A Flaw Reconstruction Method in Heterogeneous Media with Image-based FIT and Time Reversal Approach*

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Abstract
Most of commercial ultrasonic phased array systems implement B– and C–scan methods based on flight time and amplitude of flaw echoes. However these methods stand on the fundamental ray–tracing theory in a homogeneous media and are not directly applicable to heterogeneous media because of complicated phenomena of wave propagation. Time reversal techniques are adaptive methods that can be used in nondestructive evaluation to improve flaw detection in heterogeneous media. Here we propose a simulation–aided flaw imaging method combined with the time reversal approach. Scattered waves from a flaw are recorded with an array transducer and the time-reversed waves are re-emitted in the numerical simulation. The simulation is based on the finite integration technique and image-based modeling. The re-emitted waves propagate through the heterogeneous media and focus on the flaw. The flaw shape can be estimated visually through the focal point of the ultrasonic wave in the simulation.

Key words : Nondestructive Testing, Ultrasonic Wave, Image-based FIT Modeling, Time Reversal Method, Flaw Reconstruction

1. Introduction

In recent years, there are a lot of research reports on nondestructive flaw imaging with ultrasonic array transducer. Ultrasonic phased array transducers have the advantage of beam steering and focusing with various elements. In actual phased array system, B– and C–scan methods based on flight time and amplitude of echoes are incorporated for the flaw imaging. On the other hand, the synthetic aperture focusing technique (SAFT\(1\),\(2\)) has been widely recognized as an advanced tool with high imaging resolution and sensitivity. The sampling phased array\(3\) is similar technique in terms of coherently superposing waveforms at a target pixel. However these methods stand on the ray–tracing theory in a homogeneous media and are not directly applicable to heterogeneous media because of the complicated phenomena of wave propagation. To overcome this difficulty, Fink et.al\(4\)–\(6\) has already proposed the time reversal method. In the time reversal approaches, echoes from a flaw (target) are recorded by an array transducer, and the time-reversed waves are re-emitted into the medium. The re-emitted waves propagate back through the same medium and focus on the target. For the general nondestructive testing, the array transducer is mainly located at a limited area. If the time-reversal operation is only performed on a limited area, a small part of the target is estimated, thus we can obtain the limiting reversal and focusing quality.

Here we propose a simulation–aided flaw inversion method combined with the time reversal approach. Unlike the conventional time reversal method, it does not require programmable echo generators and allows us to use a commercial electronic scanner for the ultrasonic testing.

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This is because the time reversed waves are re-emitted in the simulation for our approach. To assist our inversion, it is desired to adopt an efficient ultrasonic simulation tool. The simulator requires realistic modeling of the target and stable calculation with a high accuracy. Here the image-based finite integration technique (FIT) is developed for the simulation. The FIT\(^7\),\(^8\) is a grid-based spatial discretization method that works in conjunction with a leap-frog time marching scheme. In the image-based modeling approach\(^9\), geometries of targets are properly determined by an actual object image, e.g., a cross-sectional picture, X-ray CT data, etc., and then the processed image is directly fed into the wave simulation by the FIT. The image-based FIT code is optimized for the parallel computation\(^10\). In our inversion, the measured waveforms with an array transducer are time-reversed and re-emitted in the image-based FIT simulation. The simulation shows a focusing process of the re-emitted ultrasonic wave toward a flaw. The shape and location of the flaw can be estimated visually through the focal point of the ultrasonic wave in the simulation.

2. Time Reversal Method

The time reversal method produces an acoustic image of an initial source. The source is a reflective target after being illuminated. This method works even if there is a heterogeneous material between the target and a transducer. Since Fink\(^4\) has already reported the detail of the theory, we summarized the theory as below.

We introduce a fundamental solution (Green's function) \(G\), where \(G(r_0, t_0|r, t)\) is the elastodynamic field produced in \(r\) at time \(t\) by an impulsive source located in \(r_0\) and excited at time \(t_0\). The reciprocity theorem can be written as:

\[
G(r_0, t_0|r, t) = G(r_0, t|r, t_0).
\]  

Here we consider an array transducer with the total element number \(M\) for re-emission. The

![Image](image-url)

Fig. 1 The first step consists in illumining of a target by ultrasonic transmission. The target acts as a source and generates a scattered wave that propagates through the heterogeneous medium and is distorted. The scattered waves are recorded by an array transducer (a). In the last step the array transducer generates the time-reversed wave. This wave propagates through the medium and focuses on the target (b).
focusing process consists in three steps. First, ultrasonic beam are transmitted into flaws (target) and scattered. This process is called “target illumination”. By this process, a target located in \( r_0 \) behaves as an acoustic source. If we assumed that the illumination are performed by the impulse, the flaw echo recorded at No.\( i \) array element is written as the integral form over the transducer surface \( S_i \)

\[
h_i(r_0, t) = \int_{S_i} G(r_0, t_0| r, t) dr.
\]  

(2)

In the second step, these signals are recorded at each transducer element as illustrated in Fig.1(a). Note that the signals recorded by each transducer element can have very different shapes. These signals are time-reversed and we obtain new input \( h_i(r_0, T-t) \). In the third step, these input signals are re-emitted through the same transducer element as shown in Fig.1(b). In order to compute the total acoustic field at \( r_0 \), we consider the individual wave field produced by the No.\( i \) element. This field results from the convolution product of the received impulse response \( h_i(r_0, t) \) and the time reversal signal \( h_i(r_0, T-t) \), and can be written as

\[
h^D_i(r_0, t, T) = h_i(r_0, t) \otimes h_i(r_0, T-t)
\]  

(3)

The maximum of the wave field in \( r_0 \) appears at time \( T \). The total acoustic field is obtained by the simultaneous emission of the individual time reversal wave from all the elements.

\[
\sum_{i=1}^{M} h^D_i(r_0, t, T)
\]  

(4)

In our method, \( h_i(r_0, t) \) is measured at each element of the array transducer, and the time reversal wave is emitted in the image-based FIT simulation. In the process of the simulation, the emitted wave can be focused into the target. Since the accuracy of the flaw estimation directly depends on the performance of the simulation, rigorous modeling of the target and high reliability of the simulation are required here. We have already shown the validation of the image-based FIT for heterogeneous and anisotropic materials\(^{(11), (12)}\) and isotropic thin plate\(^{(13)}\). We will show the short introduction in the next section.

3. Image–Based FIT Modeling

3.1. Finite Integration Technique (FIT)

Here we use the Cartesian coordinates \((x_1, x_2, x_3)\). The starting point of the FIT for ultrasonic wave is the integral form of the linear governing equations, the Cauchy equation of motion:

\[
\int_V \rho(x) \frac{\partial u_j}{\partial t}(x, t) dV = \int_S \tau_{ij}(x, t)n_j(x) dS + \int_V f_j(x, t) dV
\]  

(5)

and the equation of deformation rate:

\[
\int_V \frac{\partial \epsilon_{ij}}{\partial t}(x, t) dV = \int_S c_{ijkl}(x)\epsilon_{ij}(x, t)n_k(x) dS
\]  

(6)

![Fig. 2 3D FIT discretization in ultrasonic wave analyses. The velocity vector components \( v_i (i = 1, 2, 3) \) and the stress tensor components \( \tau_{ij} (i, j = 1, 2, 3) \) are located at different positions inside the grid cell.](image)
where \( \vec{v} \) is the particle velocity vector, \( \tau \) is the stress second rank tensor, \( \rho \) is the density, \( \vec{n} \) is the outward normal vector, and \( \vec{f} \) is the body force vector. In Eq.(6), \( \vec{c} \) is the stiffness tensor of rank four. In the case of ultrasonic wave propagation in liquid, we can use above equations but have to set \( \tau_{ij} = 0 \) (\( i \neq j \)).

According to the equations above, FIT performs integrations over certain control volumes \( V(= \Delta x^3) \) and over the surfaces \( S(= \Delta x^2) \), assuming constant \( \vec{n} \) and \( \tau \) within \( V \) and on each of the surfaces \( S \). This method produces staggered grids and leads to a very stable and efficient numerical code allowing an easy and flexible treatment of various boundary conditions. The spatial grid of the three dimensional (3D) FIT code for ultrasonic wave is shown in Fig.2.

### 3.2. Time Discretization

In the time domain, stress components \( \tau \) is allocated at half-time steps, while the velocities \( \vec{v} \) is at full-time steps. The following time discretization yields an explicit leap frog scheme:

\[
[t_i]^n = [t_i]^{n-1} + \Delta t [t_i]^{n+\frac{1}{2}} \\
[\tau_{ij}]^{n+\frac{1}{2}} = [\tau_{ij}]^{n-\frac{1}{2}} + \Delta t [\tau_{ij}]^{n+\frac{1}{2}}
\]

(7)

where \( \Delta t \) is the time interval and the superscript \( n \) denotes integer number of the time step. Therefore the FIT repeats the operations of Eq.(7) from \( n=1 \) to \( N \) by means of adequate initial and boundary conditions. A specific stability condition and adequate spatial resolution must be fulfilled to calculate the FIT accurately\(^{(11)}\).

### 3.3. Image–Based Modeling

In our simulation, geometries of targets are determined from an actual object image such as digital picture, X-ray CT, CAD data, and so on. After processing of the image, pixel data for the 2D case or voxel data for the 3D case are fabricated for the FIT. Here voxel is “volumetric pixel”, representing a value on a regular grid in 3D space. The pixel or voxel data are directly used as the computational cell in the FIT. Namely the voxel (pixel) size is set to be equivalent to the cell size of the FIT. The numerical modeling with this simplified pre-process is called “the image-based modeling\(^{(9)}\).” A procedure of the 3D image-based modeling is shown using a 3D curve measurement system “TRiDY\(^{(14)}\).” The TRiDY consists of a projector that flashes narrow bands of black and white light onto the model’s face and a camera. Then the camera captures the shapes based on the distortion from the lights. As shown in Fig.3, a 3D shape of a corroded steel plate is reconstructed from surface profiles which are measured at multiple angles. After unnecessary parts for calculation are trimmed away, the rest part is discretized into a voxel data.

### 3.4. Simulation of 3D Ultrasonic Wave Propagation in Corroded Steel Plate

A simulation of ultrasonic wave propagation is demonstrated using the model in Fig.3. The dimensions of the model are as follows: the width=approx.94mm, cross section = approx. 6.5mm \( \times \) 20mm. Material parameters are that the longitudinal wave velocity \( c_L = 5800m/s \), the transverse wave velocity \( c_T = 3100m/s \) and the density \( \rho = 7800kg/m^3 \). We choose \( \Delta x = 0.05mm, \Delta t=4 \) nano-second (ns) and \( N=5400 \). Total number of the voxel is approx. 70 million. The ultrasonic wave with the center frequency 1MHz is radiated from an area of left-hand side of the model as shown in Fig.3. Figure 4 shows the absolute value of the displacement vector \( |\vec{u}(= \int \vec{u} dt)\). In the plate, ultrasonic wave propagates as the guided wave with dispersions. At the center part of the model, it is difficult to transmit the ultrasonic wave to the other side due to the local thinning of the plate. The calculation time was approximately 1 hour with parallel computers\(^{(15)}\) with 64 CPUs (Flat MPI).

As shown in the above example, we are capable of carrying out simulations with a realistic model of the target. The accuracy of the image-based FIT has been already verified in a welding inspection\(^{(12)}\) and a plate inspection using SH waves\(^{(13)}\).
4. Verification of the Simulation-aided Flaw Inversion Method

Here we check the accuracy of the simulation-aided flaw inversion method by means of experimentally measured waveforms. The experimental setup is shown in Fig.5. The measurement of the ultrasonic wave is conducted with a commercial electronic scanner “Hitachi ES-3100”. The array transducer consists of total 72 elements, its element pitch is 0.8mm (bottom side in Fig.5). Figure 6 shows a reflected signal from the bottom surface of a metal when we transmit ultrasonic wave with one of the array elements in Fig.5. The peak frequency is about 2.7MHz.

As a target of the shape reconstruction, we consider a slit at the bottom side in a specimen (aluminum, \( c_L = 6368 \text{m/s}, \rho = 2700 \text{kg/m}^3 \)) as shown in Fig.7. To increase the complexity of the wave field, we set a medium “polystyrene” between the transducer and specimen. Therefore the ultrasonic wave propagates in two media, aluminum and polystyrene. Polystyrene is an acoustically different material from aluminum, and its properties are \( c_L = 2323 \text{m/s} \) and \( \rho = 1000 \text{kg/m}^3 \). The ultrasonic beam is transmitted to the slit from the array elements No.1–24. After this illumination process of the slit due to the ultrasonic beam, the scattered waveforms from the slit are recorded individually at each element of No.25–26, etc.
72, namely $M=48$. In the next step, the recorded 48 waveforms are time-reversed and emitted in the 2D image-based FIT. The simulation area is limited in the square surrounded by aqua color in Fig.7. Snapshots of ultrasonic wave propagation in the 2D image-based FIT are shown in Fig.8. Figure 8 shows the absolute value of the displacement vector $|u| = |\int \nu dt|$. Time reversal waves are shown in Fig.8(a) and arrive at the interface between the polystyrene and the aluminum in Fig.8(c). The emitted waves results in a constructive summation of all individual waves. The focal point of the waves shows both tip and corner of the slit in Fig.8(e). From the visualization result of wave propagation, it is possible to estimate the size of the slit.

5. Conclusions

Here we proposed a simulation-aided flaw imaging method based on the time reversal approach. In the first process, we illuminate the target flaw by an ultrasonic beam, and then scattered waves from the flaw are recorded with an array transducer. Next the time reversal waves of scattered waves are emitted into the image-based FIT simulation. In the simulation, the emitted waves propagate and focus on the flaw even in inhomogeneous and complex media. The flaw shape can be estimated visually through the focal point of ultrasonic waves. The performance of our inversion method was checked with an experimental measurement. As a future work, we will apply our inversion method to more complicated material.

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Fig. 7  Aluminum specimen with an artificial slit. Ultrasonic wave is transmitted from elements of the left side in the transducer. Elements of the right side in the transducer receive scattered wave from the slit.

Fig. 8  Snapshots of ultrasonic wave propagation when the time reversal waves are emitted into the image-based FIT simulation.

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