Correlation Based Fluidelastic Analysis of Tube Arrays in Two-Phase Flow*

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In single phase flow, the fluid forces may be considered to be fully correlated along the tube span. This is not the case for two-phase flow. For tube lengths much longer than the typical linear dimension of the two phase flow unit structure (slug, bubble etc.), forces along the tube span are only partially correlated. In this paper, an unsteady flow model taking into account partial spanwise correlation of the fluid forces is presented. The model is applied to fluidelastic instability analysis of a long span heat exchanger tube model. It is shown that the effect of partial correlation is to reduce the effective net fluid forces which thus raises the effective instability threshold. Another important result is the introduction of additional coupling between modes that would otherwise be uncoupled in the fully correlated case. This has important implications for heat exchanger tubes which have closely spaced modes. Lastly, partial correlation is found to eliminate multiple instability boundaries at low values of the mass-damping parameter; this is a practically significant result.

Key Words: Fluidelastic Instability, Correlation, Tube Arrays, Two-Phase Flow

1. Introduction

Single phase flow studies are a precursor to the ultimate practical problem of fluidelastic instability in heat exchangers where the flow consists of two co-existing fluid phases. In principle, the existing unsteady flow models can be applied to two-phase flow induced vibration. However, the models become intractably complex for the case of U-bend tubes in two-phase flow.

One of the most important factors differentiating two-phase flow fluid forces from their single phase flow counterparts is the correlation along the tube span. Single phase flow models are derived assuming perfect correlation over the whole tube. It is this

assumption that makes it possible to start with a force balance relation on an infinitesimal (2D) element and derive governing equations applicable to the complete tube in 3D.

Physically, the effect of partial correlation is closely similar to the correlation of turbulence forces. Hence, at any given time the fluid forces acting at a certain location on the tube span is correlated to nearby forces over only a small fraction of the tube length. In general, this spatial correlation is time dependent; in other words, the true correlation of fluid forces is both space and time dependent. Clearly this two-dimensional correlation is quite complicated and at present only an approximation can be attempted.

The concept of force correlation is a fundamental part of the analysis of turbulence excitation. By modeling the turbulence force field as a stationary random process, structural response spectra can be derived. We now briefly review this procedure in order to introduce some of the underlying concepts.

Consider a tube subjected to a random force $f(s, t)$ per unit length. The total instantaneous force acting on the tube is

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$$F(t) = \int_0^l f(s, t) ds. \quad (1)$$

Where l is the tube length. The mean square load is defined by Ref. (1),

$$\bar{F}^2 = \iint f(s_1, t) f(s_2, t) ds_1 ds_2. \quad (2)$$

For a homogeneous random force, the time averaged force at any location s along the span is independent of s . This implies that for any two locations

$$\overline{f(s_1, t) f(s_2, t)} = \bar{f}_0^2. \quad (3)$$

When Eq. (3) holds, we can write,

$$\overline{f(s_1, t) f(s_2, t)} = \bar{f}_0^2 \gamma(\Delta_{12}), \quad (4)$$

where

$$\Delta_{12} = |s_1 - s_2|. \quad (5)$$

γ is the correlation function which only depends on the separation Δ_{12} between any two given points in the case of homogeneous turbulence. A commonly used form for γ is

$$\gamma(s_1, s_2) = e^{-\sigma \Delta_{12}}, \quad (6)$$

where σ is a factor usually determined experimentally. Introducing the correlation function into Eq. (2) we have

$$\bar{F}^2(t) = J^2 \bar{f}_0^2. \quad (7)$$

where the joint acceptance J is given by

$$J^2 = \iint \gamma(\Delta_{12}) ds_1 ds_2. \quad (8)$$

Now, since, $|J| < 1$, Eq. (7) implies that the effect of partial correlation is to reduce the effective magnitude of the fluid force.

In this paper, we extend the concept of partial correlation to the case of motion dependent unsteady fluid forces. Preliminary discussion of this work may be found in Nakamura et al.⁽²⁾. The starting point is the unsteady flow model developed by Chen⁽³⁾ using some experimental data from Tanaka and Takahara⁽⁴⁾.

Nomenclature

- $c, \bar{c}$: structural damping
- D : tube diameter
- EI : flexural rigidity
- f_{ij}, g_{ij} : unsteady fluid forces
- J : joint acceptance
- l : tube length
- l_c : correlation length
- m : tube mass per unit length
- R, R_0 : tube and U-bend radius, respectively
- s : position along tube span
- $U(s, t)$: upstream flow velocity
- x : cross-flow tube displacement (out of plane for U-bend)
- y : Inflow tube displacement (In-plane for U-bend)
- z : U-bend inplane rotational displacement

Fig. 1 U-bend tube model and local array section

α : angular position at U-bend (Fig. 1)

$\alpha_{ij}, \beta_{ij}, \sigma_{ij}, \tau_{ij}$: unsteady fluid-force coefficients

δ : logarithmic decrement of damping ($= c / 2m\omega_n$)

$\varphi_0(s)$: normalized density distribution function along tube span

Φ : inter-tube model pattern

γ : correlation

$\rho(s, t)$: fluid density

ρ_0 : average density along tube span

ω : frequency

$\phi(s), \phi(s), \eta(s)$: mode shape functions

2. Theory

2.1 Tube equations of motion

The physical model considered is the U-bend region of a steam generator, as shown in Fig. 1. To model the array dynamics, a small kernel of neighboring tubes is considered. For the inline array considered in this paper, the minimal kernel consists of five tubes as depicted in Fig. 1. Tube displacements are given by, x -out-of-plane, (y, z) -in-plane radial and tangential displacements, respectively. Two geometrical coordinates α and s are defined as shown in Fig. 1.

Out of plane tube displacement (x) for tube ' i ' is given by the following equation:

$$\frac{EI}{R_0^4} \frac{\partial^4 x_i}{\partial \alpha^4} + c \frac{\partial x_i}{\partial t} + m \frac{\partial^2 x_i}{\partial t^2} = \sum_{j=1}^{N(=5)} f_{ij}(u_j, v_j, \dot{u}_j, \dot{v}_j, \ddot{u}_j, \ddot{v}_j). \quad (9)$$

In the drag (radial y -) direction we have

$$\frac{EI}{R_0^4} \left(\frac{\partial^4 y_i}{\partial \alpha^4} + \frac{\partial^3 z_i}{\partial \alpha^3} \right) + \bar{c} \frac{\partial y_i}{\partial t} + m \frac{\partial^2 y_i}{\partial t^2} = \sum_{j=1}^{N(=5)} g_{ij}(u_j, v_j, \dot{u}_j, \dot{v}_j, \ddot{u}_j, \ddot{v}_j), \quad (10)$$

while along the tube axis (z -direction) the governing equation is

$$\frac{EI}{R_0^4} \left(\frac{\partial^2 z_i}{\partial \alpha^2} - \frac{\partial^3 y_i}{\partial \alpha^3} \right) + \hat{c} \frac{\partial z_i}{\partial t} + m \frac{\partial^2 z_i}{\partial t^2} = 0. \quad (11)$$

$\hat{c}$ represents an average damping for in-plane (y or z -) displacement. Note that the damping term in (10), (11) is only approximate; in reality, cross-coupling terms will be present. In Eqs. (9), (10), f_{ij} and g_{ij} are the generalized tube (j) motion dependent unsteady fluid forces. The coordinates (u, v) represent tube displacements relative to the flow. This coordinate system is fixed in space, the origin being at the tube zero displacement position. In general, the following relation holds,

$$\begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} = [T] \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}. \quad (12)$$

The mapping $[T]$ is dependent on how the approach flow velocity (U) is defined. By defining U as the local velocity normal to the tube axis (hence in the y -direction), and assuming small tube displacements (which is the case here), $[T]$ simply becomes the identity matrix.

Introducing the notation

$$\frac{EI}{R_0^4} \frac{\partial^n}{\partial \alpha^n} = EI_{,n}, \quad (13)$$

Eqs. (9), (10) may be combined into the matrix equation

$$\begin{aligned} & \begin{pmatrix} EI_{,4} & 0 & 0 \\ 0 & EI_{,4} & EI_{,3} \\ 0 & -EI_{,3} & EI_{,2} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} + \begin{pmatrix} c & 0 & 0 \\ 0 & \hat{c} & 0 \\ 0 & 0 & \hat{c} \end{pmatrix} \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{pmatrix} \\ & + \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{z}_i \end{pmatrix} \\ & = \sum_{j=1}^N \left\{ -\rho\pi R^2 \begin{bmatrix} \alpha_{ij} & \sigma_{ij} & 0 \\ \tau_{ij} & \beta_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \ddot{u}_i \\ \ddot{v}_i \\ \ddot{w}_i \end{pmatrix} \right. \\ & \quad - \frac{\rho U^2}{\omega} \begin{bmatrix} \alpha'_{ij} & \sigma'_{ij} & 0 \\ \tau'_{ij} & \beta'_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{u}_i \\ \dot{v}_i \\ \dot{w}_i \end{pmatrix} \\ & \quad \left. + \rho U^2 \begin{bmatrix} \alpha''_{ij} & \sigma''_{ij} & 0 \\ \tau''_{ij} & \beta''_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \right\}. \quad (14) \end{aligned}$$

By introducing the tube displacement vector

$$x_i = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}, \quad (15)$$

and structural matrices such as

$$[M_i] = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}, \quad (16)$$

as well as fluid force matrices

$$\begin{aligned} [F_{ij}^A] &= -\rho\pi R^2 \begin{bmatrix} \alpha_{ij} & \sigma_{ij} & 0 \\ \tau_{ij} & \beta_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ [F_{ij}^A] &= -\frac{\rho U^2}{\omega} \begin{bmatrix} \alpha'_{ij} & \sigma'_{ij} & 0 \\ \tau'_{ij} & \beta'_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ [F_{ij}^{A''}] &= \rho U^2 \begin{bmatrix} \alpha''_{ij} & \sigma''_{ij} & 0 \\ \tau''_{ij} & \beta''_{ij} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (17)$$

The equations for a kernel of N tubes may be expressed in compact form as

$$\begin{aligned} & [M_1]\{\ddot{X}_1\} + [C_1]\{\dot{X}_1\} + [K_1]\{X_1\} \\ & = \sum_{j=1}^N \{ [F_{1j}^A][T]\{\ddot{X}_j\} \\ & \quad + [F_{1j}^A][T]\{\dot{X}_j\} + [F_{1j}^{A''}][T]\{X_j\} \} \\ & \quad \vdots \\ & [M_i]\{\ddot{X}_i\} + [C_i]\{\dot{X}_i\} + [K_i]\{X_i\} \\ & = \sum_{j=1}^N \{ [F_{ij}^A][T]\{\ddot{X}_j\} \\ & \quad + [F_{ij}^A][T]\{\dot{X}_j\} + [F_{ij}^{A''}][T]\{X_j\} \} \\ & \quad \vdots \\ & [M_N]\{\ddot{X}_N\} + [C_N]\{\dot{X}_N\} + [K_N]\{X_N\} \\ & = \sum_{j=1}^N \{ [F_{Nj}^A][T]\{\ddot{X}_j\} \\ & \quad + [F_{Nj}^A][T]\{\dot{X}_j\} + [F_{Nj}^{A''}][T]\{X_j\} \}. \end{aligned} \quad (18)$$

Equation (18) may be combined to give the global equations

$$\begin{aligned} & [\tilde{M}]\{\ddot{X}\} + [\tilde{C}]\{\dot{X}\} + [\tilde{K}]\{X\} \\ & = [\tilde{F}^A]\{\ddot{X}\} + [\tilde{F}^A]\{\dot{X}\} + [\tilde{F}^{A''}]\{X\}. \end{aligned} \quad (19)$$

The various vectors and matrices in Eq. (19) are defined as follows:

$$\{x\} = \begin{Bmatrix} \{X_1\} \\ \vdots \\ \{X_N\} \end{Bmatrix} = \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \\ \vdots \\ x_N \\ y_N \\ z_N \end{Bmatrix}, \quad (20)$$

$$[\tilde{M}] = \begin{bmatrix} [M_1] & & \\ & \ddots & \\ & & [M_N] \end{bmatrix}, \quad (21)$$

and similarly for $[\tilde{C}]$ and $[\tilde{K}]$.

$$[\tilde{F}^A] = [F^A][\tilde{T}]; \quad (F^A) = \begin{bmatrix} [F_{11}^A] & \cdots & [F_{1N}^A] \\ \vdots & [F_{ij}^A] & \vdots \\ [F_{N1}^A] & \cdots & [F_{NN}^A] \end{bmatrix},$$

$$[\tilde{T}] = \begin{bmatrix} [T] & & \\ & \ddots & \\ & & [T] \end{bmatrix}. \quad (22)$$

Next, the following standard Galerkin modal decomposition is introduced

$$X_i(s, t) = \sum_{m=1}^N A_{im}(t) \psi_m(s). \quad (23)$$

Substituting Eq. (23) into Eq. (18) we obtain

$$[\tilde{M}][\Psi][\ddot{A}] + [\tilde{M}][\Psi][\dot{A}] + [\tilde{M}][\Psi][A]$$

$$\begin{aligned} & -[\tilde{F}^A][\Psi][\dot{A}] - [\tilde{F}^A][\Psi][\dot{A}] \\ & -[\tilde{F}^A][\Psi][A] = 0, \end{aligned} \quad (24)$$

where

$$[\Psi] = \begin{pmatrix} \psi_{11} & \psi_{12} & \dots & \psi_{1M} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \vdots & & \\ 0 & & \ddots & & \vdots & & \\ 0 & & & & \vdots & & \\ 0 & \dots & \dots & \dots & 0 & 0 & \dots & \psi_{N1} & \psi_{N2} & \dots & \psi_{NM} \end{pmatrix}, \quad (25)$$

and

$$\{A\} = \{A_{11}, \dots, A_{1M}, A_{21}, \dots, A_{2M}, \dots, A_{N1}, \dots, A_{NM}\}^T. \quad (26.a)$$

Furthermore we have

$$\begin{aligned} A_{jm} &= \{a_{jm} \quad b_{jm} \quad c_{jm}\}^T, \\ \psi_m &= \{\psi_m \quad \phi_m \quad \eta_m\}^T. \end{aligned} \quad (26.b)$$

Note that N is the number of tubes and M the number of modes.

2.2 Constrained mode formulation

In view of the large size of the resulting system of Eq. (24), a constrained mode analysis is introduced to reduce the system order to a manageable size. Only the pertinent aspects of the analysis are presented here. For a detailed exposition of the constrained mode reduction, we refer the reader to the work of Price et al.⁽⁶⁾ We reduce the system to essentially a 1-tube kernel by relating the displacements of tubes 2–5 to the displacements of tube 1; in essence, we fix the phase relation relative to tube 1. By fixing the phase relationship, the displacement of a tube located at column p , row q , can be expressed as

$$\begin{aligned} x_{q,p} &= x_0 e^{(\lambda t + i p \phi_x + i q \theta_x)} \\ y_{q,p} &= y_0 e^{(\lambda t + i p \phi_y + i q \theta_y)}. \end{aligned} \quad (27)$$

In Eq. (27), $\phi_x(\phi_y)$ is the phase difference between the $x(y)$ motion of cylinders in the same row, $\theta_x(\theta_y)$ is the phase difference between the $x(y)$ motion of cylinders in the same column. Considering tube one to be located at (p, q) leads to

$$x_1 = x_{q,p}, \quad y_1 = y_{q,p}. \quad (28)$$

Based on this, the displacements for the downstream tube '3' (Fig. 2) become

$$\begin{aligned} x_3 &= x_{q-1,p} = x_1 e^{-i\theta_x} \\ y_3 &= y_{q-1,p} = y_1 e^{-i\theta_y}, \end{aligned} \quad (29)$$

where we have taken $(p, q) = (0, 0)$ for tube 1. Similar expressions can be derived for the remaining tubes 2, 4 and 5. The z -direction displacement can also be treated in the same manner. This displacement relationship is depicted in Fig. 2.

2.3 1 tube kernel equations

By introducing the constrained mode formulation, the resulting equations are significantly simplified. Rather than solve the complete system of Eqs. (18), (24), we need only consider the equations for tube 1. The constrained mode formulation furnishes us with the otherwise unknown displacements of the remain-

Fig. 2 5 tube kernel for inline array

ing tubes 2–5, by relating them to tube 1 displacements via Eqs. (27), (28).

$$\{X_j\} = [\Psi] \{A_j\}, \quad (30)$$

a standard Galerkin procedure applied only to tube 1 motion in Eq. (18) yields

$$\begin{aligned} & \int_0^1 \{[\Psi]^T [M_1] [\Psi] \{\dot{A}_1\} + [\Psi]^T [C_1] [\Psi] \{\dot{A}_1\} \\ & + [\Psi]^T [K_1] [\Psi] \{A_1\}\} ds \\ & = \sum_{j=1}^5 \left\{ \int_0^1 [\Psi]^T [F_{1j}^A] [\Psi] \{\dot{A}_j\} ds \right. \\ & + \int_0^1 [\Psi]^T [F_{1j}^B] [\Psi] \{A_j\} ds \\ & \left. + \int_0^1 [\Psi]^T [F_{1j}^C] [\Psi] \{A_j\} ds \right\}. \end{aligned} \quad (31)$$

This may be written in compact form as

$$\begin{aligned} & [\bar{M}][\dot{A}] + [\bar{C}][\dot{A}] + [\bar{K}][A] \\ & = \sum_{j=1}^5 \{[\bar{F}_{1j}^A][\dot{A}_j] + [\bar{F}_{1j}^B][\dot{A}_j] + [\bar{F}_{1j}^C][A_j]\}, \end{aligned} \quad (32)$$

where for instance

$$[\bar{M}] = \int_0^1 [\Psi]^T [M_1] [\Psi] ds, \quad (33)$$

and similarly for the other matrices.

To describe the inter-tube modal pattern (the constrained mode), matrices $[\Phi_j]$ are introduced such that

$$\{A_j\} = [\Phi_j] \{A_1\}, \quad j=2, 3, 4, 5. \quad (34)$$

Introducing $[\Phi_j]$ into Eq. (32) gives

$$\begin{aligned} & [\bar{M}][\dot{A}_1] + [\bar{C}][\dot{A}_1] + [\bar{K}][A_1] \\ & = \sum_{j=1}^5 \{[\bar{F}_{1j}^A][\dot{A}_1] + [\bar{F}_{1j}^B][\dot{A}_1] + [\bar{F}_{1j}^C][A_1]\}, \end{aligned} \quad (35)$$

where

$$\begin{aligned} & [\bar{F}_{1j}^A] = [\bar{F}_{1j}^A] [\Phi_j], \quad [\bar{F}_{1j}^B] = [\bar{F}_{1j}^B] [\Phi_j], \\ & [\bar{F}_{1j}^C] = [\bar{F}_{1j}^C] [\Phi_j]. \end{aligned} \quad (36)$$

The detailed procedure to determine the matrices in Eq. (35) will now be outlined. The critical part of the

analysis is the evaluation of the integrals in Eq.(31) which result from the Galerkin procedure. A two mode model will be used here as an example. Note however that the analysis is applicable to any number of modes.

Considering only the first two structural modes, the modal matrix and generalized coordinate vector become

$$[{}_1\Psi] = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_1 & \eta_2 \end{bmatrix} \quad (37)$$

and

$$\{A_1\} = \{a_{11} \ a_{12} \ b_{11} \ b_{12} \ c_{11} \ c_{12}\}^T. \quad (38)$$

The modal fluid inertia force acting on tube 1 due to the acceleration of tube 2 is given by

$$[F_{12}^A][{}_2\Psi]\{\ddot{A}_2\} = -\rho\pi R^2 \begin{bmatrix} \alpha_{12} & \sigma_{12} & 0 \\ \tau_{12} & \beta_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \{\ddot{X}_2\}. \quad (39)$$

Application of the Galerkin procedure to this term leads to the matrix

$$[\bar{F}_{12}^A] = \int_0^l [{}_1\Psi]^T [F_{12}^A][{}_2\Psi] ds, \quad (40)$$

where

$$\begin{aligned} & [{}_1\Psi]^T [F_{12}^A][{}_2\Psi] \\ &= -\rho\pi R^2 \begin{bmatrix} \phi_1 & 0 & 0 \\ \phi_2 & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \eta_1 \\ 0 & 0 & \eta_2 \end{bmatrix} \begin{bmatrix} \alpha_{12} & \sigma_{12} & 0 \\ \tau_{12} & \beta_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_1 & \eta_2 \end{bmatrix} \\ &= -\rho\pi R^2 \begin{bmatrix} \alpha_{12}\phi_1^2 & \alpha_{12}\phi_1\phi_2 & \sigma_{12}\phi_1\phi_1 & \sigma_{12}\phi_1\phi_2 & 0 & 0 \\ \alpha_{12}\phi_1\phi_2 & \alpha_{12}\phi_2^2 & \sigma_{12}\phi_2\phi_1 & \sigma_{12}\phi_2\phi_2 & 0 & 0 \\ \tau_{12}\phi_1\phi_1 & \tau_{12}\phi_2\phi_1 & \beta_{12}\phi_1^2 & \beta_{12}\phi_1\phi_2 & 0 & 0 \\ \tau_{12}\phi_1\phi_2 & \tau_{12}\phi_2\phi_2 & \beta_{12}\phi_1\phi_2 & \beta_{12}\phi_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (41)$$

In Eq.(40), (41) we have assumed that the structural modes for tubes 1 and 2 are identical, i.e. $[{}_1\Psi] = [{}_2\Psi]$; (note, however, that in the actual steam generator this is not strictly true.) An identical procedure leading to equations such as Eq.(41) is applicable to all the matrices in Eq.(32).

2.4 Inclusion of correlation in the fluid forces

We finally come to the primary goal of this analysis. This pertains to the evaluation of integrals of the type appearing in Eq.(40). In the U-bend region of the steam generator, fluid forces along the tube span will only be partially correlated. Existing models have considered non-uniform flow velocity (and density) distribution along the span. The non-uniform distribution causes reduced correlation due to a mismatch between tube mode shapes and the shape

of the flow velocity (and density) distribution. However, in two-phase flow, spatial-temporal correlation of the flow itself is equally important.

In the U-bend region, flow velocity (U) and fluid density (ρ) vary both spatially and temporally, hence, in general

$$\rho = \rho(s, t), \quad U = U(s, t). \quad (42)$$

Since it is practically unrealistic to consider both time and space effects (in any case time dependence is unknown in general), the following simplified model is considered. Using the density as an example, the time average value is expressed as

$$\overline{\rho(s, t)} = \rho_0 \varphi_0(s). \quad (43)$$

At the same time, the temporal effect is introduced in the form of a correlation function (see Eqs. (5), (6))

$$\gamma(s_1, s_2) = \gamma(\Delta_{12}) = e^{-\Delta_{12}/l_c}, \quad (44)$$

where l_c is an effective correlation length which must be determined experimentally. To keep the analysis simple, we have assumed that the effect of partial temporal correlation is (indirectly) manifested in l_c . In other words $l_c = l_c(\gamma_t(\Delta_{12}^t))$; the suffix 't' indicates time. Without this assumption, it is necessary to express γ as the function $\gamma(s_1, s_2, t_1, t_2)$ in Eq.(44).

The treatment leading to Eq.(44) above is much like the analysis considered in turbulence excitation (see Nakamura et al.⁽⁶⁾). Note, however, that the correlation length is significantly different (hence, in this case, $l_c/l \approx 3/40$ for a 4 000 mm long tube). The existence of $\gamma(\Delta_{12})$ is strictly dependent on the homogeneity of the fluid forces. This is an inherent assumption in the method which makes a first approximation of the correlation effect possible^{(1),(7)}. A more complex formulation, while desirable, cannot be justified due to lack of information regarding the true nature of the fluid homogeneity.

The detailed calculation procedure for one term of the fluid inertia force matrix will now be outlined. The fluid added mass matrix $[\bar{F}_{ij}^A]$ is the inertia induced on tube i due to acceleration of tube j . The case $(i, j) = (1, 2)$ was presented in Eq.(40), (41) in expanded form. Similarly to Eq.(40) we can write, in general

$$[\bar{F}_{ij}^A] = \int_0^l [{}_i\Psi]^T [F_{ij}^A][{}_j\Psi] ds. \quad (45)$$

Let ξ_{ij} represent any of the unsteady fluid force coefficients $\alpha_{ij}, \dots, \tau_{ij}$. The contribution of a coefficient ξ_{ij} to $[\bar{F}_{ij}^A]$ is

$${}_{ij}f_k^A(s, t) = -\pi R^2 \xi_{ij} \sum_{m=1}^M \int_0^l \psi_k(s) \rho(s, t) {}_j\psi_m(s) ds. \quad (46)$$

Equation (46) represents the contribution to the fluid inertia force acting on mode k of tube i caused by the acceleration of the M modes of tube j (in the appropriate direction depending on ξ_{ij}). A direct evaluation

of Eq.(46) would lead to time dependent fluid forces. From a stability analysis point of view, this would be cumbersome to handle. To overcome this drawback, the correlation function is introduced. The purpose here is to replace the time-dependent fluid forces by some appropriate time-averaged forces. Letting $\gamma(\Delta_{12})$ represent the correlation of density variation along the tube length, the mean square average force can be estimated as follows:

Using the time averaged density $\rho_0\varphi_0(s)$ defined in Eq.(43), define a function

$$_{ij}\tilde{h}_m(s) = -\pi R^2 \xi_{ij} \rho_0 \varphi_0(s) \psi_m(s) \quad (47)$$

The mean square value of the fluid force component in Eq.(46) then becomes

$$\begin{aligned} \overline{(f_{kp}^A)^2} = & \sum_{m=1}^M \sum_{n=1}^M \left\{ \int_0^l \int_0^l \gamma(\Delta_{12}) \psi_k(s_1) \tilde{h}_m(s_1) \right. \\ & \times (s_1) \psi_p(s_2) \tilde{h}_n(s_2) ds_1 ds_2 \left. \right\} \end{aligned} \quad (48)$$

In Eqs.(47), (48), $\tilde{h}(\cdot)$ indicates a time average while $f(\cdot)$ indicates a spatial average. Equation (48) indicates that the effect of partial correlation is to introduce coupling between modes k and p of tube i by interacting with modes m and n of tube j , which would not occur in the case of perfect correlation. This can be confirmed by noting that when modes k and p are orthogonal, the integral of their product is zero. The correlation function $\gamma(\Delta_{12})$, however, destroys this orthogonality. Furthermore, since $|\gamma(\Delta_{12})| < 1$, the magnitude of the net fluid force is reduced.

2.5 Solution of the governing equations

Integrals of the type of Eq.(48) are evaluated for all the elements of the matrices $[\bar{F}_{ij}^A]$, $[\bar{F}_{ji}^A]$ and $[\bar{F}_{ij}^A]$. Using these together with Eq.(36), the final fluid force matrices in Eq.(35) are determined.

Equation (35) is then converted into the following eigenvalue form

$$[\lambda[P] + [Q]]\{W\} = 0, \quad (49)$$

where

$$[P] = \begin{bmatrix} [0] & [\bar{M}] \\ [\bar{M}] & [\bar{C}] \end{bmatrix}, \quad [Q] = \begin{bmatrix} -[M] & [0] \\ [0] & [\bar{K}] \end{bmatrix}, \quad (50)$$

and for instance

$$[\bar{M}] = [M] - \sum_{j=1}^5 [\bar{F}_{ij}^A], \quad (51)$$

with similar expressions for $[\bar{C}]$ and $[\bar{K}]$. The matrices $[P]$ and $[Q]$ are functions of the reduced velocity U/fD (among other parameters) through the unsteady fluid force coefficients ξ_{ij} . A standard eigenvalue analysis of Eq.(49) as U/fD is incremented yields the fluidelastic instability boundary.

3. Results and Discussion

To form the matrices $[P]$ and $[Q]$, experimental data measured by Tanaka and Takahara⁽⁴⁾ were used. The results of an eigenvalue analysis of Eq.(49) are

presented in this section. Two cases were considered for analysis, a single flexible tube in an otherwise rigid array and a fully flexible array approximated by a one-tube constrained mode kernel. The tube considered was a typical steam generator U-bend tube. Tube and flow parameters are given below:

$$\begin{aligned} f_x &= 10 \text{ Hz} \\ f_y &= f_z = 234 \text{ Hz} \\ 0.01 &\leq \frac{m\delta}{\rho D^2} \leq 100 \\ l &= 4000 \text{ mm} \\ l_c &= 300 \text{ mm} \end{aligned}$$

Figure 3 shows the model tube considered. To keep the analysis simple, several assumptions were made. The tube was considered to be straight, rigid and flexibly mounted with frequencies as listed above. The time averaged flow velocity and density are independent of s hence $\varphi_0(s)=1$ in Eq.(47) for instance. Time dependence is introduced via the correlation function γ .

For the constrained mode analysis, the inter-tube modal pattern shown in Fig. 4 was employed. A single modal pattern was selected for all values of mass-damping parameter $m\delta/\rho D^2$ to enable a consistent evaluation of the effect of partial correlation. Figure 5 shows the stability boundary for a single flexible cylinder both for partially correlated ($\gamma < 1$) fluid forces and the ideal perfectly correlated ($\gamma = 1$) case. For low $m\delta/\rho D^2$ (essentially liquid flow), the correlation γ has little effect on the instability velocity. Starting near $m\delta/\rho D^2 = 0.25$, the effect of γ becomes apparent. Thus for $m\delta/\rho D^2 = 10$, the instability velocity considering correlation is approximately double

Fig. 3 Tube model used for computations

Fig. 4 Inter-tube modal pattern (experimentally determined by Tanaka and Takahara⁽⁴⁾)

Fig. 5 Single flexible tube stability boundaries

Fig. 6 Stability boundaries for a flexible array constrained to a 1-tube kernel

the critical velocity with fully correlated fluid forces. This represents a 100% increase in the instability velocity. For $m\delta/\rho D^2 > 100$, dynamic instability disappears altogether.

For the flexible array, the predicted stability boundaries are shown in Fig. 6. The most significant result is the total elimination the lower stability boundary (for low $m\delta/\rho D^2$) when $\gamma < 1$ is introduced. This is a very significant result from a practical point of view; the possibility of existence of the more "dangerous" lower instability boundary is all but eliminated. This result supports, though from a different approach, the findings of Paidoussis et al.⁽¹⁵⁾ that multiple instability regions are very sensitive to array asymmetries. This "fragility" of the boundaries now appears to be true relative to spatial-temporal correlation of fluid forces as well.

The stability ratio (SR) is usually used when performing stability analysis of actual steam generators. The ideal SR is given by

$$SR = \kappa \frac{U}{U_c} \quad (52)$$

where U is the effective operating flow velocity while

Fig. 7 Single tube flexible versus flexible array SR variation

U_c is the predicted instability velocity. The factor κ accounts for uncertainties in the estimation of the various parameters used to determine U_c . $SR < 1$ would, of course, be the desired operating condition. Figure 7 shows the effect of γ on SR. The mass-damping parameter for actual steam generators falls within the range $1 < m\delta/\rho D^2 < 10$. In this range, SR decreases by between 10 and 50%. This, of course, is good news for the designers since lower SR implies increased stability. In the foregoing results, the correlation length ratio $l_c/l = 0.075$ was used. Admittedly much more information regarding l_c is needed. The effect of varying l_c for a fixed $m\delta/\rho D^2 (= 30)$ is presented in Fig. 8. For l_c/l ratios below 25% the instability velocity is dramatically affected by the value of l_c chosen.

Finally to put the present findings in perspective, comparison is made with previous experimental results in Fig. 9. The experimental data in Fig. 9 is, however, from tests on short span tubes in single phase flow. The comparison made here is therefore simply to indicate the trend in stability behavior and the effect of reduced correlation. Due to lack of published data on long-span tubes, direct comparison is presently not possible. The predicted stability boundary falls within the range of experimental data. The correlation effect is most significant at high mass-damping parameter values.

4. Concluding Remarks

A first attempt to account for partial correlation along the tube span has been presented. The overall conclusion is that partial correlation has a stabilizing effect. Another finding is that modes which are uncoupled in the fully correlated case become coupled; in effect, energy redistribution over all modes occurs. A lot still remains to be done. Estimation of γ is possibly most critical for improved accuracy of the

Fig. 8 Effect of varying l_c/l on the instability velocity

Fig. 9 Comparison with published data (from Price⁽⁹⁾)

analysis. Other fundamental questions also need to be addressed. For instance an obvious effect not considered here is the fact that for two-phase flow correlation is time dependent. Hence, γ should strictly take the form, $\gamma = \gamma(s_1, s_2, t_1, t_2)$ with the temporal effect included. The assumption of homogeneity also leaves room for improvement in the long run.

Despite these obvious shortcomings, significant findings were made. The effect of γ is more important for multiple flexible tubes where SR changes by 50% for l_c/l of 0.5; for a single flexible tube, the same change in SR does not occur until l_c/l falls to 0.15. The correlation effect was found to be mainly important for high $m\delta/\rho D^2$. However, an important result for low $m\delta/\rho D^2$ was the apparent elimination of

multiple instability regions. This is, practically, a very significant result.

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