Stability of a Soft Plate in Channel Flow*

(Aerodynamic Aspects of Palatal Flutter)

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The present paper gives a computational method for the flutter analysis of a soft plate placed in two-dimensional subsonic channel flow, remarking the palatal flutter in snoring. The computations were carried out first for the preliminary case of a hard plate oscillating in windtunnel and then for the case of a soft plate oscillating with standing- or travelling-wave mode, simulating the palatal oscillation. The results obtained for a hard plate are in good agreement with those for the windtunnel wall effects, showing the validity of the present method. The results for a soft plate show that the palatal flutter can be caused by the oscillation of travelling-wave mode, and that the flutter is slightly promoted by the channel walls but slightly suppressed by the hard plate attached fore the soft plate. The effect of mechanical damping is also discussed.

Key Words: Flutter, Snoring, Palate, Subsonic Flow, Windtunnel

1. Introduction

Recently, attention has been progressively paid on the mechanism of the generation of snoring, which would be sometimes fatal. The snoring is considered to be caused by the self-excited oscillation of the soft palate in narrow airway. Huang⁽¹⁾ carried out experimental and theoretical studies on snoring to analyze its mechanism from the aeroelastic point of view. His theoretical study, however, was just to apply the ordinary flutter analysis of an isolated aerofoil in incompressible flow, taking no account of the effects of airway-wall and hard palate.

The wall effects have been studied for the interference of windtunnel wall on a hard aerofoil in both steady and unsteady cases. Extending the method for an isolated aerofoil, Runyan et al.⁽²⁾ carried out a theoretical analysis on a thin aerofoil oscillating in subsonic flow and predicted the wall effects quantitatively. Further, for the same problem, Lane and Friedman⁽³⁾ presented a sophisticated analysis, in which the Fourier transform was applied to the kernel function in the aerodynamic integral equation, giving more exact results.

The present paper gives a computational method for the flutter of rigid and soft plates placed at arbitrary positions in two-dimensional subsonic channel flow. The kernel function in the aerodynamic integral equation is transformed to a simpler form by the conversion of the series of Hankel functions into that of exponential functions⁽⁴⁾, which was successfully applied to the analysis of sound propagation through a blade row⁽⁵⁾. This conversion leads to much easier calculations in subsonic flow as well as in incompressible flow as a limiting case. The computations were carried out only in the case where a plate was placed on the channel axis, first for a hard plate oscillating in windtunnel and then for a soft plate oscillating with travelling-wave mode, remarking the palatal flutter. The effect of plate position will be discussed in the following paper.

2. Formulation

Figure 1 shows a fundamental model, in which a thin plate placed at an arbitrary position in two-dimensional channel oscillates with small amplitude in subsonic, inviscid and isentropic flow. Here, U is the flow velocity, a_∞ is the speed of sound, M is the Mach number ($= U/a_\infty$), ρ_∞ is the density of fluid, b is the half chord length of plate, d is the channel height, and (x, y) and (ξ, η) are the coordinates, in which (ξ, η) is related to the plate position. Further, p is the pressure and t is the time.

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Fig. 1 Model and Nomenclatures

Using the Euler and continuity equations and neglecting the small terms of higher orders, the governing equation of the flow is given as

$$(1-M^2) \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} - \frac{1}{a_\infty^2} \frac{\partial^2 \Phi}{\partial t^2} - \frac{2M}{a_\infty} \frac{\partial^2 \Phi}{\partial x \partial t} = 0 \quad (1)$$

where $\Phi = -(p - p_\infty)/\rho_\infty U^2$ is the acceleration potential.

Assume that the plate oscillates steadily with a frequency ω , then it can be represented by a sheet of doublets of strength $A(\xi)\exp(i\omega t)$, with their axes perpendicular to the chord. The doublet strength is directly proportional to the pressure jump across the plate. That is,

$$A(\xi)\exp(i\omega t) = \frac{p_u - p_l}{\rho_\infty U^2} \quad (2)$$

where p_u and p_l denote the pressures on the upper and lower surfaces of the plate.

Equation (1) gives the acceleration potential induced by a doublet at (ξ, η) as

$$\Phi_d = \frac{U^2 A(\xi)}{4\sqrt{1-M^2}} \frac{il(y-\eta)}{r} H_1^{(2)}(lr) \times \exp\left[i\left\{\omega t + \frac{Mk}{1-M^2}(x-\xi)\right\}\right] \quad (3)$$

where

$$r^2 = \frac{(x-\xi)^2}{1-M^2} + (y-\eta)^2$$

$$k = \frac{\omega}{a_\infty}, \quad l^2 = \frac{k^2}{1-M^2}$$

and $H_n^{(2)}$ is the Hankel function of the second kind.

Assume that the perturbations vanish far upstream of the plate, and from the Euler equation, the upwash velocity induced by a doublet at (ξ, η) is given as

$$v_d(x, y; \xi, \eta) = \frac{1}{U} \exp\left(-i \frac{kx}{M}\right) \times \int_{-\infty}^x \frac{\partial \Phi_d}{\partial y} \exp\left(i \frac{kx}{M}\right) dx \quad (4)$$

Substitute Eq. (3) into Eq. (4), then⁽⁶⁾

$$v_d(x, y; \xi, \eta) = -i \frac{UA(\xi)}{4\sqrt{1-M^2}} \exp(i\omega t) \times \left[\exp\left\{i \frac{Mk(x-\xi)}{1-M^2}\right\} \cdot \left\{ \frac{l}{r}(x-\xi) H_1^{(2)}(lr) + i \frac{k}{M} H_0^{(2)}(lr) \right\} + \frac{k^2}{M^2} \exp\left\{-i \frac{k}{M}(x-\xi)\right\} \right] \times \int_{-\infty}^x H_0^{(2)}(lr) \exp\left\{i \frac{k(x-\xi)}{M(1-M^2)}\right\} dx \quad (5)$$

In the present case, the images of the plate due to the wall reflection should be considered such that the doublets of strength $A(\xi)\exp(i\omega t)$ are on $(\xi, \eta+2md)$ and $-A(\xi)\exp(i\omega t)$ on $(\xi, -\eta+2md)$, where $m=0, \pm 1, \pm 2, \dots$. Hence, the upwash velocity induced by a doublet in channel flow is

$$v(x, y; \xi, \eta) = \sum_{m=-\infty}^{+\infty} \{v_d(x, y; \xi, \eta+2md) - v_d(x, y; \xi, -\eta+2md)\} \quad (6)$$

which consists of the series of the Hankel function.

Applying the conversion of the series of the Hankel functions into that of the exponential functions^{(4),(5)}, Eq. (6) is transformed as

$$v(x, y; \xi, \eta) = \frac{UA(\xi)}{d} \exp(i\omega t) \times \sum_{\nu=1}^{\infty} \left[K_\nu^{(1)} \exp\left\{i \frac{Mk}{1-M^2}(x-\xi) - q\sqrt{\nu^2-w^2}\right\} + K_\nu^{(2)} \{1 + \text{sgn}(x-\xi)\} \right] \sin \nu p_y \sin \nu p_\eta \quad (7)$$

where

$$p_y = -\frac{\pi y}{d}, \quad p_\eta = -\frac{\pi \eta}{d}$$

$$q = \frac{\pi|x-\xi|}{d\sqrt{1-M^2}}, \quad w = \frac{kd}{\pi\sqrt{1-M^2}}$$

$$K_\nu^{(1)} = \frac{M\nu^2}{\sqrt{\nu^2-w^2} \{ \text{sgn}(x-\xi) \cdot M\sqrt{\nu^2-w^2} - iw \}}$$

$$K_\nu^{(2)} = \frac{(1-M^2)w^2}{w^2 + M^2(\nu^2-w^2)}$$

The details are given in Ref. (7).

Carrying out further rearrangement of Eq. (7), the induced velocity on the plate ($y=\eta$) is then given by

$$v(x, \eta; \xi, \eta) = \frac{UA(\xi)}{d} \exp(i\omega t) \times \left[\exp\left\{i \frac{Mk}{1-M^2}(x-\xi)\right\} (S_0 + S_1 + S_2) + \{1 + \text{sgn}(x-\xi)\} \exp\left\{-i \frac{k}{M}(x-\xi)\right\} \cdot S_3 \right] \quad (8)$$

where

$$S_0 = \frac{1}{4} \text{sgn}(x-\xi) \cdot \sinh q \left\{ \frac{1}{\cosh q - 1} - \frac{1}{\cosh q - \cos 2p_\eta} \right\}$$

$$S_1 = i \frac{w}{4M} \log \left(\frac{\cosh q - \cos 2p_\eta}{\cosh q - 1} \right)$$

$$S_2 = \sum_{\nu=1}^{\infty} \left[\frac{M\nu^2}{\sqrt{\nu^2-w^2}} \frac{\exp(-q\sqrt{\nu^2-w^2})}{\text{sgn}(x-\xi) \cdot M\sqrt{\nu^2-w^2} - iw} \right]$$

$$\begin{aligned}
& -\left\{\operatorname{sgn}(x-\xi)+i\frac{w}{M\nu}\right\}\exp(-\nu q)\Big\}\sin^2\nu p_\eta \\
S_3 &= \frac{\pi}{4}a\left\{\cosh\pi a-\frac{\cosh a(\pi-2p_\eta)}{\sinh\pi a}\right\}; \\
a^2 &\equiv\left(\frac{1-M^2}{M^2}\right)w^2
\end{aligned}$$

Here, S_0 , S_1 and S_2 are obtained by the Taylor expansion of the $K_\nu^{(1)}$ term in the RHS of Eq.(7) with respect to w , giving the order of magnitude $O(w^0)$, $O(w^1)$ and $O(w^2)$, respectively. So, S_0 gives the quasi-steady component. S_3 is related to the $K_\nu^{(2)}$ term in the RHS of Eq.(7), which describes the shed wake vortex conveyed downstream by the primary flow. In the case of low oscillating frequency such that $w \ll 1$, S_2 can be neglected and only S_0 , S_1 and S_3 of closed forms can be considered, so the calculation is easily carried out with much less time.

Integrating Eq.(8) over the plate gives the velocity induced on $y=\eta$ by the whole doublets of the plate.

$$v(x, \eta) = \int_{-b}^b v(x, \eta; \xi, \eta) d\xi \quad -b \leq x \leq b \quad (9)$$

Assuming that the plate is oscillating with the displacement $f(x)\exp(i\omega t)$, the boundary condition is given as

$$v(x, \eta) = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\{f(x)\exp(i\omega t)\} \quad (10)$$

From Eqs.(8) to (10), the doublet distribution $A(\xi)$ can be obtained for a given mode of oscillation. According to the usual way of calculation, the doublet distribution function $A(\xi)$ is expressed by the Glauert series, which satisfies the singularity at the leading edge and the Kutta condition at the trailing edge of the plate. That is,

$$\begin{aligned}
A(\xi) &= A_0 \tan \frac{\theta}{2} + 2 \sum_{n=1}^{\infty} A_n \sin n\theta; \\
\frac{\xi}{b} &= \cos \theta, \quad 0 \leq \theta \leq \pi
\end{aligned} \quad (11)$$

Calculations are carried out for a finite number (N) of control points, such that the doublet is distributed discretely at $\theta = m\pi/N$ ($m=1, 2, 3, \dots, N$) and the boundary conditions are satisfied at $\theta = (m-1/2)\pi/N$, and the first N coefficients of the Glauert series are obtained.

The unsteady lift and moment about the mid-chord of the plate are given as

$$\left. \begin{aligned} \frac{L}{\pi\rho_\infty U^2 b} &= -(A_0 + A_1)\exp(i\omega t) \\ \frac{M}{\pi\rho_\infty U^2 b^2} &= -\frac{1}{2}(A_0 + A_2)\exp(i\omega t) \end{aligned} \right\} \quad (12)$$

In the case of a hard plate, the aeroelastic instability is predicted directly from these results of lift and moment. In the case of a soft plate accompanied by the deformation of its profile, however, the instability could be predicted by the energy input integrated over the plate. That is,

$$\begin{aligned}
E &= \int_0^{2\pi/\omega} \int_{-b}^b [A(\xi)\exp(i\omega t)]_R \\
&\times \left[\frac{\partial}{\partial t}\{f(\xi)\exp(i\omega t)\}\right]_R d\xi dt
\end{aligned} \quad (13)$$

where the suffix R denotes the real part of complex function. E is the aerodynamic work done to keep a given oscillation of the plate, so $E > 0$ gives positive aerodynamic damping to suppress the oscillation and $E < 0$ gives negative damping to cause flutter.

As is seen from Eq.(7), $A(\xi)$ should vanish at $w=1.0$, that corresponds to the condition of acoustic resonance of the channel flow. In the present paper, only the sub-resonance condition ($w < 1.0$) will be considered, when the induced perturbations diminish far upstream. The acoustic resonance frequency, that is for $w=1.0$, is given as

$$\omega_{res} = \frac{\pi a_\infty}{d} \sqrt{1-M^2} \quad (14)$$

3. Results

3.1 Oscillation of a hard plate

To confirm the validity of the present method, the case of a single hard flat-plate allocated on the channel axis ($\eta=d/2$) was studied first.

Figure 2 shows a comparison of the calculated results of the plate oscillating in pitch, $a\exp(i\omega t)$, about the midchord, where ω/ω_{res} corresponds to w . It is seen that the present results agree quite well with the others, particularly with the Lane and Friedman's⁽³⁾ which is considered to be more accurate than the Runyan et al.'s⁽²⁾. Comparing with the case of an isolated aerofoil ($d=\infty$), the channel wall acts to increase the magnitudes of lift and moment, accompanied by the slight increase of phase lag. As the oscillating frequency approaches the acoustic resonance frequency, however, the lift and moment vanish, accompanied by the remarkably increasing phase lag.

The dotted lines give the results obtained by the approximate method, which neglects S_2 in Eq.(8). The approximation is likely to hold for w being less than 0.2.

The one-degree-of-freedom oscillation of the hard plate is stable except near the resonance condition, so no further descriptions are given here.

3.2 Oscillation of a soft plate

Here will be considered the case corresponding to the palatal flutter in snoring. In the human upper airway anatomy, the palate consists of hard and soft parts, so here consider a soft plate (l_s) attached with a stationary hard plate (l_h), which are allocated on the channel axis, as shown in Fig. 3, where $2b = l_s + l_h$.

3.2.1 Mode of oscillation Figure 4(Upper) shows an example of the real oscillation of a soft plate obtained by Huang⁽¹⁾. The soft plate oscillates in a

Fig. 2 Comparison of Calculations
Upper: Lift ($d/b=7.6$)/Lift ($d/b=\infty$)
Lower: Phase of Lift to Pitching Motion

travelling-wave mode, which has the amplitude similar to the first mode of cantilever oscillation but with a phase lag.

Then, it is assumed that the soft plate oscillates with the first cantilever mode, accompanied by some phase lag, as shown in Fig. 4 (Lower). That is,

$$\left. \begin{aligned} f(\tilde{x}) \exp(i\omega t) &= f_s(\tilde{x}) \exp\left\{i\omega\left(t - \frac{\tilde{x}}{V}\right)\right\} \\ f_s(\tilde{x}) &= C \left(\frac{\cosh \beta \tilde{x} - \cos \beta \tilde{x}}{\cosh \beta l + \cos \beta l} - \frac{\sinh \beta \tilde{x} - \sin \beta \tilde{x}}{\sinh \beta l + \sin \beta l} \right) \quad 0 \leq \tilde{x} \leq 2b \end{aligned} \right\} \quad (15)$$

where $\tilde{x}$ is the distance from the fixed end of the soft plate, V the travelling-wave velocity, and $\beta l_s = 0.60\pi$, which satisfies $\cos \beta l_s = -1/\cosh \beta l_s$. The phase lag at

Fig. 3 Palate Model

Fig. 4 Travelling-Wave Oscillation of a Soft Plate
Upper: Real Oscillation⁽¹⁾
Lower: Eq. (15)

the free end is given by $\phi = \omega l_s/V$. The local amplitude is normalized by the free-end amplitude.

3.2.2 Flutter of a soft plate Applying Eq. (15) to Eq. (10), the air force distribution is obtained, and then the energy input E can be calculated by Eq. (13). Neglecting the mechanical damping, the stability of the plate is estimated by E , or whether E is positive or not.

Figure 5 gives the energy input E (arbitrary scale) against the dimensionless (reduced) frequency $\omega l_s/U$ in the case of $d/l_s=0.5$ with no hard plate ($l_h=0$). E increases monotonically as $\omega l_s/U$ increases in the frequency range considered. The oscillation of standing-wave mode ($\phi=0$) is always stable ($E>0$), but the travelling-wave mode ($\phi>0$) brings about negative energy input or negative damping ($E<0$)

Fig. 5 Energy Input in Travelling-Wave Oscillation ($M=0.01$ and 0.5 , $d/l_s=0.5$, $l_h=0.0$)

Fig. 6 Effect of Mach Number ($d/l_s=0.5$, $l_h=0.0$)

below a critical reduced frequency. That is, for a given oscillation of travelling-wave mode, flutter may occur beyond a critical flow velocity. As the phase lag increases, E decreases to shift downward nearly in parallel, so the critical (flutter) reduced frequency increases.

The effect of the Mach number is mostly to increase the magnitude of E . In low Mach number range, where the oscillating frequency is much smaller than the resonance frequency, it is recognized that the magnitude of E is likely to be governed by the Prandtl-

Fig. 7 Effect of Channel Height. $\circ$: Huang⁽¹⁾ ($M=0.1$, $l_h=0.0$, $\phi=2.0$)

Fig. 8 Effect of Hard Plate ($M=0.1$, $d/l_s=0.5$, $\phi=2.0$)

Glauert rule, so there is no marked change of the flutter frequency for $M < 0.5$, as shown in Fig. 6. Hence, in the following, M is taken as 0.1, particularly remarking the palatal flutter.

Huang showed that the free end of the plate oscillates with the phase lag of about two radians behind the fixed end, nearly irrespective of its length. So, the phase lag will be taken here as $\phi=2.0$.

Figure 7 shows the effect of the channel height on the flutter reduced frequency $(\omega l_s/U)_f$ in the case with no hard plate, predicting that the wall may promote the flutter slightly. The flutter reduced frequency $(\omega l_s/U)_f$ is around 1.90, while the travelling-wave velocity is nearly equal to the flow velocity, $V=0.95U$. The circles in Fig. 7 are the experimental results of the flutter reduced frequency obtained by Huang for a thin plastic sheet⁽¹⁾. He also estimated the

travelling-wave velocity to be nearly equal to the flow velocity. His results agree well with those of the present calculation. This agreement would be attributed to the use of a thin plastic sheet by Huang, which may have small mechanical damping.

Figure 8 shows the effect of the hard plate (l_h) attached fore the soft plate in the case of $d/l_s=0.5$. It is seen that the hard plate acts as the spoiler to suppress the flutter of the soft plate.

3.2.3 Consideration on the effect of mechanical damping Huang also carried out an acoustic experiment on snoring, using a natural-rubber plate. He did not give the critical reduced frequency and the travelling-wave velocity explicitly, but these are estimated as 0.75 and 0.38, respectively. These are much smaller than, or nearly one third as much as, those for the case of the plastic plate.

Observing the free oscillation of a natural-rubber plate, it is strongly damped out with large logarithmic decrement, such as $\delta=0.5^{(8)}$. From the consideration on a simple mass-spring system, the energy dissipation for one cycle of the oscillation of unit amplitude is approximately estimated as

$$W \sim \left(\frac{m}{\pi \rho_\infty l^2} \right) \left(\frac{\omega l}{U} \right)^2 \delta$$

where m and l are the mass and the characteristic length of the oscillating body.

Applying this mechanical damping to the present calculation, the flutter will occur when $E+W<0$ holds. For the case of the natural-rubber plate (e.g. 100 mm long and 1 mm thick), the mass ratio ($m/\pi \rho_\infty l_s^2$) is in the order of 5 and the logarithmic damping coefficient δ is about 0.5. Assume $W=2(\omega l_s/U)^2$, then from E for $\phi=2.0$ in Fig. 5, the flutter reduced frequency $(\omega l_s/U)_f$ and the travelling-wave velocity $(V/U)_f$ is estimated as 0.70 and 0.35, respectively, which are comparable to the Huang's results for the natural-rubber plate. This suggests that the mechanical damping gives a predominant effect on the flutter of the soft palate.

4. Conclusions

The present study gives a simple computational method for the flutter analysis of hard and soft plates placed in two-dimensional subsonic channel flow, and the results obtained are summarized as follows.

(1) The wall effects obtained for a oscillating hard plate are in good agreement with those by the

other methods, showing the validity of the present method.

(2) A soft plate is free from flutter when oscillating with a standing wave of the first cantilever mode. However, when the plate oscillates with a travelling-wave mode, energy may be supplied from the flow to the plate to cause flutter below a critical reduced frequency.

(3) The channel wall acts to promote the flutter, whereas the hard plate attached fore the soft plate acts to suppress the flutter, although both effects are not so significant.

(4) The mechanical damping could play a predominant role on the palatal flutter, so it should be estimated strictly.

The above mentioned results suggest that the palatal flutter could be predicted approximately by the analysis of an isolated soft plate, although the snoring may be caused by the palatal flutter in channel flow.

Finally, the present method could be easily applied to the various cases, such as the ground-effect on high-speed transportation, fish propulsion, etc.

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