Applications of experimental stress separation technique using thermoelasticity and photoelasticity to fracture mechanics

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Abstract: This paper describes an experimental study on full-field stress separation from thermoelasticity and photoelasticity measurements and its application to estimation of stress intensity factor and the J -integral. Thermoelastic stress analysis (TSA) and photoelastic stress analysis (PSA) have been developed as full-field visualization methods of stress distribution. Only the sum of principal stresses can be measured by TSA, while only the difference of principal stresses can be measured by PSA. In this study, the hybrid stress separation measurement technique developed by the present authors using both of these methods was applied for determining distribution of all individual stress components in a center-cracked plate subjected to mechanical load. Stress intensity factor and the J -integral were calculated from the obtained stress distribution. In addition to the conventional calculation method, near-tip exclusive domain integral method was proposed, in which the J -integral was evaluated without using degraded experimental stress distribution data near the crack tip. It was found that these fracture mechanics parameters can be evaluated with good accuracies by the present technique.

Key words: *Thermoelasticity, Photoelasticity, Hybrid Technique, Stress Separation, Crack, Fracture Mechanics, Stress Intensity Factor, J-Integral*

1. INTRODUCTION

Remarkable progress in numerical stress analyses such as finite element method (FEM) and boundary element method (BEM) enabled us to evaluate accurate stress distribution of loaded structural components even with complicated shapes. In the actual situations, however, we often encounter difficulties in conducting numerical stress analyses in actual components, because loading conditions or boundary conditions of the objective regions cannot be easily prescribed. Experimental techniques of stress evaluation are then very important. Thermoelastic stress measurement and photoelastic stress measurement have been widely used as full-field experimental stress evaluation methods. However these methods have limitations in the determination of individual stress components. Only the sum of principal stresses can be determined by thermoelastic stress measurement and only the difference of principal stresses can be determined by photoelastic stress measurement. In order to solve this problem, many researchers have developed stress separation techniques [1-5]. Especially remarkable progress can be found in the development of stress separation techniques based on inverse analyses [4-5]. In these techniques unknown boundary values were estimated by an inverse analysis using values of the sum of principal stresses measured by thermoelastic method. A direct problem was solved to compute individual stress components using the boundary values determined by the inverse analysis. Kishimoto et al. [5]

developed computational techniques to regularize the ill-posed inverse problems and improved the accuracy of the inverse analyses.

Combined use of thermoelasticity and photoelasticity enables us to develop an experimental stress separation technique. Patterson [6] examined the practicability of the hybrid use of thermoelasticity and photoelasticity for obtaining the maximum and minimum values of the principal stresses. In this study, the thermoelastic and photoelastic hybrid technique was developed for full-field separation measurement of the all individual stress components in plane-stress condition. First, stress separation of a loaded plate with a circular hole was investigated to show the feasibility of the proposed technique. Next, the proposed technique was applied to the stress separation measurement of a cracked plate to evaluate fracture mechanics parameters, such as stress intensity factors and the J -integral. For the J -integral calculation, near-tip exclusive domain integral method was proposed, in which the J -integral was evaluated without using degraded experimental stress distribution data near the crack tip.

2. PRINCIPLES OF STRESS SEPARATION

2.1. Thermoelastic stress analysis (TSA)

Dynamic stress change causes very small temperature change under the quasi-adiabatic condition in solid. This phenomenon is called thermoelastic effect and is

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described by the following equation that relates temperature change (ΔT) to a change in the sum of the principal stresses ($\Delta \sigma$) under cyclic loading.

$$\Delta T = -\frac{\alpha}{\rho C_p} T \Delta \sigma \quad (1)$$

α : Coefficient of thermal expansion

ρ : Mass density

C_p : Specific heat at constant pressure

T : Absolute temperature

For measuring a very small thermoelastic temperature change, lock-in thermography with less than 5mK resolution is needed. The lock-in thermography correlates the load-induced infrared signal with the reference-loading signal enabling the noise reduction and the measurement of very small temperature changes due to the thermoelastic effect. Calibration relation between the sum of principal stresses and the infrared radiation is required for obtaining the absolute stress values. In this experiment, calibration points, where uniform nominal stresses were supposed to be applied, were chosen on the specimen. The stress values calculated from the geometry of the employed specimen and the applied load were calibrated with the measured infrared radiation values.

2.2. Photoelastic stress analysis (PSA)

Coating photoelastic stress measurement system with a gray-field reflection polariscope, which can measure the birefringence induced by strained plastic coatings installed on the specimen surface, was employed in this experiment. The rotating analyzer in the gray-field reflection polariscope analyzed the direction of the principal stress from the intensity of the transmitted light, which increased as the polarizer aligned with the major axis of the elliptical polarization and decreased as the polarizer aligned with its minor axis. Calibration relation between the difference of principal stresses and the intensity of light is required for obtaining the absolute stress values. In this experiment, calibration points, where uniform nominal stresses were supposed to be applied, were again chosen on the specimen. The stress values calculated from the geometry of the employed specimen and the applied load were calibrated with the measured

Fig.1 Dimensions of specimen and area for stress separation.

light intensity.

2.3. Stress separation by Mohr's stress circle

Mohr's stress circle can be drawn, as the sum of principal stresses are determined by TSA, and the difference of principal stresses and the direction of the principal stress are determined by PSA. All of the stress components can be determined from the Mohr's stress circle.

3. STRESS MEASUREMENT OF A PLATE WITH A CIRCULAR HOLE

The feasibility of the present stress separation technique was evaluated by an experiment using a 2024 aluminum alloy plate specimen with a circular hole shown in Fig. 1. Thermoelastic stress measurement was carried out under cyclic tensile load. The maximum load and minimum load were 2940N (19.6 MPa in nominal stress) and 0N, respectively. The loading frequency was set to be 3Hz. Photoelastic stress measurement was carried out under the static tensile load of 2940N. The area of stress separation was set in ACDF in Fig. 1. The sum of principal stresses, the difference of principal stresses and the direction of principal stresses were obtained for 31×31 grid points in ACDF from TSA and PSA measurement,

(a) Distribution of σ_x

(b) Distribution of σ_y

(c) Distribution of τ_{xy}

Fig. 2 Experimental result of stress separation for a specimen with a hole.

Fig.3 Dimensions of specimen with a center crack.

and the calculation of stress separation based on Mohr's stress circle was made at each grid point. Two-dimensional numerical analysis using BEM was carried out for evaluating the accuracy of the present stress separation technique.

Distributions of the stress components σ_x , σ_y and τ_{xy} obtained by the present technique are shown in Fig. 2. The results of experimental stress separation are shown on the left hand side, and the BEM results are shown on the right hand side in Fig. 2. It is found that the experimentally obtained stress distributions agree with those computed by BEM, demonstrating the feasibility of the present stress separation technique. Along the edge of the circular hole, however, the experimentally obtained stress values deviate from the actual values, due to the degraded accuracy of TSA by the edge effect. A slight difference is also found in the values of σ_x and σ_y on AF and FD. This is caused by the error in PSA. This makes measured differences of principal stresses larger than actual values.

4. STRESS MEASUREMENT OF A PLATE WITH A CENTER CRACK

An aluminum alloy plate specimen with a center crack, shown in Fig. 3, was employed for the experimental study. Through-thickness crack of 16mm in length was introduced to the plate by electro-discharge machining. Thermoelastic stress measurement was carried out under cyclic tensile load. The maximum load and the minimum load were 7840N (16.3 MPa in nominal stress) and 0N, respectively. The loading frequency was 5Hz. Photoelastic stress measurement was carried out under the static tensile load of 7840N. The area of stress separation was set in ACDF in Fig. 3. The sum of principal stresses, the difference of principal stresses and the direction of principal stresses were obtained for 80×200 grid points in ACDF from TSA and PSA measurement, and the calculation of stress separation was made at each grid point. Two-dimensional BEM analysis was again carried out for evaluating the accuracy of stress separation. Quarter-point singular elements were used at the crack tips in BEM analysis.

Obtained distributions of the stress components σ_x , σ_y and τ_{xy} are shown in Fig. 4. The experimental stress

Fig.4 Experimental result of stress separation for a specimen with a center crack.

separation result is shown in the left hand side, and the BEM result is shown in the right hand side in Fig. 4. It is found that the stress distributions obtained by the experimental stress separation agree with those computed by BEM analyses. Slight differences are found between experimental stress values and numerical ones. Degradation of accuracy of PSA results may be attributed to the residual stresses in the specimen and the unsatisfactory installation of the photoelastic coating on the specimen.

5. EXPERIMENTAL EVALUATION OF FRACTURE MECHANICS PARAMETERS

Fracture mechanics parameters were calculated using

Fig. 5 A coordinate near the crack tip and J -integral path.

the individual stress components obtained by the present stress separation technique.

5.1. Stress intensity factor

First, mode I stress intensity factor K_I was calculated based on the stress extrapolation method. Under the mode I loading, σ_y near the crack tip is given as the following equation using the coordinate system shown in Fig. 5.

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2)$$

K_I can be determined by the following equation extrapolating the σ_y values along the forward line of the crack ($\theta=0$).

$$K_I = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_y|_{\theta=0} \quad (3)$$

Calculated value of K_I is shown in Table 1, as well as the theoretically obtained value [7]. It is found that K_I value can be evaluated from experimentally obtained stress distribution with the error less than 5%.

Table 1 Estimation of stress intensity factor

	K_I (MPa $\sqrt{\text{m}}$)	Error %
Experimental Value	2.778	4.7
Exact Value	2.653	

5.2. J -integral by line integral method

The J -integral was evaluated based on the path integral method. The J -integral is given as the following line integral along an arbitrary counterclockwise path Γ shown in Fig. 5.

$$J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} d\Gamma \right) \quad (4)$$

where W is the strain energy density, T_i is traction on Γ . Numerical integration was carried out along Γ using individual stress values obtained by the stress separation. In Eq. (4), $\partial u_y / \partial x$ cannot be directly obtained by the present stress separation technique. Therefore, it was assumed that u_y takes a uniform value on AF and CD, and u_y on the integral path was determined by integrating the strain value in y -direction based on the uniform value of u_y . The J -integral was obtained for 35 different paths

around the crack tip. The most accurate J value is shown in Table 2, as well as the theoretical value obtained from the stress intensity factor K_I . It is found that J value can be evaluated from experimentally obtained stress distribution accurately based on the line integral method.

Table 2 Estimation of the J -integral by line integral

	J (J/m ²)	Error %
Experimental Value	96.25	0.02
Exact Value	96.268	

5.3. J -integral by domain integral method

In the foregoing paragraph, it was shown that J value can be evaluated accurately using the line integral method. However the line-integral method can be applied under the assumption that u_y takes a uniform value on AF and CD. The J -integral evaluated based on the domain integral method without the assumption of displacement u_y was proposed for the J -integral calculation using numerical data on stress distribution around crack [8]. The J -integral is given by the following equation for domain A surrounded by boundary Γ , provided that no force was applied on the crack face.

$$J = \int_A \left\{ (-W \delta_{ij} + \sigma_{ij} u_{i,1}) q_{,j} + (-W_{,1} + \sigma_{ij} \varepsilon_{ij,1}) q \right\} dA \quad (5)$$

where q is an arbitrary variable defined in the domain A which satisfying the following equations.

$$\begin{cases} q = 1 & \text{at crack tip} \\ q = 0 & \text{on } \Gamma \end{cases} \quad (6)$$

In this study, the J -integral was calculated for the elliptical domain with minor axis a and major axis b , whose center coincides with the crack tip $O(O_x, O_y)$. The variable q given by the following equation was employed for the elliptical domain integral.

$$q(x, y) = 1 - \left\{ \frac{(x - O_x)^2}{a^2} + \frac{(y - O_y)^2}{b^2} \right\} \quad (7)$$

The numerical integration was carried out for 25 different domains using individual stress values obtained by the stress separation. The most accurate J -integral value is shown in Table 3, as well as the theoretical value. It is found that J -integral value can be evaluated from experimentally obtained stress distribution with the error less than 2%.

Table 3 Estimation of the J -integral by domain integral

	J (J/m ²)	Error %
Experimental Value	94.96	1.36
Exact Value	96.268

Fig. 6 Near tip exclusive domain employed for the J -integral by domain integral method.

5.4. J -integral by near-tip exclusive domain integral method

It was shown in the foregoing paragraph that the J -integral by the domain integral method was effectively employed without any assumptions concerning displacement used in the line-integral method. However the accuracy of the J -integral evaluation was slightly degraded compared with that of the line-integral method, even though the same individual stress values obtained by the stress separation were used. This seems to be caused by the fact that error in the experimentally obtained stress values was large near the crack tip where the stress values showed steep distribution.

To take the both benefits of the line-integral method and domain-integral method, the near-tip exclusive domain integral method is newly proposed for the J -integral evaluation. For instance, a near tip exclusive domain is defined as the inter-area between two ellipses. The same equation as Eq.(5) can be applied for the domain integration for the near tip exclusive domain, using an arbitrary variable q which satisfying the following condition.

$$\begin{cases} q = 1 & \text{at inner boundary } \Gamma_i \\ q = 0 & \text{at outer boundary } \Gamma_o \end{cases} \quad (8)$$

The J -integrals were evaluated for the near tip exclusive domain defined as the donut-like inter area between two circles as shown in Fig. 6. Then the arbitrary variable q was defined by the following equation.

$$q(x, y) = \frac{r_o^2 - \{(x - O_x)^2 + (y - O_y)^2\}}{r_o^2 - r_i^2} \quad (9)$$

$$\begin{cases} q = 1 & \text{at inner boundary } \Gamma_i \\ q = 0 & \text{at outer boundary } \Gamma_o \end{cases}$$

The numerical integration was carried out for 25 different near tip exclusive donut-like domains defined by the combinations of r_i and r_o using individual stress values obtained by the stress separation. The most accurate J value is shown in Table 4, as well as the theoretical value. It is found that J value can be evaluated accurately from experimentally obtained stress distribution demonstrating the effectiveness of the present near tip exclusive domain integral method. The following attractive advantages of this method should be emphasized. (1) The J -integral can be calculated from experimentally obtained data on the distribution of individual stress components without any measurement and assumption of the displacement. (2) J value can be evaluated accurately without using degraded data near the crack tip.

Table 4 Estimation of the J -integral by near-tip exclusive domain integral

	J (J/m ²)	Error %
Experimental Value	96.19	0.08
Exact Value	96.268	

6. CONCLUSIONS

In this study, the thermoelastic and coating photoelastic hybrid technique was developed for full-field separation measurement of all the individual stress components. Stress separation of a loaded plate with a circular hole can be conducted with good accuracy demonstrating the feasibility of the proposed technique. Furthermore, the proposed technique was applied to the stress separation measurement of a cracked plate. Fracture mechanics parameters, i.e., stress intensity factors and the J integral were evaluated. For the J -integral calculation, near-tip exclusive domain integral method was newly proposed. It was found that fracture mechanics parameters were accurately evaluated using the obtained stress distributions.

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