Effects of Helicity and System Rotation on Decaying Homogeneous Turbulence*

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To consider the nonreflection property of the turbulence statistics in a rotating system, a new method has been proposed to introduce turbulence helicity into an isotropic turbulent field for direct numerical simulation (DNS). Effects of the helicity and the rotation on turbulence statistics and vortex structure are investigated by theoretical analysis and DNS of the homogeneous decaying turbulence. Although DNS results show that both the rotation and the helicity inhibit energy decay, theoretical analysis reveals the occurrence of very different mechanisms for the rotation and the helicity for producing the inhibition effect. The helicity directly decreases the energy transfer while the rotation suppresses the energy transfer due to the so-called scrambling effect. The other effect of system rotation is that rotation elongates the vortex structure along the rotation axis, however, the presence of the helicity appears to weaken this tendency.

Key Words: Turbulence, Numerical Analysis, Computational Fluid Dynamics, Helicity, System Rotation, Complex Helical Waves Decomposition, Scrambling Effect

1. Introduction

Rotating flows have numerous industrial applications in the fields of engineering, geophysics, astrophysics and meteorology. The turbulence properties of the rotating flows in blade passages of radial pumps and gas or steam turbines determine the efficiency of these devices. Therefore, the study of rotating turbulence is interesting from the viewpoints of both turbulence modeling and fundamental research^{(1),(2)}. To date, a number of researchers have studied rotating turbulence that includes decaying homogeneous turbulence⁽³⁾⁻⁽⁶⁾, homogeneous turbulent shear flow⁽⁷⁾⁻⁽⁹⁾, turbulent channel flow⁽¹⁰⁾⁻⁽¹⁴⁾ and turbulent Couette flow⁽¹⁵⁾ with theoretical, experimental and numerical methods. Furthermore, in order to develop a model for the rotating turbulence, it is necessary to elucidate

the turbulence structure of the turbulent field. Helicity, an important parameter for evaluating the turbulence structure, and the relationship between the helicity and the vortex structure are investigated in this study.

With the development of computers, numerical simulation has become one of the main tools for turbulence research. In particular, direct numerical simulation (DNS) plays a significant role in the investigation of turbulence structure and turbulence modeling. When DNS of homogeneous turbulence is considered, one needs an initial turbulent velocity field. Conventionally, a fully symmetric velocity field in which the turbulence statistics satisfy the rotation and reflection symmetries⁽¹⁶⁾ is generated as an initial velocity field for the DNS of homogeneous decaying turbulence^{(17),(18)}. Helicity is evidently zero if the turbulence statistics possess the reflectional symmetry property. Consequently, a fully symmetric turbulent flow field does not include mean helicity. However, when the turbulence is subjected to uniform system rotation, the reflection symmetry of the statistics is destroyed by the rotation⁽¹⁹⁾ and the flow field becomes an axisymmetric turbulent field. Hence, the turbu-

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lence subjected to uniform system rotation allows the coexistence of the shell symmetric kinetic energy and helicity spectrum. Lesieur⁽²⁰⁾ reported that isotropy possesses rotation symmetry only, and it is referred to as semi-isotropy in some studies⁽⁵⁾. Hence, the definition of semi-isotropy seems to fit more suitably for actual rotating turbulence than that of isotropy. Using the turbulence closure theory of eddy-damped quasi-normal Markovian (EDQNM), Andre and Lesieur⁽²¹⁾ investigated the effects of the helicity on the turbulence, and indicated that the helicity inhibits the kinetic energy transfer of the homogeneous turbulence in the stationary system. Kraichnan⁽²²⁾ also predicted the same effect by theoretical analysis. Based on the definition of the helicity spectrum, i.e., $\langle \tilde{\mathbf{u}} \cdot \tilde{\boldsymbol{\omega}}^* \rangle / 2$, Polifke and Shtilman⁽²³⁾ proposed a random phase method to introduce the helicity into a turbulent field. Murakami et al.⁽²⁴⁾ also used this method to study the effect of helicity on turbulence. However, this method needs much realization to obtain an effective field with less helicity and no detailed discussion on the effects of the helicity was included in their studies.

In the present study, a simple and convenient method is proposed to introduce the helicity into an initial isotropic turbulent velocity field. DNS of rotating homogeneous decaying turbulence is carried out with and without the initial helicity. Then, the effects of the system rotation and the helicity on the turbulence are investigated by theoretical analysis and based on DNS results.

Nomenclature

Roman symbols

- $E(k)$: energy spectrum function
 $e(\mathbf{k})$: energy spectrum
 $\mathbf{e}^1, \mathbf{e}^2$: basis vectors of Craya-Herring frame
 F : flatness factor of velocity gradient
 $H(k)$: helicity spectrum function
 $h(\mathbf{k})$: helicity spectrum
 He : helicity, $\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle / 2$
 I : complex unit, $\sqrt{-1}$
 K : turbulence kinetic energy
 $\mathbf{k}$: wave number vector
 k : magnitude of wave number $\mathbf{k}$
 $L_{ij,k}$: integral lengthscale
 $L_{33,3}$: longitudinal integral length scale
 $L_{11,3} + L_{22,3}$: transverse integral length scale
 L^* : $(L_{11,3} + L_{22,3}) / L_{33,3}$
 $N(\pm \mathbf{k})$: basis vectors of complex helical waves decomposition
 $\mathbf{n}^k$: unit vector along the direction x_k
 $\mathbf{p}$: wave-number vector

p : fluctuating pressure, or magnitude of wave number $\mathbf{p}$

P_{ij} : projection operator, $\delta_{ij} - k_i k_j / k^2$

$\mathbf{q}$: wave-number vector

q : magnitude of wave number $\mathbf{q}$

q_0 : initial kinetic energy (at $t=0$)

$\mathbf{r}$: position vector

Re_λ : Taylor microscale Reynolds number

S : skewness factor of velocity gradient

s, s_k, s_p, s_q : sign of helicity mode

$T(k)$: energy transfer function

$\mathbf{u}$: velocity vector

$Z(\mathbf{k})$: complex deviator spectrum

Greek symbols

α : relative helicity coefficient

δ_{ij} : Kronecker delta tensor

ε_{ijl} : alternating tensor

λ : Taylor microscale

ν : kinetic viscosity of working fluid

ξ_{+1}, ξ_{-1} : helical modes

Π : modified pressure

ρ : fluid density

$\boldsymbol{\omega}$: vorticity vector

ω : magnitude of vorticity vector $\boldsymbol{\omega}$

$\boldsymbol{\Omega}$: angular velocity vector

Ω : magnitude of angular velocity vector $\boldsymbol{\Omega}$

Other symbols

$\langle \rangle$: ensemble average

$(\hat{})$: coefficient of Fourier transform

2. Formulation and Generation of Initial Helicity

2.1 Basic equations

The incompressible Navier-Stokes (NS) and the continuity equations in a rotating system are used as the fundamental equations of this study.

$$\frac{\partial \mathbf{u}}{\partial t} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla \Pi - \boldsymbol{\omega} \times \mathbf{u} + \nu \nabla^2 \mathbf{u} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where $\Pi = p/\rho - (\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})/2 + \mathbf{u} \cdot \mathbf{u}/2$. By applying the Fourier transform and the continuity condition to Eq. (1), the NS equation in wave space is obtained.

$$\left(\frac{\partial}{\partial t} + \nu k^2 \right) \tilde{\mathbf{u}}(\mathbf{k}) + \frac{2\boldsymbol{\Omega} \cdot \mathbf{k}}{k^2} [\mathbf{k} \times \tilde{\mathbf{u}}(\mathbf{k})] = - \sum_{\mathbf{k}-\mathbf{p}-\mathbf{q}=0} \left(1 - \frac{\mathbf{k} \cdot \mathbf{k}}{k^2} \right) [\tilde{\boldsymbol{\omega}}(\mathbf{p}) \times \tilde{\mathbf{u}}(\mathbf{q})] \quad (3)$$

Here, $\mathbf{k}, \mathbf{p}$ and $\mathbf{q}$ are the wave-number vectors and $k^2 = \mathbf{k} \cdot \mathbf{k}$. The second term on the left-hand side of this equation is the Coriolis term, and it is a linear term. In order to analyze the linear effect of the Coriolis term more clearly, we adopt the complex helical waves decomposition^{(25),(26)} to reform the NS equation in wave space. The solenoidal condition implies that the complex vector $\tilde{\mathbf{u}}(\mathbf{k})$ is located on the plane normal to the wave vector $\mathbf{k}$, and the following

decomposition is useful:

$$\begin{aligned}\bar{\mathbf{u}}(\mathbf{k}) &= \xi_{+1}(\mathbf{k})\mathbf{N}(\mathbf{k}) + \xi_{-1}(\mathbf{k})\mathbf{N}(-\mathbf{k}) \\ &= \sum_{s=\pm 1} \xi_s(\mathbf{k})\mathbf{N}(s\mathbf{k}).\end{aligned}\quad (4)$$

The components ξ_{+1} and ξ_{-1} are respectively the plus and minus helical modes of the complex helical-wave decomposition. The orthogonal basis $\mathbf{N}(s\mathbf{k})$ is defined as

$$\mathbf{N}(s\mathbf{k}) = \mathbf{e}^2(\mathbf{k}) - I s \mathbf{e}^1(\mathbf{k}). \quad (5)$$

Here,

$$\mathbf{e}^1 = \mathbf{k} \times \mathbf{a} / |\mathbf{k} \times \mathbf{a}|, \quad \mathbf{e}^2 = \mathbf{k} \times \mathbf{e}^1 / |\mathbf{k}| \quad (6)$$

and the wave vector $\mathbf{k}$ form a direct orthonormal frame called Craya-Herring frame⁽²⁰⁾, where $\mathbf{a}$ is an arbitrary constant vector. Substituting the complex helical-wave decomposition term into Eq.(3), the evolution equation of the helical modes ξ_{s_k} ($s_k = \pm 1$) is

$$\begin{aligned}\left(\frac{\partial}{\partial t} + \nu k^2\right)\xi_{s_k} &= \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} (s_p p - s_q q) g^* \xi_{s_p}^* \xi_{s_q}^* \\ &+ I s_k g_k \xi_{s_k}.\end{aligned}\quad (7)$$

The right-hand side has been written in symmetric form in terms of $\mathbf{p}$ and $\mathbf{q}$. The triplet (s_k, s_p, s_q) is $(\pm 1, \pm 1, \pm 1)$ and $g^* = -(\mathbf{N}_{s_p}^* \times \mathbf{N}_{s_q}^* \cdot \mathbf{N}_{s_k}^*)/4$ is a geometrical factor dependent on the shape and the orientation of the wave vector triad $(\mathbf{k}, \mathbf{p}, \mathbf{q})$ which satisfies the condition $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$. Here, $g_k = 2\Omega \cos \theta_k$ and θ_k is the angle between $\mathbf{k}$ and Ω . In the stationary frame, since $g_k = 0$, Eq.(7) is written in the following form:

$$\left(\frac{\partial}{\partial t} + \nu k^2\right)\xi_{s_k} = \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_p, s_q} (s_p p - s_q q) g^* \xi_{s_p}^* \xi_{s_q}^* \quad (8)$$

The corresponding energy evolution equations (Eqs. (7) and (8)) have the same form as

$$\begin{aligned}\left(\frac{\partial}{\partial t} + 2\nu k^2\right)\langle \xi_{s_k} \xi_{s_k}^* \rangle &= \langle \sum_{\mathbf{k}+\mathbf{p}+\mathbf{q}=0} \sum_{s_k, s_p, s_q} 2(s_p p - s_q q) \text{Re}[g^* \xi_{s_k}^* \xi_{s_p}^* \xi_{s_q}^*] \rangle.\end{aligned}\quad (9)$$

Equation (9) shows that, even in the rotating frame, there is no explicit rotation term in the kinetic energy evolution equation.

2.2 Spectral tensor

A general form of the spectral tensor⁽⁵⁾ can be expressed as

$$\begin{aligned}\hat{U}_{ij}(\mathbf{k}) &= \langle \hat{u}_i^*(\mathbf{k}) \hat{u}_j(\mathbf{k}) \rangle = e(\mathbf{k}) P_{ij}(\mathbf{k}) \\ &+ \text{Re}[Z(\mathbf{k}) N_i(\mathbf{k}) N_j(\mathbf{k})] + I \varepsilon_{ijk} h(\mathbf{k}) / k^2,\end{aligned}\quad (10)$$

where $e(\mathbf{k})$, $h(\mathbf{k})$ and $Z(\mathbf{k})$ are respectively the spectra of energy, helicity and complex deviator, defined as

$$\begin{aligned}e(\mathbf{k}) &= \frac{1}{2} \langle \bar{\mathbf{u}}(\mathbf{k}) \cdot \bar{\mathbf{u}}^*(\mathbf{k}) \rangle \\ &= \langle \xi_{+1}(\mathbf{k}) \xi_{+1}^*(\mathbf{k}) + \xi_{-1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k}) \rangle, \\ h(\mathbf{k}) &= \frac{1}{2} \langle \bar{\mathbf{u}}(\mathbf{k}) \cdot \bar{\boldsymbol{\omega}}^*(\mathbf{k}) \rangle \\ &= \langle k [\xi_{+1}(\mathbf{k}) \xi_{+1}^*(\mathbf{k}) - \xi_{-1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k})] \rangle, \\ Z(\mathbf{k}) &= 2 \langle \xi_{+1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k}) \rangle.\end{aligned}\quad (11)$$

The complex deviator denotes a part of the anisotropy on the plane perpendicular to $\mathbf{k}^{(5),(24)}$. For isotropic turbulence [$Z(\mathbf{k})=0$] with rotation symmetry only, the spectral tensor can be written as

$$\hat{U}_{ij}(\mathbf{k}) = e(k) P_{ij}(\mathbf{k}) + I \varepsilon_{ijk} h(k) / k^2 \quad (12)$$

where $e(\mathbf{k})$ and $h(\mathbf{k})$ are replaced by the corresponding isotropic forms $e(k) = E(k)/(4\pi k^2)$ and $h(k) = H(k)/(4\pi k^2)$, respectively. If the reflectional symmetry is also satisfied, the spectral tensor for isotropic turbulence is simplified to:

$$\hat{U}_{ij}(\mathbf{k}) = e(k) P_{ij}(\mathbf{k}). \quad (13)$$

However, since the reflectional symmetry is destroyed by the system rotation, the general spectral tensor for the isotropic rotating turbulence has the form of Eq. (12).

Using the helical modes ξ_{+1} and ξ_{-1} , one obtains the shell symmetry forms of the spectrum as

$$\begin{aligned}e^{++}(k) &= \langle \xi_{+1}(\mathbf{k}) \xi_{+1}^*(\mathbf{k}) \rangle = \frac{E^{++}(k)}{4\pi k^2}, \\ e^{--}(k) &= \langle \xi_{-1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k}) \rangle = \frac{E^{--}(k)}{4\pi k^2},\end{aligned}\quad (14)$$

where the definitions of $E^{++}(k)$ and $E^{--}(k)$ are

$$\begin{aligned}E^{++}(k) &= \frac{1}{2} \left[E(k) + \frac{H(k)}{k} \right], \\ E^{--}(k) &= \frac{1}{2} \left[E(k) - \frac{H(k)}{k} \right].\end{aligned}\quad (15)$$

2.3 Generation of initial helicity

In the present study, we propose a simple and useful method to obtain a velocity field with an arbitrary distribution of the helicity spectrum. The advantage of this method is that only the amplitudes of the helical modes are changed with a scalar parameter, the kinetic energy of the field is kept constant.

Based on Eq.(14), it can be assumed that,

$$\begin{aligned}\xi_{+1}(\mathbf{k}, t) &= \left[\frac{E^{++}(k, t)}{4\pi k^2} \right]^{1/2} \exp(I\theta_1), \\ \xi_{-1}(\mathbf{k}, t) &= \left[\frac{E^{--}(k, t)}{4\pi k^2} \right]^{1/2} \exp(I\theta_2),\end{aligned}\quad (16)$$

where θ_1 and θ_2 are the phase angles of ξ_{+1} and ξ_{-1} respectively, and vary between 0 to 2π . Here, we introduce a relative helicity coefficient

$$\alpha(\mathbf{k}) = \frac{h(\mathbf{k})}{ke(\mathbf{k})} = \frac{\langle \xi_{+1}(\mathbf{k}) \xi_{+1}^*(\mathbf{k}) - \xi_{-1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k}) \rangle}{\langle \xi_{+1}(\mathbf{k}) \xi_{+1}^*(\mathbf{k}) + \xi_{-1}(\mathbf{k}) \xi_{-1}^*(\mathbf{k}) \rangle}. \quad (17)$$

Here, $\alpha = +1$ and -1 correspond to the presence of the plus and minus modes only and the helicity takes the maximum and minimum values, respectively. $\alpha = 0$ corresponds to a non-helicity field. Equations (15) and (17) yield, at a certain time t_0 ,

$$H(k, t_0) = \alpha k E(k, t_0), \quad |\alpha| \leq 1. \quad (18)$$

In accordance with Eq.(18), the helical modes are brought into:

$$\begin{aligned}\xi_{+1}^{\alpha}(\mathbf{k}, t_0) &= \left[\frac{(1+\alpha)E(k, t_0)}{8\pi k^2} \right]^{1/2} \exp(I\theta_1), \\ \xi_{-1}^{\alpha}(\mathbf{k}, t_0) &= \left[\frac{(1-\alpha)E(k, t_0)}{8\pi k^2} \right]^{1/2} \exp(I\theta_2).\end{aligned}\quad (19)$$

A direct deduction of this equation yields:

$$\begin{aligned}\xi_{+1}^{\alpha}(\mathbf{k}, t_0) &= (1+\alpha)^{1/2} \xi_{+1}^0(\mathbf{k}, t_0), \\ \xi_{-1}^{\alpha}(\mathbf{k}, t_0) &= (1-\alpha)^{1/2} \xi_{-1}^0(\mathbf{k}, t_0)\end{aligned}\quad (20)$$

The superscripts α and 0 denote that the relative helicity coefficients have nonzero and zero values, respectively. If an initial field at a certain time t_0 is known, i.e., the helical modes ξ_{+1}^0 and ξ_{-1}^0 are known, one finds it easy to introduce the helicity into this field through Eq.(20) by only changing the parameter α . The corresponding kinetic energy of Eq.(20) is

$$e^{\alpha}(\mathbf{k}, t_0) = e^0(\mathbf{k}, t_0) + \alpha \langle |\xi_{+1}^0|^2 - |\xi_{-1}^0|^2 \rangle. \quad (21)$$

This means that, when the helicity is generated, a part of the extra energy $\alpha \langle |\xi_{+1}^0|^2 - |\xi_{-1}^0|^2 \rangle$ is also introduced into the initial field. The kinetic energy is conserved only when the field satisfies the condition $|\xi_{+1}^0| = |\xi_{-1}^0|$. However, in the numerical simulation in particular, it is difficult to satisfy the condition exactly when we use a finite number of Fourier modes. In order to conserve the kinetic energy of the system, we reform Eq.(20) into the following forms:

$$\begin{aligned}\xi_{+1}^{\alpha}(\mathbf{k}, t_0) &= (1+\alpha\beta)^{1/2} \xi_{+1}^0(\mathbf{k}, t_0), \\ \xi_{-1}^{\alpha}(\mathbf{k}, t_0) &= (1-\alpha)^{1/2} \xi_{-1}^0(\mathbf{k}, t_0), \\ \xi_{+1}^{\alpha}(\mathbf{k}, t_0) &= (1+\alpha)^{1/2} \xi_{+1}^0(\mathbf{k}, t_0), \\ \xi_{-1}^{\alpha}(\mathbf{k}, t_0) &= (1-\alpha/\beta)^{1/2} \xi_{-1}^0(\mathbf{k}, t_0),\end{aligned}\quad \begin{cases} (0 \leq \alpha \leq 1) \\ (-1 \leq \alpha \leq 0) \end{cases} \quad (22.1) \quad (22.2)$$

where $\beta = (\xi_{-1}^0)^2 / (\xi_{+1}^0)^2$. In this paper, we use Eq.(22) to introduce the helicity into a given isotropic turbulent field. Then, the velocity field in Fourier space is computed by using the definition of the complex helical-wave decomposition given by Eq.(4).

3. Initial Condition

In this study, we perform DNS of the decaying turbulence subjected to system rotation by using a new DNS algorithm⁽⁶⁾, which integrates the Coriolis term analytically. The initial flow field for the DNS is obtained by a pre-computation. The initial flow field of the precomputations is produced through Eq.(19) with $t_0=0$, $\alpha=0$, and θ_1 and θ_2 are random numbers varying between 0 to 2π . The initial energy spectrum is⁽²⁷⁾

$$E(k, 0) = \frac{32}{3} \sqrt{\frac{2}{\pi}} q_0 \frac{k^4}{k_p^5} \exp\left(-2 \frac{k^2}{k_p^2}\right), \quad (23)$$

with $q_0=13.5$ and $k_p=8$. Then, the precomputation is performed to $t_0=2.5$ [s]. At $t_0=2.5$ [s], reliable velocity gradient skewness and power-law decay for the turbulent kinetic energy are obtained. Finally, the turbulent velocity field at t_0 is adopted as the initial flow field without the helicity and the initial flow field with helicity is generated by using Eq.(22) and $\alpha \neq 0$.

Table 1 Statistics of initial turbulent field

α	K_0	ε_0	He_0	Re_λ	S	F
0	0.2013	0.0567	0.0067	53.4	-0.496	4.680
1	0.2013	0.0567	1.6990	53.4	-0.080	4.258
-1	0.2013	0.0567	-1.6990	53.4	-0.061	4.310

Fig. 1 Evolution of velocity gradient skewness factor

The computational region is $(2\pi \times 2\pi \times 2\pi)$ and periodic conditions are assumed in all directions. The spectral mode number is 128 in each direction. The kinetic viscosity and the time increment are set as $\nu=0.001$ [m²/s] and $\Delta t=0.01$ [s], respectively. DNS of decaying homogenous turbulence for $\alpha=0$ and 1 is carried out with $\Omega=0, 0.5, 1, 2$ and 5 [rad/s], respectively. In order to investigate the influence of the minimum helicity, DNS of $\alpha=-1$ with $\Omega=0$ and 2 is also submitted. The statistics of the initial field at $t_0=2.5$ [s] are presented in Table 1. Here, K_0 [m²/s²], ε_0 [m²/s³] and He_0 [m/s²] are the initial kinetic energy, energy dissipation rate and helicity, respectively.

Table 1 shows that there is slight initial helicity in the initial turbulent field for the case of $\alpha=0$. This comes from the turbulent field obtained by the precomputation. The corresponding relative helicity coefficient of this turbulent field is $\alpha_0=0.00268$. In the present study, this small α_0 value can be ignored and considered as $\alpha=0$. Additionally, Fig.1 also shows that the absolute value of the velocity gradient skewness factor decreases with increasing the value of α . This can be explained by the definition of the skewness factor. The definition of the skewness factor for the velocity gradient is given by

$$S = \frac{\langle \left(\frac{\partial u_i}{\partial x_j} \right)^3 \rangle}{\left(\langle \left(\frac{\partial u_i}{\partial x_j} \right)^2 \rangle \right)^{3/2}}. \quad (24)$$

Batchelor and Townsend⁽²⁸⁾ have shown that, for isotropic three-dimensional turbulence, the skewness factor S is equivalent to

$$S = \left(\frac{135}{98} \right)^{1/2} \left\langle \omega_i \omega_j \frac{\partial u_i}{\partial x_j} \right\rangle / D^{3/2}, \quad (25)$$

with $D = \langle \omega^2 \rangle / 2$. Using the homogeneous condition, $\langle \partial(\) / \partial x_i \rangle = 0$, Eq.(25) can be brought into a vector form:

$$S = \left(\frac{135}{98} \right)^{1/2} \langle \boldsymbol{\omega} \cdot \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) \rangle / D^{3/2}. \quad (26)$$

Helicity is defined as $\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle / 2$, and it takes the maximum absolute value when the velocity vector $\mathbf{u}$ is parallel or antiparallel to the vorticity vector $\boldsymbol{\omega}$. On the other hand, while $\mathbf{u}$ is perpendicular to $\boldsymbol{\omega}$, the helicity and the term $\mathbf{u} \times \boldsymbol{\omega}$ take the minimum and the maximum absolute values, respectively. One can find from Eq.(26) that, when the helicity is increased, $\mathbf{u}$ becomes parallel or antiparallel to $\boldsymbol{\omega}$ and the value of $|\mathbf{u} \times \boldsymbol{\omega}|$ decreases. Hence, the absolute value of the skewness factor decreases with increasing the helicity.

4. Effects of System Rotation and Helicity

4.1 Effects of system rotation

Figure 2 shows the evolution of the turbulence kinetic energy with system rotation. It shows that the kinetic energy decays with time and the decay rate is decreased by the system rotation. The effects of the system rotation on the turbulence are discussed analytically as follows.

The system for the helical modes corresponding to the single triplet (s_k, s_p, s_q) have the form (from Eq. (7) with viscosity omitted),

$$\begin{aligned} \dot{\xi}_{s_k} &= (s_p p - s_q q) g^* \xi_{s_p}^* \xi_{s_q}^* + I s_k g_k \xi_{s_k}, \\ \dot{\xi}_{s_p} &= (s_q q - s_k k) g^* \xi_{s_q}^* \xi_{s_k}^* + I s_p g_p \xi_{s_p}, \\ \dot{\xi}_{s_q} &= (s_k k - s_p p) g^* \xi_{s_k}^* \xi_{s_p}^* + I s_q g_q \xi_{s_q}. \end{aligned} \quad (27)$$

Here, a dot denotes the time derivative. The first term on the right-hand side (RHS) is the nonlinear term and the second term is a linear term introduced

by the system rotation. The system without rotation introduced by Waleffe⁽²⁶⁾ can be recovered from Eq. (27) with $g_k = g_p = g_q = 0$. In general, following Eq. (27), the turbulence kinetic energy is cascaded from the low to high wave numbers due to the nonlinear interactions among wave vectors $(\mathbf{k}, \mathbf{p}, \mathbf{q})$. However, one can find from the following analysis that system rotation affects the nonlinear interactions significantly, although this effect is implicit. When the rotation is strong enough, the effect of the nonlinear interactions can be ignored, compared with the influence of the Coriolis term. According to the linear system of Eq.(27), the linear solution corresponding to the helical mode ξ_{s_k} is

$$\xi_{s_k}(\mathbf{k}, t) = \xi_{s_k}(\mathbf{k}, 0) \exp(I s_k g_k t). \quad (28)$$

In order to investigate the nonlinear interactions in the rotating case, we introduce the following assumption⁽⁵⁾:

$$\xi_{s_k}(\mathbf{k}, t) = a_{s_k}(\mathbf{k}, t) \exp(I s_k g_k t), \quad (29)$$

where $a_{s_k}(\mathbf{k}, t)$ expresses the nonlinear contribution. One can recover the linear solution of Eq.(28) with $a_{s_k}(\mathbf{k}, t) = \xi_{s_k}(\mathbf{k}, 0)$. Then, substitution of Eq.(29) into Eq.(27) yields the equation for the nonlinear component $a_{s_k}(\mathbf{k}, t)$:

$$\begin{aligned} \dot{a}_{s_k}(\mathbf{k}, t) &= (s_p p - s_q q) g^* a_{s_p}^* a_{s_q}^* \\ &\times \exp[-I(s_k g_k + s_p g_p + s_q g_q)t] : \end{aligned} \quad (30)$$

This means that if $s_k g_k + s_p g_p + s_q g_q \neq 0$, then the term $\exp[-I(s_k g_k + s_p g_p + s_q g_q)t]$ provides a strong rotation to the nonlinear interactions with the time evolution and the nonlinear effect is averaged out over a long time-scale. This rotation effect is called the 'scrambling effect' in the literature⁽⁴⁾. Hence, in addition to the limit $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$ on the triad interaction (nonlinear interaction), the following limit also acts on

$$s_k g_k + s_p g_p + s_q g_q = 0, \quad (31)$$

or equivalently,

$$2\Omega \cdot (s_k \mathbf{k}/k + s_p \mathbf{p}/p + s_q \mathbf{q}/q) = 0. \quad (32)$$

The triad interaction that satisfies Eq.(32) is called the 'resonant triad interaction'⁽⁶⁾. A direct deduction from the above analysis is that only the resonant triad interactions contribute to the energy transfer in a rotating system while all the triad interactions in a nonrotating system contribute to the energy transfer. Hence, the transfer rate in a nonrotating system is generally larger than that in the rotating system. Figure 3 shows the kinetic energy transfer function $T(k, t)$ with $\alpha = 0$. $T(k, t)$ is defined as

$$\begin{aligned} T(k, t) &= \int \left[-P_{ijm}(\mathbf{k}) \right. \\ &\times \int_{\mathbf{k}-\mathbf{p}=\mathbf{q}=0} \text{Im} \langle \hat{u}_i(\mathbf{k}, t) \hat{u}_j(\mathbf{p}, t) \\ &\times \hat{u}_m(\mathbf{q}, t) \rangle d\mathbf{p} d\mathbf{q} \Big] dA(\mathbf{k}), \end{aligned} \quad (33)$$

where $P_{ijm}(\mathbf{k}) = k_m P_{ij}(\mathbf{k}) + k_j P_{im}(\mathbf{k})$, and $\int dA(\mathbf{k})$

Fig. 2 Evolution of turbulence kinetic energy

Fig. 3 Evolution of energy transfer function

Fig. 4 Effect of rotation on energy transfer function

Fig. 5 Evolution of helicity

Fig. 6 Effect of helicity on energy transfer function

denotes the integration about the shell of radius k . Figure 4 shows that the absolute value of $T(k, t)$ decreases with the increase of the rotation rates. This DNS result is also consistent with the result of previous analysis. Since the absolute value is decreased by rotation, the energy cascade from the low to high wave numbers is inhibited.

In addition, since Eq. (29) is an exact expression of the helical mode ξ_{sn} which includes the nonlinear effect, one finds that not only the kinetic energy, but also the helicity and the complex deviator can be determined by this equation. With the same analysis of energy transfer, one can deduce that there is scrambling effect on the evolutions of h and $|Z|$, respectively. In other words, the decay of the helicity and the complex deviator are also inhibited by system rotation. As an example, the evolution of the helicity He is presented in Fig. 5. It shows that the decay rate of the helicity decreases with the increase of the rotation

rate.

4.2 Effect of helicity

As shown in Fig. 2, even with the same rotation rate, the energy decay of the cases with helicity is slower than that of the cases without the mean helicity. That is, the presence of helicity also inhibits the energy decay. Figure 6 shows the effect of the helicity on $T(k, t)$. With the increase of the absolute value of α , the value of $T(k, t)$ decreases gradually. In particular, the reverse cascade is presented in the cases of $|\alpha|=1$. In accordance with the definition of $T(k, t)$, the effect of the helicity on $T(k, t)$ can be explained as follows. Equation (33) can be rewritten in a more symmetric form,

$$T(k) = 2 \int \text{Re} \langle \hat{\lambda}(\mathbf{k}) \cdot \hat{\mathbf{u}}^*(\mathbf{k}) \rangle dA(\mathbf{k}). \quad (34)$$

Here, $\lambda = \mathbf{u} \times \boldsymbol{\omega}$ is the Lamb vector. From this equation, considering the definition of the helicity, with the increase of the helicity, velocity $\mathbf{u}$ and vorticity $\boldsymbol{\omega}$

Fig. 7 Isosurface of vorticity ($\Omega=5$ [rad/s])

travel parallel to each other, which leads us to deduce that the value of λ decreases. Hence, the energy transfer function will decrease with the increase of the helicity. It is consistent with the DNS result shown in Fig. 5.

4.3 Turbulence structure

Figure 7 presents the isosurface of the absolute value of the turbulent vorticity. It is well known that two-dimensionalization is one of the effects of system rotation. That is, the vortical structure is stretched and elongated along the rotation axis by system rotation. As shown in Fig. 7(c), such a vortical structure does appear in the rotating system without the initial helicity ($\alpha=0$). On the other hand, the two-dimensionalization is obviously weakened by the presence of helicity [Fig. 7(d), $\alpha=1$]. The effect of the helicity on the vortical structure of the rotating system can be

demonstrated by the integral length scale. In Fig. 8, the evolution of the ratio L^* of the transverse integral length scale to the longitudinal one is presented. The integral length scales $L_{ij,k}$ and L^* are defined as

$$L_{ij,k} = \int_0^\infty \langle u_i(\mathbf{x}) u_j(\mathbf{x} + r\mathbf{n}^k) \rangle dr / \langle u_i(\mathbf{x}) u_j(\mathbf{x}) \rangle, \quad (35)$$

$$L^* = (L_{11,3} + L_{22,3}) / L_{33,3}. \quad (36)$$

The value of L^* in the figure is normalized by its initial value at $t-t_0=0$. The ratio of the integral lengthscale in the inertial frame ($\Omega=0$) undergoes almost no change with time, irrespective of whether or not the initial field has helicity. On the other hand, in the rotating system, L^* with $\alpha=0$ increases significantly while L^* with $\alpha=1$ increases more slowly. Cambon et al.⁽⁵⁾ defined the longitudinal and the transverse two-dimensional energy components as

Fig. 8 Evolution of integral lengthscale

$$E_{33,3} = \langle u_3^2 \rangle L_{33,3} = \pi \int (e + Re[Z])|_{k_3=0} d\mathbf{k}^2, \quad (37)$$

$$E_{11,3} + E_{22,3} = \langle u_1^2 \rangle L_{11,3} + \langle u_2^2 \rangle L_{22,3} \\ = \pi \int (e - Re[Z])|_{k_3=0} d\mathbf{k}^2. \quad (38)$$

Then, these are rewritten as

$$L_{33,3} = \pi \int (e + Re[Z])|_{k_3=0} d\mathbf{k}^2 / \langle u_3^2 \rangle, \quad (39)$$

$$L_{11,3} + L_{22,3} = \pi \int (e - Re[Z])|_{k_3=0} d\mathbf{k}^2 / \langle u_1^2 \rangle, \quad (40)$$

with the assumption that $\langle u_1^2 \rangle \cong \langle u_2^2 \rangle$. From Eqs.(11) and (17), one finds that $|Z|=0$ for the cases of $|a|=1$. Hence, if the system has the maximum (plus or minus) helicity, the extent of increase of the longitudinal integral length scale is almost the same as that of the transverse integral length scale, and the two-dimensionalization by the system rotation will not appear even in the rotating system. However, when $a=0$, it is difficult to obtain a turbulent field with $Re[Z]=0$ by the numerical method. According to Eqs.(39) and (40), the increase of the integral length scale generally depends on the distribution of e and $Re[Z]$.

5. Conclusions

In order to investigate the effects of system rotation and helicity, first, a new method that introduces the helicity into an isotropic turbulent field for DNS is proposed. Then, DNS of rotating homogeneous decaying turbulence with or without initial helicity is carried out. The main conclusions of this study are as follows:

(1) Although both system rotation and helicity inhibit energy decay, the mechanisms are different. System rotation limits the triad interactions based on the 'scrambling effect,' while helicity reduces the energy transfer function directly.

(2) The scrambling effect also acts on the evolu-

tion of helicity, in the same manner as it acts on the turbulence kinetic energy.

(3) In the case without helicity, the vortex structure is apparently stretched along the rotation axis, and this tendency is weakened by the presence of initial helicity. This effect can be explained by the theoretical analysis for the integral length scale.

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