
= \dot{r} - b' \omega + \delta_{rel} + \omega \delta_{rel} + 2 \omega \dot{\delta}_{rel} \quad \text{(b)}
\]

For all of the nodes, the acceleration equations can be assembled in two forms:
\[
d = \begin{bmatrix} \mathbf{I} & -\mathbf{s} & \mathbf{T} \end{bmatrix} \begin{bmatrix} \ddot{r} \\ \dot{\delta} \\ \delta_{rel} \end{bmatrix} + \mathbf{w} \quad \text{(a)}
\]
\[
= \begin{bmatrix} \mathbf{I} & -\mathbf{b} & \mathbf{T} \end{bmatrix} \begin{bmatrix} \ddot{r} \\ \dot{\omega} \\ \delta_{rel} \end{bmatrix} + \mathbf{w}_{rel} \quad \text{(b)}
\]

where \(w = \ddot{\omega} \delta s\) and \(w_{rel} = \ddot{\omega} \delta b + 2 \ddot{\delta}_{rel}\).

The equations of motion in Eq. (5) can be transformed into a more useful form by using Eq. (9)(b) and then expressing all entities in terms of their body-fixed components:
\[
\begin{bmatrix} \mathbf{M}^\omega & 0 & 0 & 0 \\ 0 & \mathbf{M}^\omega & 0 & 0 \\ \mathbf{I} & -\mathbf{b} & \mathbf{I} & 0 \\ \mathbf{D} & -\mathbf{D} \mathbf{b} & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}} \\ \dot{\mathbf{\omega}} \\ \delta_{rel} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^\mathbf{r} \\ \mathbf{A}^\omega \\ \mathbf{A}^{\delta s} \\ \mathbf{A}^{\delta_{rel}} \end{bmatrix}
\]

This is a typical (conventional) form of the equations of motion for a rigid-deformable body. However, this form of the equations exhibits several drawbacks:

a) The number of degree-of-freedom is too large causing prohibitive computation time.
b) Some of the inertia sub-matrices or the force sub-arrays, such as \(-\mathbf{M} \dot{\mathbf{r}}\), need to be updated as the tire deforms (due to the presence of the \(\mathbf{b}\) array).
c) As the tire rolls, the boundary conditions must be moved from node to node. This requires implementation of proper logic in the computational procedure.
d) The total number of boundary nodes can change from one time-step to the next, even if the actual size (length) of the contact patch remains constant. This causes an oscillatory exchange of mass between the free and constrained portions of the tire, which will manifest itself as an artificial high frequency in the response. This is a critical phenomenon associated with a discretized tire model in a rolling scenario.

### 4.2.3 Mode Shapes Rotating with Tire

Some of the difficulties associated with a model in nodal space can be resolved if we transform some or all of the deformation degrees-of-freedom to modal space. For this purpose the nodal to modal transformations for the free nodes are expressed as [Craig and Bampton (1968)]
\[
\mathbf{\tilde{\mathbf{y}}} = \begin{bmatrix} \mathbf{\Psi}' & \mathbf{\Psi}_s \end{bmatrix} \begin{bmatrix} \mathbf{z}' \\ \mathbf{\bar{\mathbf{z}}}_s \end{bmatrix}
\]
\[
\mathbf{\tilde{\mathbf{y}}}_{rel} = \begin{bmatrix} \mathbf{\Psi}' & \mathbf{\Psi}_s \end{bmatrix} \begin{bmatrix} \mathbf{z}' \\ \mathbf{\bar{\mathbf{z}}}_{rel} \end{bmatrix}
\]

where \(\mathbf{\Psi}'\) is the matrix of normal deformation mode shapes, \(\mathbf{\Psi}_s\) is the matrix of static (also called constrained) mode shapes, and \(\mathbf{z}\) is the array of modal coordinates. In order to obtain \(\mathbf{\Psi}'\), all of the degrees-of-freedom associated with the \(\mathbf{b}\) nodes must be constrained; i.e., \(\mathbf{M}'\) and \(\mathbf{K}'\) are used to obtain \(\mathbf{\Psi}'\) by solving a generalized eigen-value problem. The static mode shapes are obtained from the stiffness matrix as \(\mathbf{\Psi}_s = -\mathbf{K}' \mathbf{\Psi}'\).

The acceleration transformation from Eq. (11) is substituted into Eq. (10), and the third row is pre-multiplied by \(\mathbf{\Psi}'\) to obtain
The transformation between the modal and nodal spaces can be achieved by using ideas from a concept that is known in the finite-element community as the Arbitrary-Lagrangian-Eulerian formulation [Donea (1983); Stein (2004)]. If the mode shapes of a tire are represented by matrix $\Psi$, the transformation between the modal and nodal coordinates is expressed as

$$\delta = \Psi z$$  \hspace{1cm} (13)

If we consider that the mode shapes do not rotate with the body-fixed frame, the time derivative of $\delta$ can be obtained by using the material derivative $d/dt = \partial / \partial t + (\partial / \partial \theta) \partial$, where the left-hand side represents the total time derivative in the Lagrangian space, the first term on the right-hand side is the time derivative in the Eulerian space, $\partial$ is the rotational coordinate about the wheel axis, and, $\theta$ is the rotational speed. The time derivative of Eq. (13) can then be expressed as

$$\delta = \Psi \dot{z} + \Psi_{s} z \dot{\theta}$$  \hspace{1cm} (14)

In this equation $\Psi_{s}$ represents the partial derivative of the mode shapes with respect to the rotational coordinate about the wheel axis. Similarly, the second time derivative of the transformation formula is expressed as

$$\delta = \Psi \ddot{z} + \Psi_{s} \dot{z} \ddot{\theta} + (\Psi_{ss} z \ddot{\theta} + 2 \Psi_{s} \dot{z} \dot{\theta})$$  \hspace{1cm} (15)

where $\Psi_{ss}$ represents the second partial derivative with respect to the rotational coordinate.

With this view of non-rotating mode shapes, if only the $f$ nodes are transformed to the modal space, they remain with the non-rotating frame. And if the $b$ nodes are kept in the nodal space, they will rotate with the tire and it becomes necessary to redefine a new set of $b$ nodes through interpolation at every time step. In order to eliminate the need for such an interpolation procedure, the $b$ nodes should also be transformed to the modal space.

The nodal to modal transformation for both sets of nodes is expressed as

$$[\delta'] = [\Psi'] [z']$$  \hspace{1cm} (16)

where

$$[\Psi'] = [\Psi \Psi_{s}]$$

The $b$ degrees-of-freedom will not be truncated since they could be constrained, and therefore, there is no need for static modes for these nodes. Matrices $\Psi'$ and $\Psi_{s}$ are obtained as it was described before. To obtain the modal matrix associated with the $b$ nodes, the $f$ nodes must be remain with a non-rotating frame, but the mass points “flow” through them.

The transformation from nodal to the new modal space can be achieved by using ideas from a concept that is known in the finite-element community as the Arbitrary-Lagrangian-Eulerian formulation [Donea (1983); Stein (2004)]. If the mode shapes of a tire are represented by matrix $\Psi$, the transformation between the modal and nodal coordinates is expressed as

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$$\delta = \Psi \ddot{z} + \Psi_{s} \dot{z} \ddot{\theta} + (\Psi_{ss} z \ddot{\theta} + 2 \Psi_{s} \dot{z} \dot{\theta})$$  \hspace{1cm} (15)

where $\Psi_{ss}$ represents the second partial derivative with respect to the rotational coordinate.

With this view of non-rotating mode shapes, if only the $f$ nodes are transformed to the modal space, they remain with the non-rotating frame. And if the $b$ nodes are kept in the nodal space, they will rotate with the tire and it becomes necessary to redefine a new set of $b$ nodes through interpolation at every time step. In order to eliminate the need for such an interpolation procedure, the $b$ nodes should also be transformed to the modal space.

The nodal to modal transformation for both sets of nodes is expressed as

$$[\delta'] = [\Psi'] [z']$$  \hspace{1cm} (16)

where

$$[\Psi'] = [\Psi \Psi_{s}]$$

The $b$ degrees-of-freedom will not be truncated since they could be constrained, and therefore, there is no need for static modes for these nodes. Matrices $\Psi'$ and $\Psi_{s}$ are obtained as it was described before. To obtain the modal matrix associated with the $b$ nodes, the $f$ nodes must be
constrained; i.e., the generalized eigen-value problem of $M^{10}$ and $K^{10}$ provides $\Psi^{10}$.

If we assume that the wheel axis coincides with the $\eta$-axis of the wheel-fixed frame, we have $\dot{\theta} = n^1 \omega$ and $\dot{\theta} = n^1 \omega$. Therefore, the velocity and acceleration transformations become

$$
\begin{bmatrix}
\delta' \\
\delta
\end{bmatrix} = \begin{bmatrix}
\Psi' & \Psi \\
0 & \Psi
\end{bmatrix}
\begin{bmatrix}
z' \\
\omega
\end{bmatrix}
+ \begin{bmatrix}
B' \\
B
\end{bmatrix}
$$

(17)

$$
\begin{bmatrix}
\delta' \\
\delta
\end{bmatrix} = \begin{bmatrix}
\Psi' & \Psi' \\
0 & \Psi
\end{bmatrix}
\begin{bmatrix}
z' \\
\omega
\end{bmatrix}
+ \begin{bmatrix}
e' \\
0
\end{bmatrix}
$$

(18)

where

$$
\begin{bmatrix}
B' \\
B
\end{bmatrix} = \begin{bmatrix}
\Psi' & \Psi' \\
0 & \Psi
\end{bmatrix}
\begin{bmatrix}
z' \\
\omega
\end{bmatrix}
$$

(19)

Substituting Eq. (9)(a) into Eq. (5) yields another form of equations of motion:

$$
\begin{bmatrix}
M' & 0 & 0 & 0 & 0 & 0 \\
0 & M^{10} & 0 & 0 & 0 & 0 \\
1 & -\hat{s} & 0 & 0 & 0 & 0 \\
1 & -\hat{s} & 0 & 0 & 0 & 0 \\
D & -D & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\omega \\
\delta \\
\delta \\
\chi
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

(22)

We substitute the transformations from Eqs. (16)-(18) in this equation, and pre-multiply the third row by $\Psi'$ and the fourth row by $\Psi''$ to obtain

$$
\begin{bmatrix}
M' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & M^{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\Psi' & \Psi' & \Psi' & \Psi' & \Psi' & \Psi' & \Psi' & \Psi' & \Psi' & \Psi' \\
\Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' & \Psi'' \\
D & -D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
r \\
\omega \\
\delta \\
\delta \\
\chi
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

(23)

Now the equation is in a form that the modal coordinates associated with the $f$ nodes can be truncated in order to reduce the size of the problem.

Although the interpretation of non-rotating mode shapes could initially appear non-intuitive, they are the most logical means for their inclusion in a rolling scenario. The only drawback of this formulation is the requirement to obtain partial derivatives of the mode shapes with respect to the rotational coordinate about the wheel axis. These derivatives can be determined from the mode shapes using the finite-difference technique. Although the computed partial derivatives could contain numerical errors, the advantages of this formulation over standard formulations by far exceed its disadvantages.

In certain conditions, there is an instability issue associated with the numerical integration of Eq. (22). In order to reveal this condition, we assume the wheel rotates about its axis with a constant angular velocity; i.e., $\dot{\theta} = 0$ and $\dot{\omega} = \omega$. We also assume that there is no contact with the ground and therefore all of the nodes are free. This eliminates the $b$ nodes and the corresponding constraints. Based on these assumptions, only the third row of the equations of motion remains:

$$
\dot{\zeta} = \Psi^{10}(M^{10}(f' - f'_c) - (w^0 + e'))
$$

(21)

After substituting for $\dot{\zeta}$ we have

$$
\dot{\zeta} = -(\Psi^{10}(\Psi^{10}\dot{\omega} + \Lambda)\zeta - 2\Psi^{10}\Psi^{10}\dot{\theta} + \Psi^{10}(M^{10}(f' - f'_c) - w^0) + \Psi^{10}(-\dot{w} + \dot{e})w')
$$

In order for these second-order differential equations to be stable, the coefficient matrix of $\dot{\zeta}$ must remain semi positive definite; i.e., the system becomes unstable if

$$
det(\Psi^{10}\Psi^{10}\dot{\omega} + \Lambda) \leq 0
$$

This condition shows the relationship between the angular velocity of the wheel and the tire natural frequencies for which the system is unstable.
4.2.5 Non-Rotating Mode Shapes: Canonical Form

In order to eliminate the numerical instability associated with Eq. (22), the nodal equations of motion are transformed to canonical form before conversion to the modal space. In canonical form Eq. (21) is expressed as

\[
\begin{bmatrix}
M^0 & 0 & 0 & 0 & 0 \\
0 & M^0 & 0 & 0 & 0 \\
1 & -\tilde{s} & 0 & 0 & 0 \\
i & -\tilde{s} & 0 & T & -D' \\
D_l & -D_l & 0 & D & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{p}} \\
\mathbf{p}' \\
\mathbf{p}'' \\
\mathbf{p}''' \\
\mathbf{p}'''
\end{bmatrix}
= \begin{bmatrix}
f' \\
\omega \Theta \\
f'' \\
\omega \dot{\Theta} \\
0
\end{bmatrix}
\]

(23)

The \( \mathbf{p} \) arrays in these equations represent the normalized momentum. The Lagrange multipliers \( \sigma \) and \( \lambda \) are related as \( \sigma = \lambda \).

The equations of motion in canonical form have several advantages over the standard (acceleration) form of the equations: they are more stable during numerical integration due to a lower so-called index; the velocity constraints at both the coordinates and the velocity levels can become violated.

Substituting Eq. (17) into Eq. (23) and pre-multiplying the third row by \( \Psi^{r'} \) results into:

\[
\begin{bmatrix}
M^0 & 0 & 0 & 0 & 0 \\
0 & M^0 & 0 & 0 & 0 \\
1 & -\tilde{s} & 0 & 0 & 0 \\
i & -\tilde{s} & 0 & T & -D' \\
D_l & -D_l & 0 & D & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{p}} \\
\Psi^{r'} \mathbf{p}' \\
\mathbf{p}'' \\
\mathbf{p}''' \\
\mathbf{p}'''
\end{bmatrix}
= \begin{bmatrix}
f' - \dot{\mathbf{f}}_g \\
\omega \Theta \\
f'' - \dot{\mathbf{f}}_g \\
\omega \dot{\Theta} \\
0
\end{bmatrix}
\]

(24)

where \( \Psi^{r'} = \Psi^{r'} \mathbf{p}' \). The time derivative of \( \Psi^{r'} \) is expressed as \( \dot{\Psi}^{r'} = \Psi^{r'} \dot{\mathbf{p}}' + \Psi^{r'} \mathbf{p}' \dot{\mathbf{p}}' \). The third row of Eq. (24) is pre-multiplied by \( \Psi^{r'} \) to obtain

\[
\begin{bmatrix}
\dot{\mathbf{p}}' \\
\dot{\mathbf{p}}'' \\
\dot{\mathbf{p}}'''
\end{bmatrix}
= \begin{bmatrix}
\dot{f}' - \dot{\mathbf{f}}_g \\
\dot{n}'' - \dot{n}_g \\
\dot{\Psi}^{r'} \left( M^{0} (f' - \mathbf{f}_g) - w' - D' \sigma \right) + \Psi^{r'} \mathbf{p}' \dot{\mathbf{p}}'
\end{bmatrix}
\]

(26)

Note that the second partial derivatives of the mode shapes are not needed in this formulation—we only need the first partial derivatives.

The equations of motion in canonical form that are transformed to modal space have reasonably good characteristics for modeling a wheel-tire assembly:

a) The equations are stable for computational purposes.
b) The number of degrees-of-freedom is lowered substantially by truncating the modes associated with the free nodes.
c) Practically all of the sub-matrices or the sub-arrays are invariants—they need to be computed once.
d) As the tire rolls, the nodes translate with the tire—there is no need for any interpolation.
e) When the tire moves vertically, the number of nodes that are in contact with the ground can be adjusted easily through the constraints.
f) The mesh of the tire does not need to be uniform about the wheel axis. The mesh can be denser at the contact patch.
g) Each node in the contact patch can be constrained in all directions, or its slip-stick response could be presented through proper friction models.
h) Since the equations of motion for the wheel and the nodes are loosely coupled, the wheel equations, even as a part of a multibody vehicle model, can be solved first before the nodal equations are solved.
i) The loose coupling between the wheel and the nodal degrees-of-freedom makes these equations a good candidate for multi-rate integration.

5. CONCLUSION

This presentation has demonstrated the possibility of adopting a high-resolution finite-element tire model for multibody vehicle dynamic simulations. Before a high-resolution FE tire model could be used in a multibody simulation, the FE model must undergo a size-reduction process. The process should take advantage of the symmetric structural characteristics of a tire. The structural symmetricity would allow a special representation of the mode shapes that would not require the nodes to rotate as the tire rolls about the wheel axis. This concept eliminates numerous procedural difficulties. By selecting a proper set of deformation modes, a model could be tailored for any specific simulation scenario, while the size of the model is reduced substantially. Furthermore, the idea of separating the nodes into the free and boundary nodes enables us to enforce the tire-ground contact model individually on each boundary node.

In summary, it appears that with recent advancements in modeling techniques, the tire no longer needs to be the weakest link in wheeled vehicle models. It is no longer a far-fetched possibility to adopt high-resolution finite-element tire models for multibody dynamics.
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REFERENCES


