Interference Effect of Stress Wave in a Finite Length Strip with Circular Cut-Outs*

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Abstract
A method of reducing the concentration of dynamic stress, which utilizes the interference effect of the stress wave, was investigated. Impact compression load is applied to a strip specimen with circular cut-outs made of polymethylmethacrylate. These cut-outs consist of a central cut-out and two additional cut-outs with varied diameter, \( d \), and distance between the cut-outs, pitch \( p \), on the strip. Dynamic elastic stresses on the minimum section of the cut-outs are calculated using the DYNA3D software, and the stress concentration factor \( K \) was also obtained. The \( K \) at the central cut-out decreased as the diameter of the added cut-outs was increased. Because of the interference of the reflected wave and the circumferential propagation of the stress wave around the edges, it is considered that the amplitude of the stress at the edge of the central circular cut-out was reduced. Therefore, this reduction method in which circular cut-outs are added is useful for the impact safety design of mechanical structures. Experiments showed similar results.

Key words: Interference Effect of Stress Wave, Circumferential Propagation of Stress Wave, Circular Cut-Outs, Concentration of Dynamic Stress, Stress Rate

1. Introduction
In thin walled structures, such as airplanes and vehicles, many cut-outs are prepared because of the necessity of weight reduction or the structural design. Research on the static stress concentration of these cut-outs\(^{(1)}\) has been systematically summarized. When an impact load is applied on a structural member, the stress wave is propagated, and a break is produced in the cut-out portion. Therefore, to solve the impact break problem, the concentration of dynamic stress should be clarified \(^{(2)-(4)}\) and it is very important to show the
method of reducing the concentration of dynamic stress.

On the other hand, many research studies\(^{(5)-(12)}\) have been carried out on the impact strength of the structural member that has imperfection as represented by the crack and notch. The authors have so far discussed the impact strength\(^{(13)-(17)}\) from the viewpoint of stress wave propagation. In the case of a notched member under impact loading, the characteristics of the stress wave propagation, in which the stress wave is propagated around the notched portion, and the existence of the interference effect caused by those stress waves were clarified\(^{(13)}\). Then, two circular cut-outs were newly added to both sides of the existing single circular cut-out. It is considered that the reduction of the concentration of dynamic stress, which utilized the circumferential propagation of the stress wave and the interference effect of the stress wave, was expected.

In this study, the impact compression load is applied to a finite length strip with a train of circular cut-outs; the new method of reducing the concentration of dynamic stress is examined from the viewpoint of stress wave propagation, and the validity is described.

**Nomenclature**

- \(B, L\) and \(t\): breadth, length and thickness of specimen
- \(d_0\) and \(d_1\): diameters of central and additional circular cut-outs
- \(d_1/d_0\): circular cut-out ratio
- \(E\): longitudinal elastic modulus
- \(K_{0a}, K_{ia}, K_{fa}\): concentration factors of dynamic strain at end of each cut-out
- \(K_{0r}, K_{ir}, K_{fr}\): concentration factors of dynamic stress at end of each cut-out
- \(p\): distance between center of central cut-out and added circular cut-out, pitch
- \(p/d_0\): ratio of pitch to diameter of central circular cut-out
- \(\varepsilon_m\): average strain amplitude of the minimum cross section on cut-out
- \(\varepsilon_{m}\): maximum strain amplitude at end of cut-out
- \(\nu\): Poisson’s ratio
- \(\rho\): density
- \(\sigma_m\): average stress amplitude of the minimum section on cut-out
- \(\sigma_{m}\): maximum stress amplitude at end of cut-out
- \(\sigma_m\): stress rate using \(\tau_m\)
- \(\sigma_{m}\): stress rate using \(\tau_m\)
- \(\tau_m\): time until stress at the end of central circular cut-out rises after impact load is applied
- \(\tau_m\): elapsed time until stress rises and becomes maximum at central cut-out, \(\sigma_{0m}\)
- \(\tau_m\): time until stress becomes maximum at central cut-out, \(\sigma_{0m}\) after impact load is applied

**Subscript**

- \(a\): average amplitude of dynamic stress and/or strain
- \(f\): free end of left site of specimen
- \(i\): impact loading site of specimen
- \(m\): maximum amplitude of dynamic stress and/or strain
- \(r\): rising of dynamic stress and/or strain
- \(0\): central circular cut-out
2. Examination Model for Interference Effect of Stress Wave

As shown in Fig. 1, the dimensions of the examination model are length $L = 500$ mm, breadth $B = 200$ mm, and thickness $t = 5$ mm. Model 1, the basic model, has a single circular cut-out, with diameter $d_0$, prepared in the center of the strip. Model 2, on which the comparison and examination of the reduction effect of concentration of dynamic stress is carried out in the loading direction, has two newly added circular cut-outs, one on each side of the central circular cut-out. Both new cut-outs had identical diameters, $d_1$. The distance between the cut-outs is the pitch $p$. The circular cut-out ratio is defined as $d_1/d_0$ and the non-dimensional pitch is defined as $p/d_0$. By using the compared results of the dynamic behavior of the stress wave in Models 1 and 2, a method of reducing the concentration of dynamic stress, which utilizes the interference effect of the stress wave, was examined. The dimension ratio of Model 1 is set to $d_0/B = 0.05, 0.1, 0.33, 0.5,$ and $0.7$. For Model 2, similarly, it is constant at $d_0/B = 0.33$, and it is referred to as $p/d_0 = 1.0$ and 2.0, and as $d_1/d_0 = 0.5, 0.65, 0.75,$ and 1.0.

3. Dynamic Numerical Analysis

Each examination model is divided using a four-node shell element, and dynamic elastic numerical analysis using the DYNA3D software is performed. Moreover, the dynamic behavior of the stress wave around the circular cut-out portions is considered. To obtain the concentration factors of dynamic stress at the end of the circular cut-out, $K_{mn}$, the symmetrical condition was set up on the $x$-axis of the strip as shown in Fig. 1, and the step wave of the nodal velocity $V_t = 1.0$ m/s was given to the strip. The material characteristics of the specimen are as follows: longitudinal elastic modulus, $E = 2.99$ GPa, Poisson’s ratio, $\nu = 0.36$, and density, $\rho = 1200$ kg/m$^3$. The elastic longitudinal propagation velocity, $C_L$, of a specimen is approximately 1600 m/s.

To analyze the impact problem by the finite element method, it is necessary to make the time increment $\Delta t$ smaller than the time to pass through the inside of the minimum element. In the DYNA3D software, the representation length of the element was set to $S$ and Eq. (1) was used to determine $\Delta t$ automatically. In this study, it is approximately $\Delta t = 2.73 \times 10^{-7}$ s.

$$\Delta t = \frac{S}{\sqrt{E/\rho}} \quad (1)$$

The dynamic stress histories of impact loading direction in each circular cut-out end at the minimum cross section are calculated. The maximum stress amplitude at the end of each circular cut-out is defined as $\sigma_{m}$, and the mean stress on the minimum cross section, when $\sigma_m$ is produced, is defined as $\sigma_s$. The concentration factor of dynamic stress $K_a$ is obtained using Eq. (2).

$$K_a = \sigma_m / \sigma_s \quad (2)$$
In addition, the mean stress $\sigma_a$ on the minimum cross section of each cut-out is obtained using Eq. (3), where $\Delta y_i$ is the width of the element in the $i$-th finite element and $\sigma_{ai}$ is the stress in the $x$-axis direction of the element.

$$\sigma_a = \frac{\sum_i (\Delta y_i \times \sigma_{ai})}{\sum_i \Delta y_i} \quad (3)$$

4. Experiments of Impact Compression

4.1 Specimen and Cemented Portion of Strain Gages

The material of the specimen is polymethylmethacrylate (PMMA). The dimensions of the specimen are the same as those described in the previous section. The edge of the circular cut-out and each contact surface were finished smoothly. The dynamic strain component at the edge of the circular cut-out being measured was taken as the $x$-axis direction. The cemented portion of the strain gage is shown in Fig. 2. When $d_0/B = 0.33$ is shown in an example of Model 2, in the case of $d_1/d_0 = 0.5$, it is $y_1 = 32$ mm, $y_2 = 30$ mm, $y_3 = 6$ mm. On the added circular cut-out, in the case of $d_1/d_0 = 0.5$, it is $y_1 = 36$ mm, $y_2 = 40$ mm, $y_3 = 7$ mm, and, in the case of $d_1/d_0 = 1.0$, it is $y_1 = 32$ mm, $y_2 = 20$ mm, $y_3 = 6$ mm. Although not shown in Fig. 2, in any case, the distance from the center of the strain gage to the edge of the circular cut-out is $y_0 = 2$ mm. The measured strain is used and the concentration factor of dynamic strain $K_a$ is obtained using Eq. (4).

$$K_a = \frac{e_m}{e_a} \quad (4)$$

The mean strain $e_a$ on the minimum cross section of the circular cut-out is obtained using Eq. (3).

4.2 Loading Method of Impact Compression and Experimental Device

The experimental device, as shown in Fig. 2, was manufactured and the dynamic strain at the end of the circular cut-out was measured. The impactor is a steel plate of 99 mm width, 271 mm height, 11.5 mm thickness, and 2 kg mass, and this is hanged by four connecting bars with the distance between fulcrums being 75 mm. As shown in Fig. 2, aluminum pipe rollers, $R_1$, were arranged in the vertical direction at the side surface of the specimen to prevent the specimen from falling. To reduce the friction between the specimen and the sidewall of the experimental device, aluminum pipe rollers, $R_2$, were placed between both. The experimental device was adjusted so that the impactor might contact with the specimen and a plane at the lowest point. The pendulum-type impactor with an arm length of 115 mm was swung down from $\theta = 50^\circ$, and impact compression load was given to the specimen. The calculated impact velocity when the impactor reaches the lowest point was $V = 0.90$ m/s. The characteristics of the aluminum pipe rollers and strain gages are listed in Fig. 2.

![Fig. 2 Experimental setup.](image-url)
5. Results and Discussion

5.1 Concentration of Dynamic stress on a Single Circular Cut-out

An example of strain histories of Model 1 is shown in Fig. 3 (a). The strain gage at the impact loading site is $G_S$, and those at the end of the circular cut-out on the minimum cross section are $G_{01}$, $G_{02}$, and $G_{03}$. The amplitude obtained by the strain gage $G_{01}$ becomes the maximum $G_{01,m}$, and $G_{0a}$ is the mean value on the minimum cross section at the same time. The concentration factor of dynamic strain $K_{0a,c}$, which is obtained using Eq. (4), is shown in Fig. 3 (b). The propagated distance of the stress wave when the strain amplitude becomes maximum is made into $L_B$, which is the position of the wave front. The non dimensional propagated distance of the stress wave $L_B$ is defined as the ratio $L_B = L_p/L$. $L_B$ is shown in Fig. 3 (b). According to Fig. 3 (b), $d_p/B$ increases, $K_{0a,c}$ decreases gently, and $L_B$ increases, taking a value larger than 1. That is, after a stress wave has phase reversed and is reflected at the free boundary of the left side of the strip, it propagates toward the central circular cut-out, as shown in Fig. 3 (b).

For Model 1, by using the maximum stress $G_{0m}$ at the end of the cut-out and mean stress $G_{0a}$ at the minimum cross section, the concentration factors of dynamic stress $K_{0a,c}$, which are obtained using Eq. (2), are shown in Fig. 4 (a). While according to Fig. 4 (a), $d_p/B$ becomes large and $K_{0a,c}$ becomes small, whereas both $G_{0m}$ and $G_{0a}$ increase. Therefore, since the risk of breakage at the end of the cut-out increases, caution with a structural member concerning safety design is required. Moreover, $L_B$ in Fig. 4 (b), which shows the non dimensional propagated distance of the stress wave, is the same as that in the experimental results.

$$L_B = \frac{L_p}{L}$$

\[ L_p \text{: length of strip} \]

Fig. 3 Experimental results (Model 1).

Fig. 4 Stress concentration factors and propagated distance of stress wave (Numerical results, Model 1).
5.2 Reduction of Stress Concentration by Interference Effect of Stress Wave

The typical examples of stress history are shown in Fig.5. Although \( d_1/d_0 \) increases to 0.50, 0.65, and 0.75 and \( \sigma_{\text{in}} \) decreases, \( \tau_m \) that takes \( \sigma_{\text{in}} \) is almost constant. If it is set to \( d_1/d_0 = 1.0 \) and each end of the cut-out touches together, the magnitude of \( \sigma_{\text{in}} \) decreases markedly, and \( \tau_m \) also becomes large. The case of \( p/d_0 = 2.0 \) shows almost the same tendency as \( d_1/d_0 = 1.0 \). \( K_{i,a}, K_{i,a} \) and \( K_{i,a} \) at the end of the cut-out are shown in Figs. 6 (a) and (b). In the cases of \( p/d_0 = 1.0 \) and 2.0, \( d_1 \) increases, and \( K_{0,a} \) becomes small, and the stress concentration becomes lower than that in the case of Model 1. However, \( K_{i,a} \) is smaller than \( K_{0,a} \) in each \( p/d_0 \). By providing a cut-out before and after the loading direction of compression to the existing single circular cut-out, it is possible to carry out the reduction of concentration of dynamic stress using the interference effect of the stress wave. This method is considered useful for the safety design of mechanical structures. The tendencies of strain concentration factors based on the experiments as shown in Fig.7 are almost the same as numerical results.

![Fig.5 Stress histories (Numerical results, Model 2, \( d_1/B = 0.33, p/d_0 = 1.0 \)).](image)

![Fig.6 Stress concentration factors (Numerical results, Model 2, \( d_1/B = 0.33 \)).](image)

![Fig.7 Strain concentration factors (Experimental results, Model 2, \( d_1/B = 0.33 \)).](image)
When Fig. 8 that shows $L_B$ is examined, $d_1/d_0$ is increasing and will increase gradually in approximately $L_B = 1$. The stress will become the maximum in a comparatively short time after impact loading. When compared with $p/d_0 = 1.0$ and 2.0 of Model 2, $L_B$ takes approximately 1 law smaller in the range in the case of $p/d_0 = 1.0$ whose $d_1/d_0$ is smaller than 1. That is, if $d_1$ becomes large, the distance between the central cut-out and added cut-outs approaches. The number of interference times per unit time of the stress wave of reversed phase and the stress wave from another side, which is reflected on each free boundary at the end of the cut-out, increases. It is considered that the stress amplitude at the end of the central cut-out was reduced by this. In $p/d_0 = 2.0$, although $L_B$ is larger than that in the case of $p/d_0 = 1.0$ by approximately 10%, if $d_1/d_0$ exceeds 0.5, the magnitude of $L_B$ is about 1 law. Therefore, for the amplitude of the stress wave to become maximum, a certain fixed time is necessary (7).

It is shown in Fig. 9 by taking as an example the stress distribution in the case of $\tau = 0.335$ ms, which took 0 ms for the dynamic behavior of the stress wave in Model 2. Around the central cut-out, the stress domain enclosed with a white line passed the free boundary of the strip upper part to the central cut-out, and propagated around the added cut-out site at left side.

Concerning the stress value at the end of the cut-out, $p/d_0 = 1.0$ is shown as an example in Fig. 10. When $d_1$ is increasing, $\sigma_{im}$ is decreased. This tendency is the same in the case of $p/d_0 = 2.0$; the difference in $p/d_0$ is not significant. $d_1$ follows $\sigma_{im}$ and $\sigma_{im}$ in becoming large, and increases them slightly. In particular, the stress value at the end of three cut-outs being close in $d_1/d_0 = 1.0$ is interesting.

Fig. 8 Propagated distance of stress wave (Numerical results, Model 2, $d_0/B = 0.33$).

Fig. 9 Circumferential propagation of stress wave around circular cut-outs (Model 2, $d_0/B = 0.33$, $p/d_0 = 1.0$, $d_1/d_0 = 0.5$).

Fig. 10 Stress at the end of circular cut-outs (Model 2, $d_0/B = 0.33$, $p/d_0 = 1.0$).
5.3 Evaluation of the Impact Strength by Stress Rate

According to Fig. 5 that shows stress histories at the end of the central cut-out, the time at which $\sigma_{0m}$ changes with geometric dimensions, such as $d_1/d_0$, and takes $\sigma_{0m}$ similarly. Then, it is considered using the stress rate on the basis of stress histories as the evaluation of the impact strength. The following term is defined in reference to Fig. 5. That is, $\tau_0$ is the time until stress at the end of a central circular cut-out rises after an impact load is applied, $\tau_m$ is the elapsed time until stress rises and becomes maximum in the central cut-out, $\sigma_{0m}$ and $\sigma_{om}$ is the time until the stress becomes maximum at the central cut-out, $\sigma_{0m}$, after an impact load is applied. The stress rate is calculated using Eqs. (5) and (6).

$$\dot{\sigma}_m = \frac{\sigma_{0m}}{\tau_m}$$  \hspace{1cm} (5)

$$\dot{\sigma}_rm = \frac{\sigma_{0m}}{\tau rm}$$  \hspace{1cm} (6)

The calculated results of stress rate are shown in Fig. 11. $\dot{\sigma}_m$ and $\dot{\sigma}_rm$ decrease as $d_1/d_0$ increases in the cases of $p/d_0=1.0$ and 2.0. When the distance at the center of the circular cut-out, $p$, increases, $\dot{\sigma}_m$ and $\dot{\sigma}_rm$ both decrease. Furthermore, $\dot{\sigma}_m$ is larger than $\dot{\sigma}_rm$. Therefore, it is considered that the impact strength of the finite length strip with circular cut-outs can be evaluated using the stress rate based on stress histories.

![Stress rate](image)

Fig. 11 Stress rate (Numerical results, Model 2, $d_i/B = 0.33$).

6. Conclusions

In this paper, a method of reducing the concentration of dynamic stress, which utilizes the interference effect of a stress wave, is investigated. The results are as follows.

(1) The concentration of dynamic stress at the end of circular cut-outs could be reduced because of the interference of the reflected wave and the circumferential propagation of the stress wave around the edges.

(2) The impact compression strength of a finite length strip with circular cut-out trains can be evaluated using the stress rate based on stress histories.

(3) This reduction method for arranging cut-outs as a train before and after the loading direction to one existing circular cut-out is useful for the impact safety design of structures.

7. References


